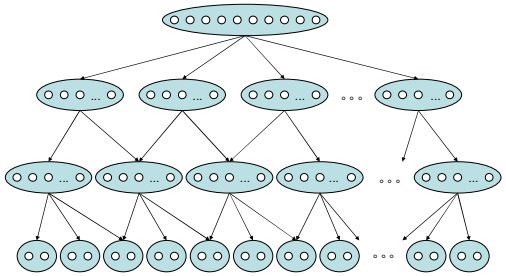
Deep Restricted Bayesian Network BESOM



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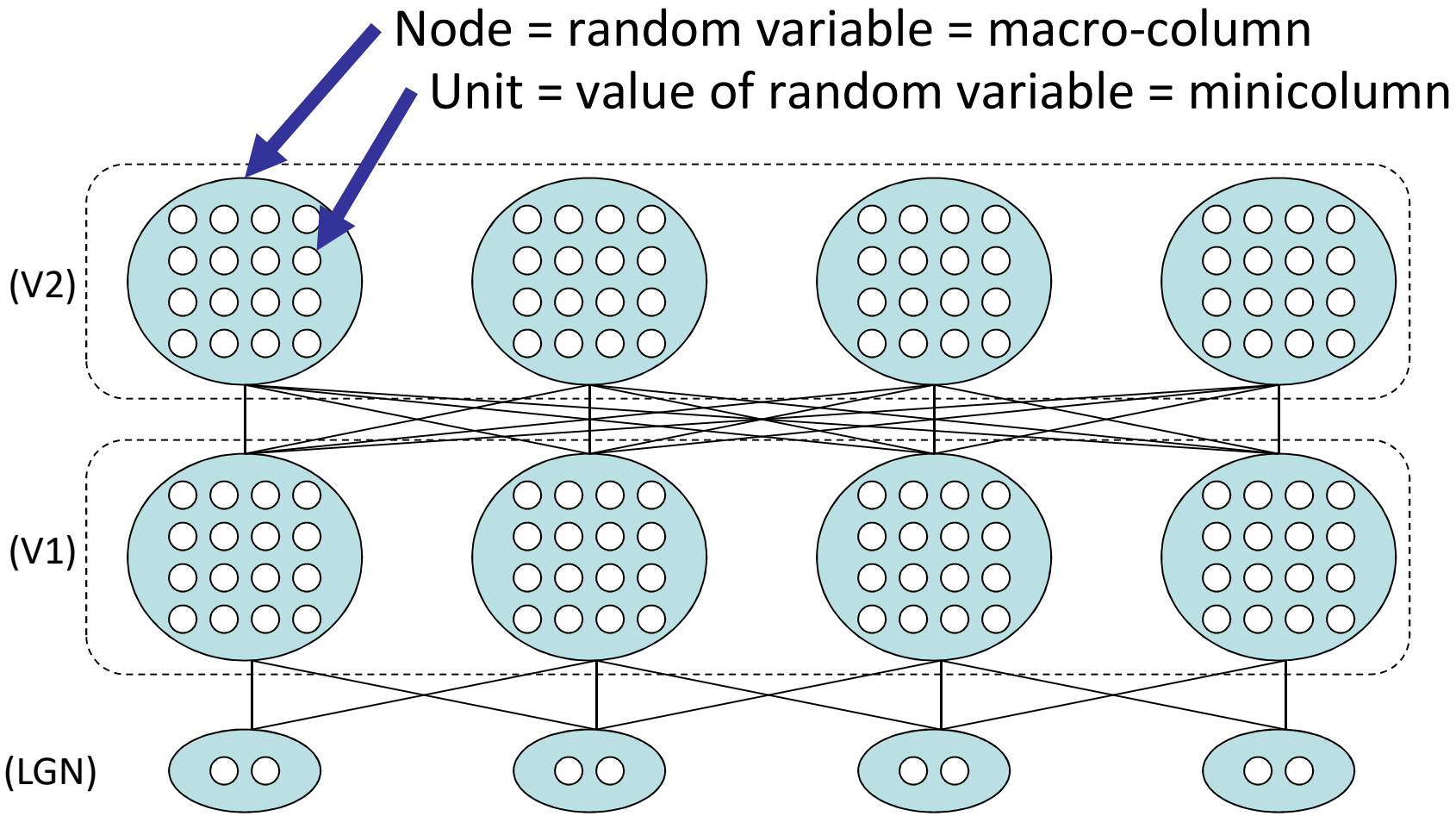
BESOM (BidirEctional Self Organizing Maps) [Ichisug 2007]

- A computational model of the cerebral cortex
  - **A model of column network**, *not* spiking neurons
- Design goals:
  - **Scalability** of computation
  - **Usefulness** as a machine learning system
  - **Plausibility** as a neuroscientific model
- As a long-term goal, we aim to reproduce functions of such as the visual areas and the language areas using this cerebral cortex model.

Architecture of BESOM model

Recognition step: The entire network behaves like a **Bayesian network**.
Learning step: Each node behaves like a **Self-organizing map**.



Outline

- Bayesian networks and the cerebral cortex
- BESOM Ver.3 and robust pattern recognition
- Toward BESOM Ver.4

Models of visual cortex based on Bayesian networks

- Various functions, illusions, neural responses and anatomical structure of the visual cortex were reproduced by Bayesian network models.
  - [Tai Sing Lee and Mumford 2003]
  - [George and Hawkins 2005]
  - [Rao 2005]
  - [Ichisugi 2007]
  - [Litvak and Ullman 2009]
  - [Chikkerur, Serre, Tan and Poggio 2010]
  - [Hosoya 2012]
  - ...

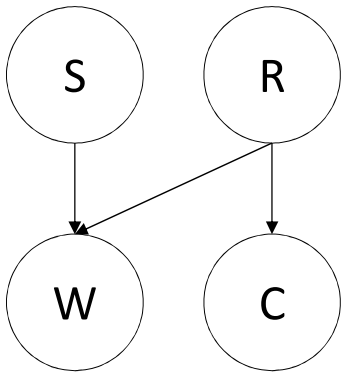
The visual cortex seems to be a huge Bayesian network with layered structure like Deep Neural Networks.

What is Bayesian networks?

– **Efficient and expressive data structure of probabilistic knowledge** [Perl 1988]

- Various probabilistic inference can be executed efficiently if a joint probability table can be factored into small **conditional probability tables (CPTs)**.

$$P(S,W,R,C) = P(W \mid S,R)P(C \mid R)P(S)P(R)$$



CPTs

P(S=yes)
0.2

P(R=yes)
0.02

S	R	P(W=yes|S,R)
no	no	0.12
no	yes	0.8
yes	no	0.9
yes	yes	0.98

R	P(C=yes|R)
no	0.3
yes	0.995

Loopy Belief Propagation

[Murphy, Weiss, Jordan 1999]

- Efficient approximate inference algorithm
  - Iterative algorithm with **local and asynchronous computation**, like brain.
  - Although there is no guarantee of convergence, it is empirically accurate.

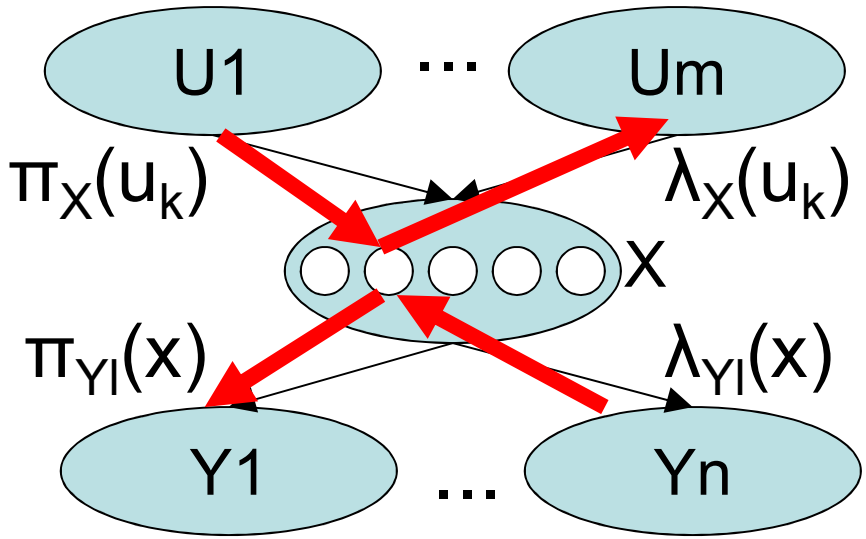
$$BEL(x) = \alpha\lambda(x)\pi(x)$$

$$\pi(x) = \sum_{u_1,\cdots,u_m} P(x \mid u_1,\cdots,u_m)\prod_k \pi_X(u_k)$$

$$\lambda(x) = \prod_l \lambda_{Y_l}(x)$$

$$\pi_{Y_l}(x) = \pi(x)\prod_{j\neq l} \lambda_{Y_j}(x)$$

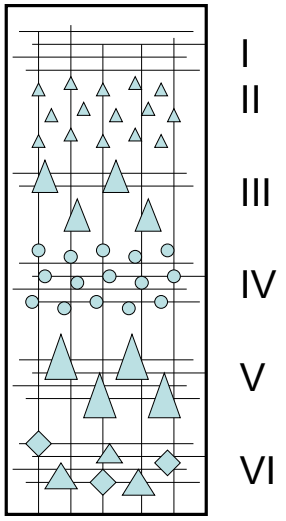
$$\lambda_X(u_k) = \sum_x \lambda(x) \sum_{u_1,\cdots,u_m/u_k} P(x \mid u_1,\cdots,u_m)\prod_{i\neq k} \pi_X(u_i)$$



Belief propagation and micro circuit of cerebral cortex

- The similarity between belief propagation and the six-layer structure of the cerebral cortex has been pointed out many times.

  [George and Hawkins 2005]
  [Ichisugi 2007]
  [Rohrbein, Eggert and Korner 2008]
  [Litvak and Ullman 2009]



Approximate Belief Propagation

[Ichisugi 2007]

$$\mathbf{l}_{XY}^{t+1} = \mathbf{z}_Y^t + \mathbf{W}_{XY}\mathbf{o}_Y^t$$

$$\mathbf{o}_X^{t+1} = \prod_{Y \in children(X)}^{\otimes} \mathbf{l}_{XY}^{t+1}$$

$$\mathbf{k}_{UX}^{t+1} = \mathbf{W}_{UX}^T \mathbf{b}_U^t$$

$$\mathbf{p}_X^{t+1} = \sum_{U \in parents(X)} \mathbf{k}_{UX}^{t+1}$$

$$\mathbf{r}_X^{t+1} = \mathbf{o}_X^{t+1} \otimes \mathbf{p}_X^{t+1}$$

$$Z_X^{t+1} = \sum_i (\mathbf{r}_X^{t+1})_i \quad (= \left\| \mathbf{r}_X^{t+1} \right\|_1 = \mathbf{o}_X^{t+1} \bullet \mathbf{p}_X^{t+1})$$

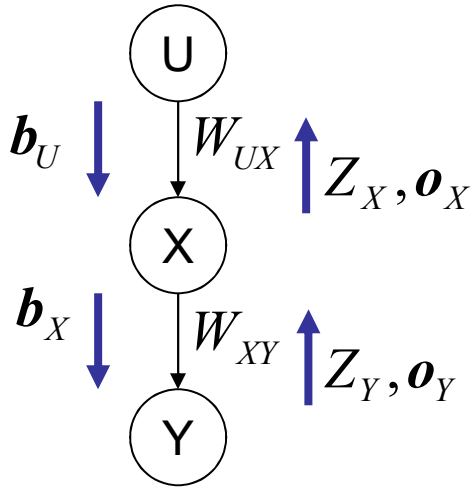
$$\mathbf{z}_X^{t+1} = (Z_X^{t+1}, Z_X^{t+1}, \cdots, Z_X^{t+1})^T$$

$$\mathbf{b}_X^{t+1} = (1/Z_X^{t+1})\mathbf{r}_X^{t+1}$$

where $\mathbf{x} \otimes \mathbf{y} = (x_1 y_1, x_2 y_2, \cdots, x_n y_n)^T$

Approximates Pearl's algorithm[Pearl 1988] with some assumptions.

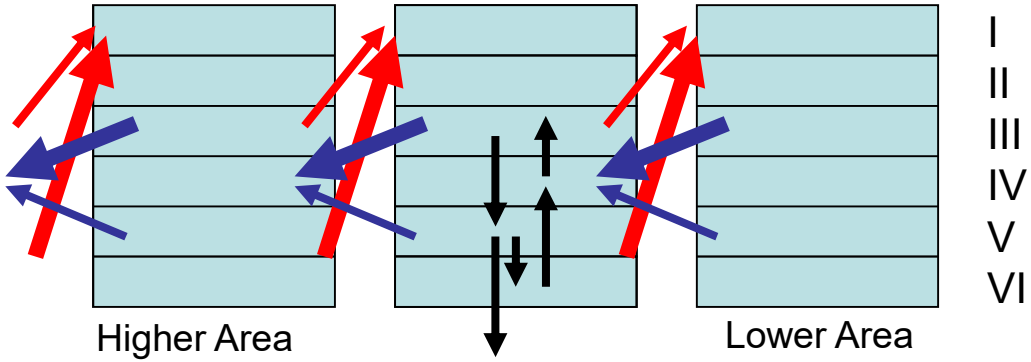
$$P(x|u_1, \ldots, u_m) = \frac{1}{m} \Sigma_{k=1}^{m} P(x|u_k)$$



Yuuji ICHISUGI, "The cerebral cortex model that self-organizes conditional probability tables and executes belief propagation", In proc. of IJCNN2007, Aug 2007.

Similarity in information flow

[Gilbert 1983]
[Pandya and Yeterian 1985]



Anatomical structure

Gilbert, C.D., Microcircuitry of the visual-cortex, Annual review of neuroscience, 6: 217-247, 1983.

Pandya, D.N. and Yeterian, E.H., Architecture and connections of cortical association areas. In: Peters A, Jones EG, eds. Cerebral Cortex (Vol. 4): Association and Auditory Cortices. New York: Plenum Press, 3-61, 1985.

$$\mathbf{l}_{XY}^{t+1} = \mathbf{z}_Y^t + \mathbf{W}_{XY}\mathbf{o}_Y^t$$

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$$\mathbf{z}_X^{t+1} = (Z_X^{t+1}, Z_X^{t+1}, \cdots, Z_X^{t+1})^T$$

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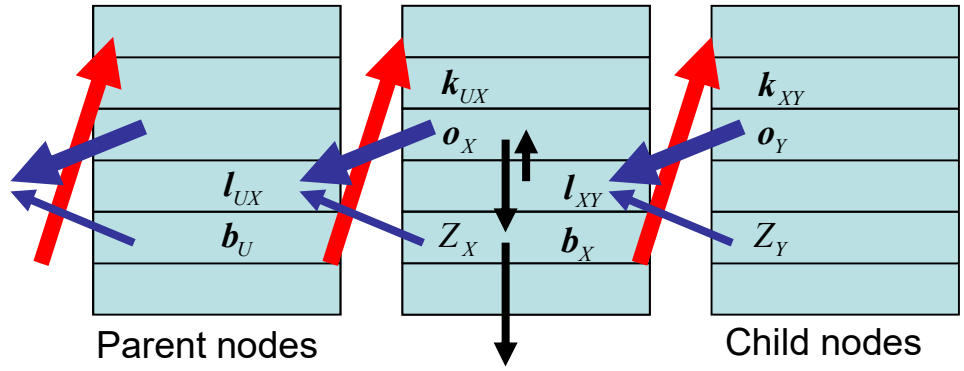
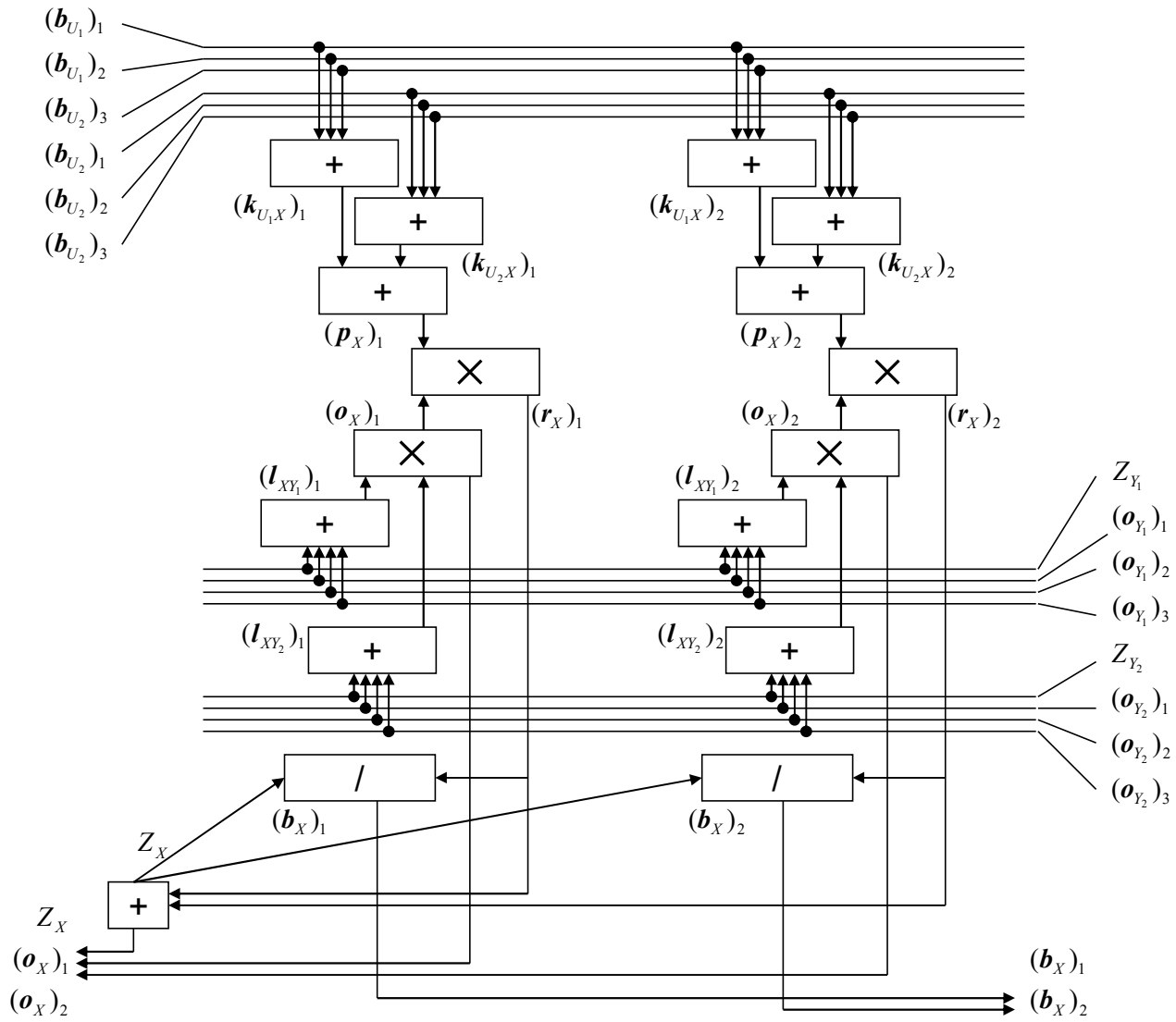
where $\mathbf{x} \otimes \mathbf{y} = (x_1 y_1, x_2 y_2, \cdots, x_n y_n)^T$

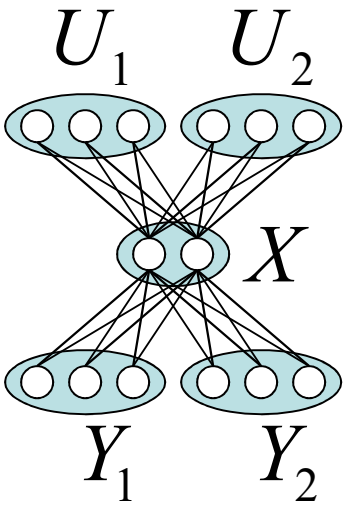


Information flow of the approx. BP

The intermediate variables of this algorithm can be assigned to each layer of the cerebral cortex without contradicting the known anatomical structure.

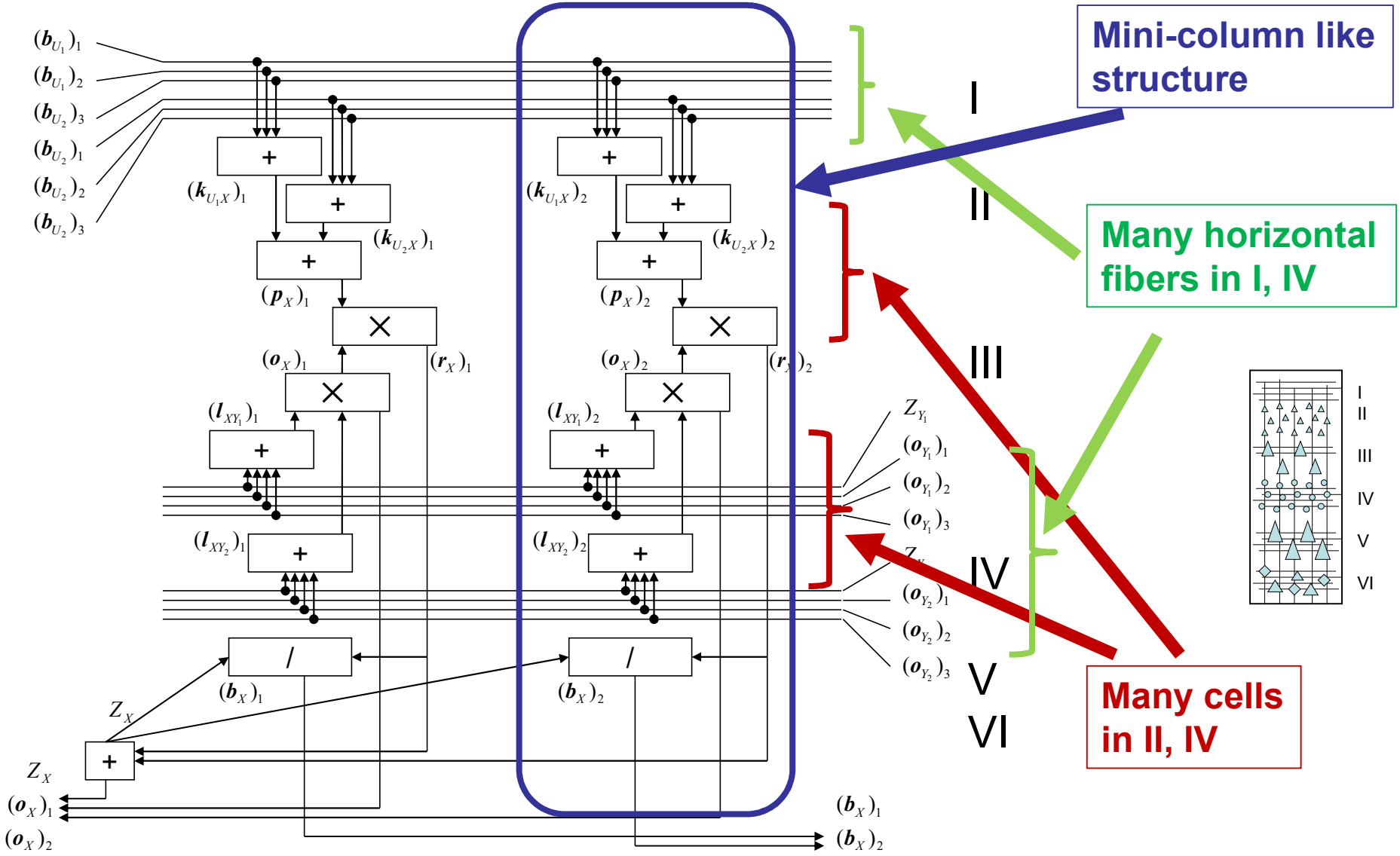
Detailed circuit that calculates the approximate BP





The left circuit calculates values of two units, x1 and x2, in node X in the above network.

Correspondence with local cortical circuit



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- Bayesian networks and the cerebral cortex
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- Toward BESOM Ver.4

Toward realization of the brain function

- If the cerebral cortex is a kind of Bayesian network, we should be able to reproduce function and performance of it using Bayesian networks.
  - As a first step, we aim to reproduce some part of the functions of the visual areas and the language areas.
  - Although there were some difficulties such as computational cost and local minimum problem, now they have been solved considerably.

BESOM Ver.3.0 features

- Restricted Conditional Probability Tables:

$$P(x|u_1,\ldots,u_m) = \frac{1}{m}\Sigma_{k=1}^{m} P(x|u_k)$$

- Scalable recognition algorithm OOBP [Ichisugi, Takahashi 2015]
- Regularization methods to avoid local minima
  - Win-rate and Lateral-inhibition penalty [Ichisugi, Sano 2016]
  - Neighborhood learning

$$\begin{aligned}
\lambda_{Y_l}^{t+1}(x) &= \beta_2 \sum_{y_l} \lambda^t(y_l)(\pi^t(y_l) - \kappa_X^t(y_l) + w(y_l,x)) \\
\lambda^{t+1}(x) &= \prod_{l=1}^{n} \lambda_{Y_l}^{t+1}(x) \\
\pi_{Y_l}^{t+1}(x) &= \beta_1 \rho^{t+1}(x)/\lambda_{Y_l}^{t+1}(x) \\
\kappa_{U_k}^{t+1}(x) &= \sum_{u_k} w(x,u_k)\pi_X^t(u_k) \\
\pi^{t+1}(x) &= \sum_{k=1}^{m} \kappa_{U_k}^{t+1}(x) \\
\rho^{t+1}(x) &= \lambda^{t+1}(x)\pi^{t+1}(x) \\
BEL^{t+1}(x) &= \alpha\rho^{t+1}(x)
\end{aligned}$$

Recognition algorithm OOBP

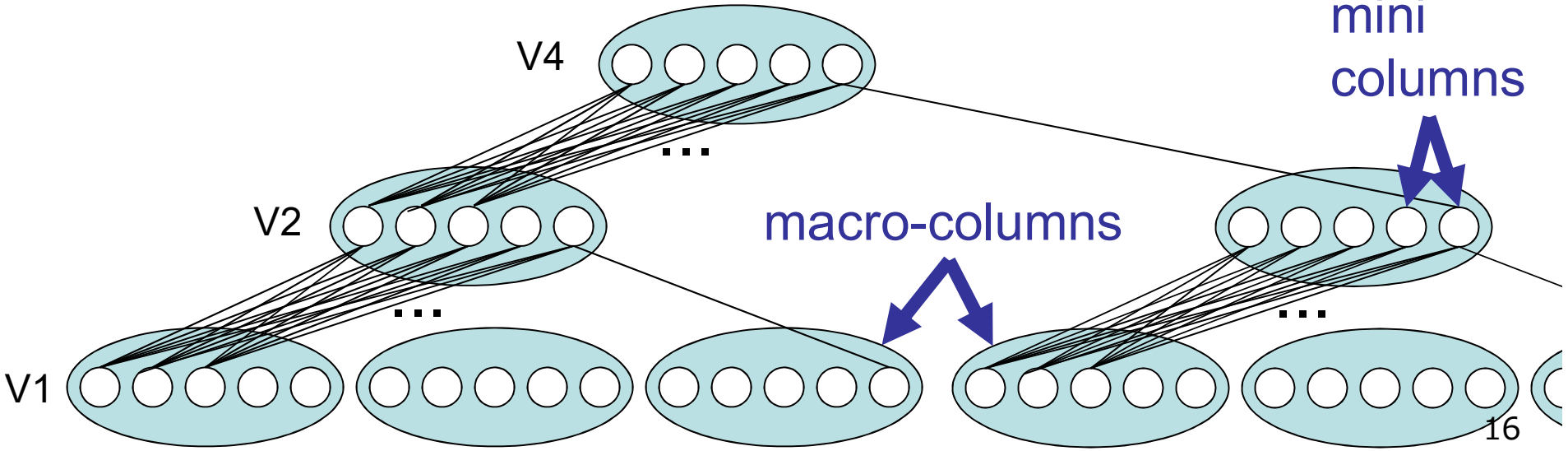
Computational amount of one step of iteration of OOBP is linear to the number of edges of the network.

Yuuji Ichisugi and Naoto Takahashi,
An Efficient Recognition Algorithm for Restricted Bayesian Networks, In proc. of IJCNN 2015.

Yuuji Ichisugi and Takashi Sano,
Regularization Methods for the Restricted Bayesian Network BESOM, In Proc. of ICONIP2016, Part I, LNCS 9947, pp.290--299, 2016.

The design of BESOM is motivated by two neuroscientific facts.

1. Each macro-column seems to be like a SOM.
2. A macro-column at a upper area receives the output of the macro-columns at the lower area.

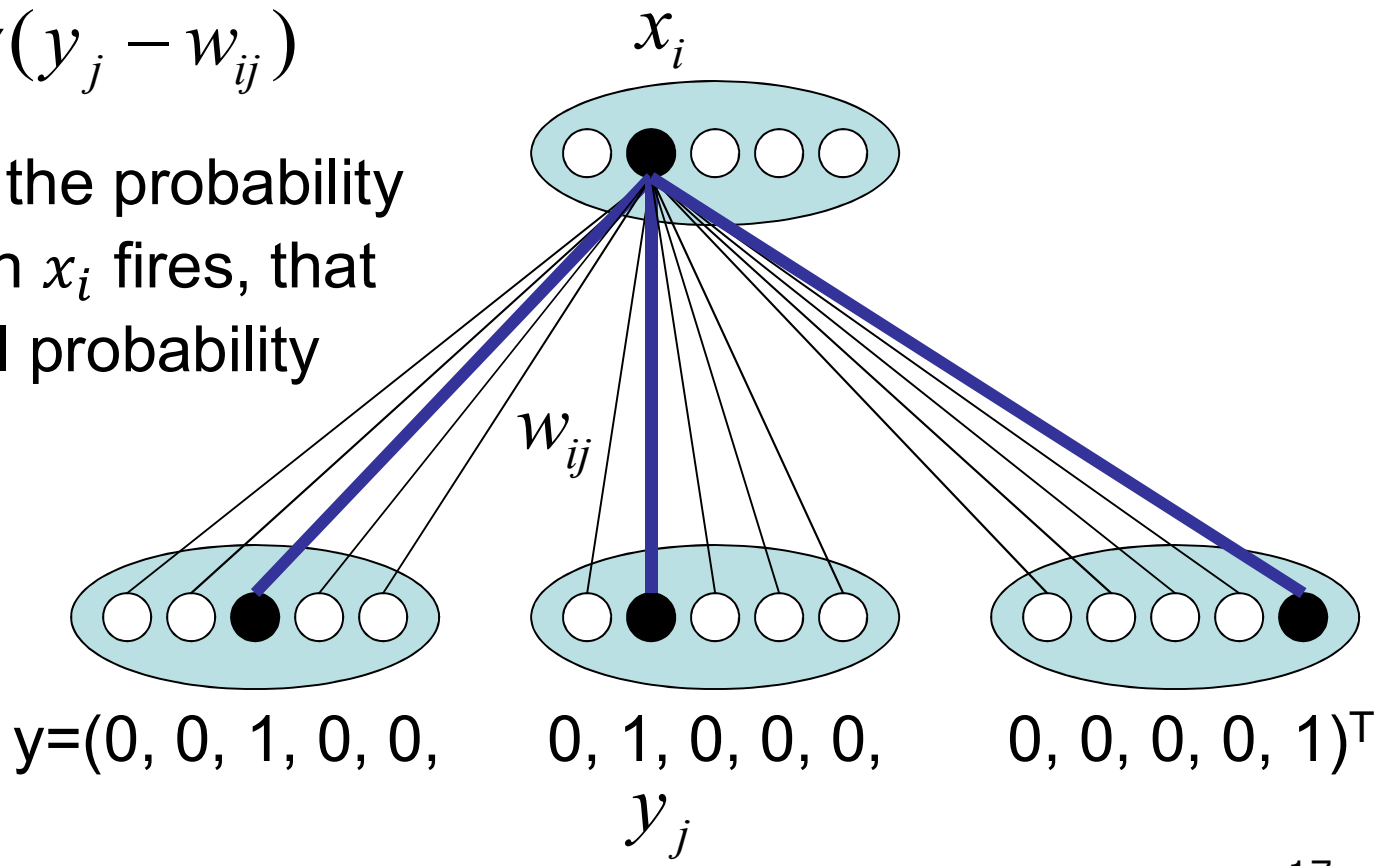


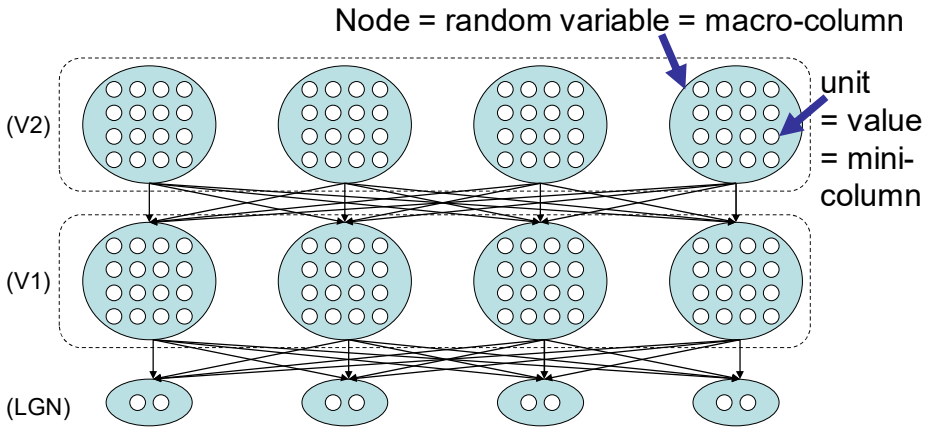
If a SOM receives input from other SOMs, they naturally become a Bayesian Network

Learning rule (without neighborhood learning)

$$w_{ij} \leftarrow w_{ij} + \alpha(y_j - w_{ij})$$

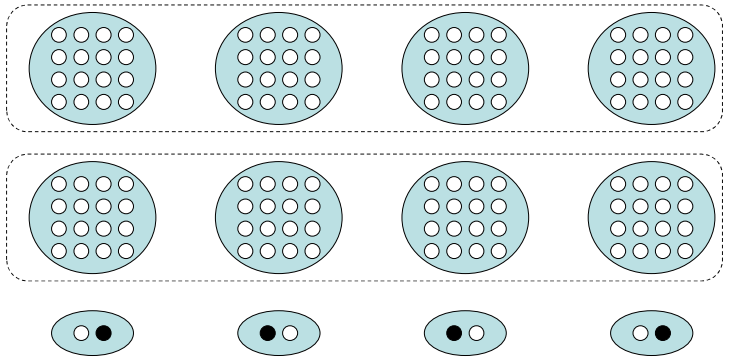
w_{ij} converges to the probability that y_j fires when x_i fires, that is, the conditional probability $P(y_j|x_i)$.





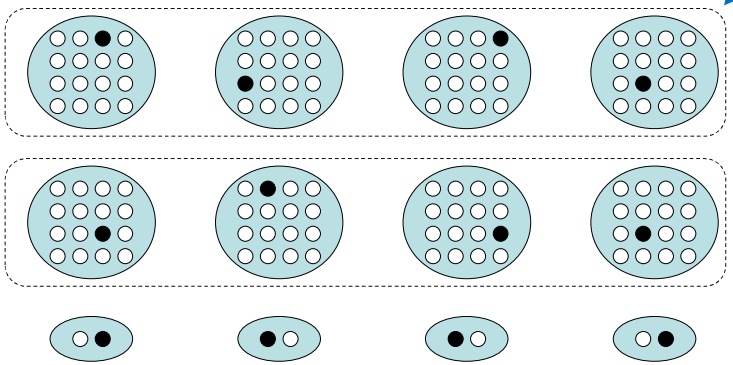
Connection weights
= Conditional probabilities
= synapse weights

Input



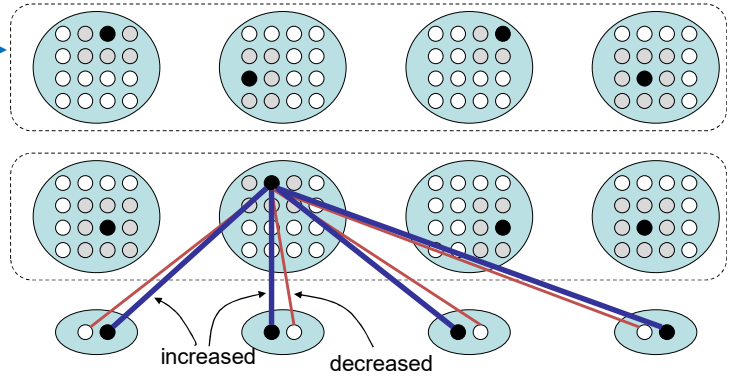
Input (observed data) is given at the lowest layer.

Recognition



Find the values with the highest posterior probability. (MAP)

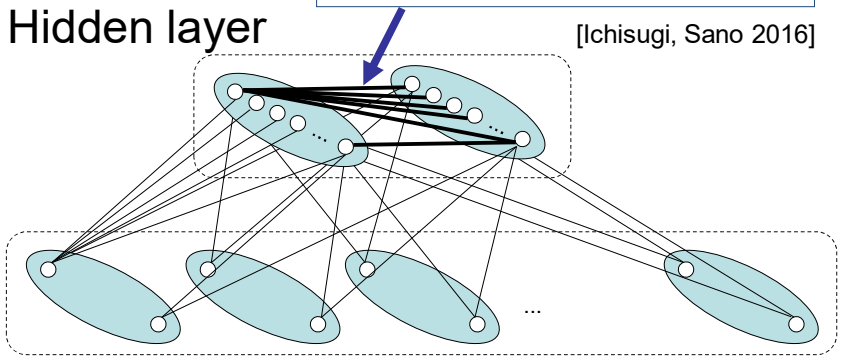
Learning



Updates connection weights with Hebb's rule.

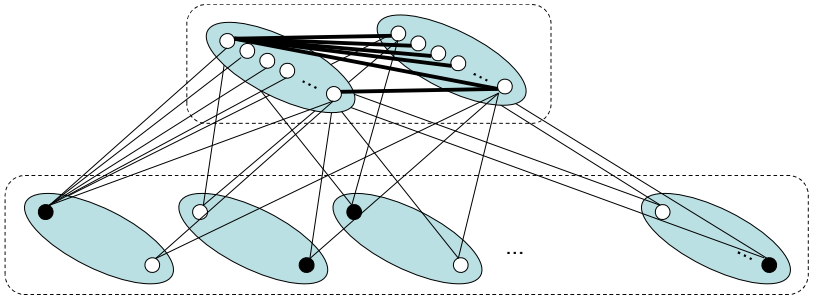
Connections of long distance lateral inhibition

[Ichisugi, Sano 2016]



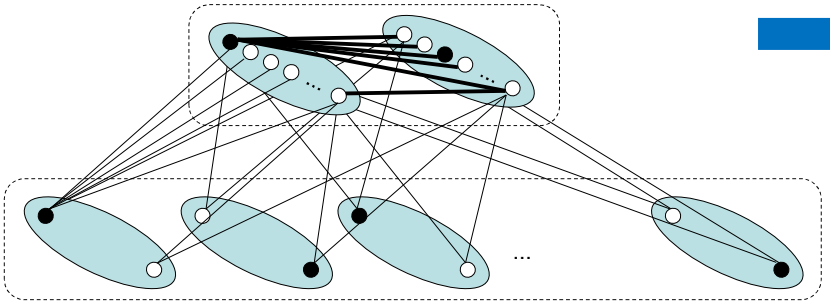
Yuuji Ichisugi and Takashi Sano,
Regularization Methods for the Restricted Bayesian Network BESOM,
In Proc. of ICONIP2016, Part I, LNCS 9947, pp.290--299, 2016.

Input



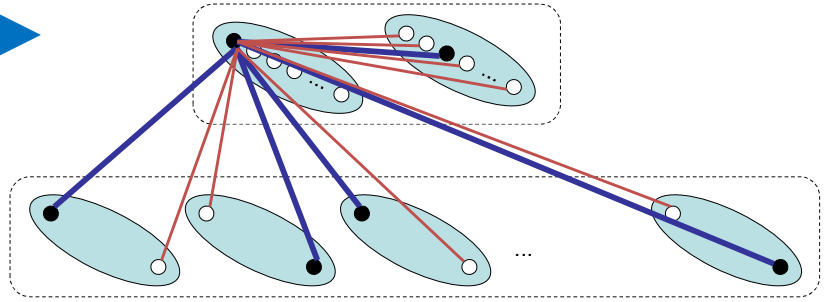
Input (observed data) is given at the lowest layer.

Recognition



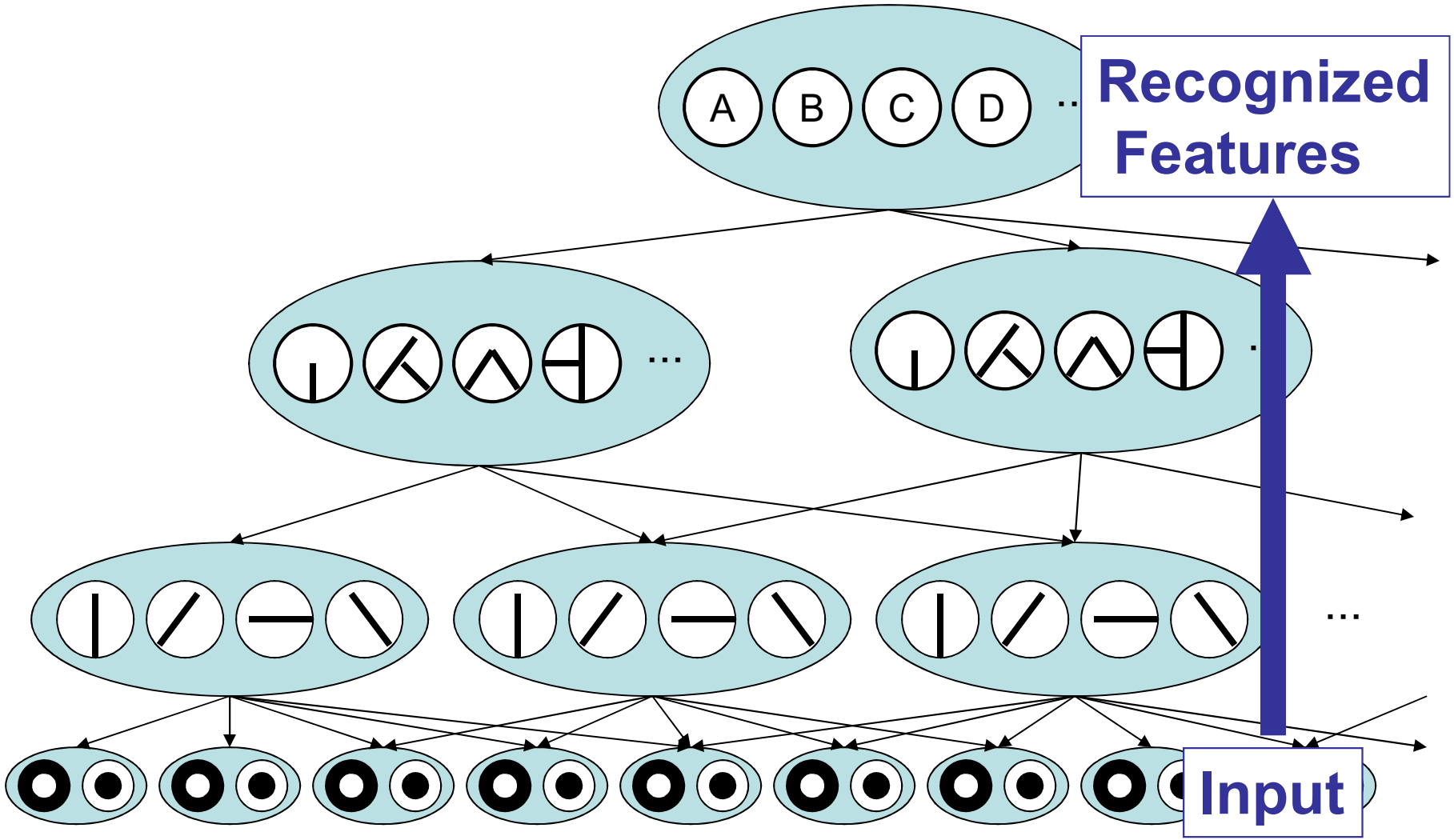
Units that receive strong lateral inhibition are less likely to become winners.

Learning

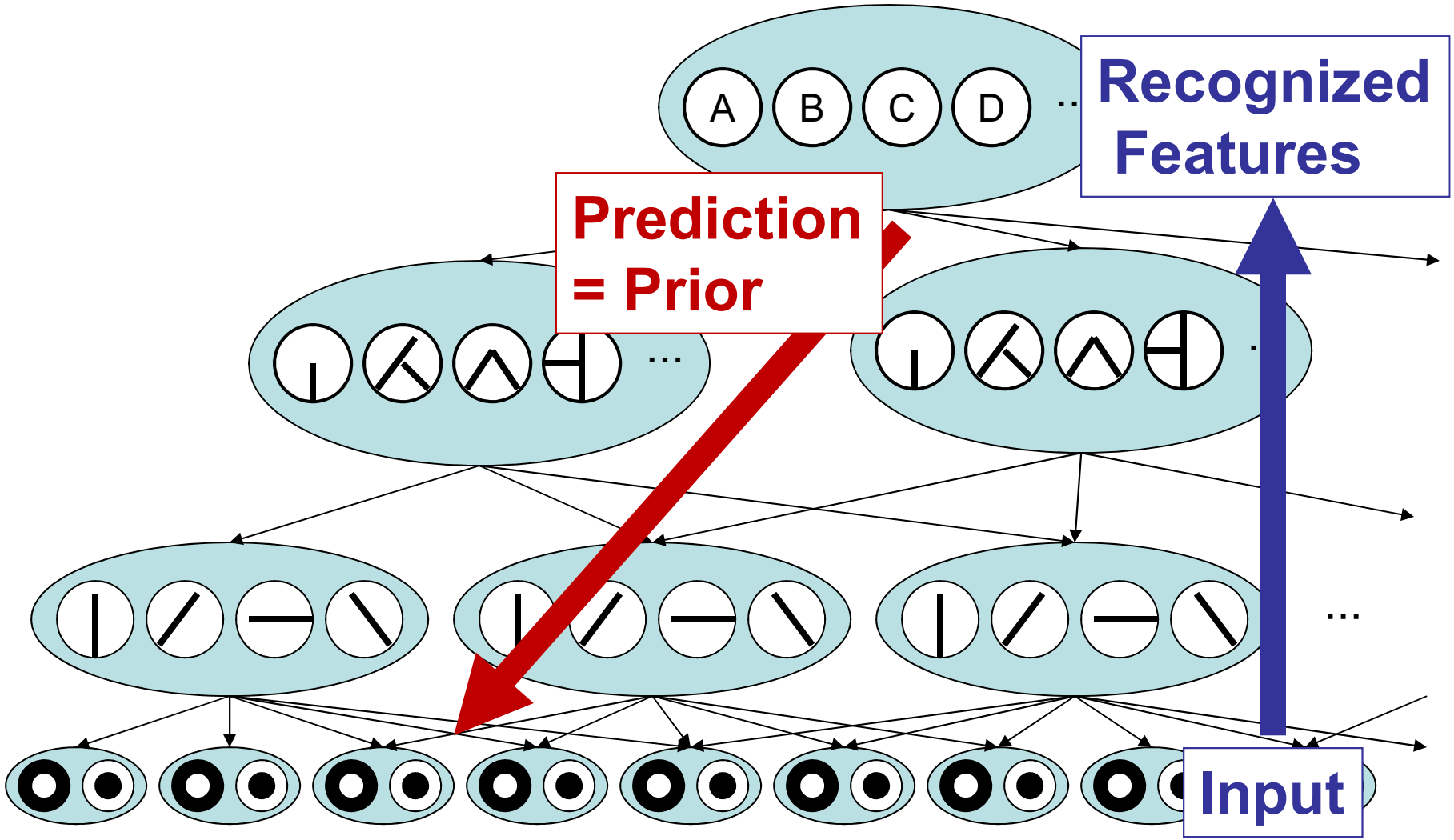


Updates connection weights with Hebb's rule.

BESOM can be used as if it were a Deep neural network.

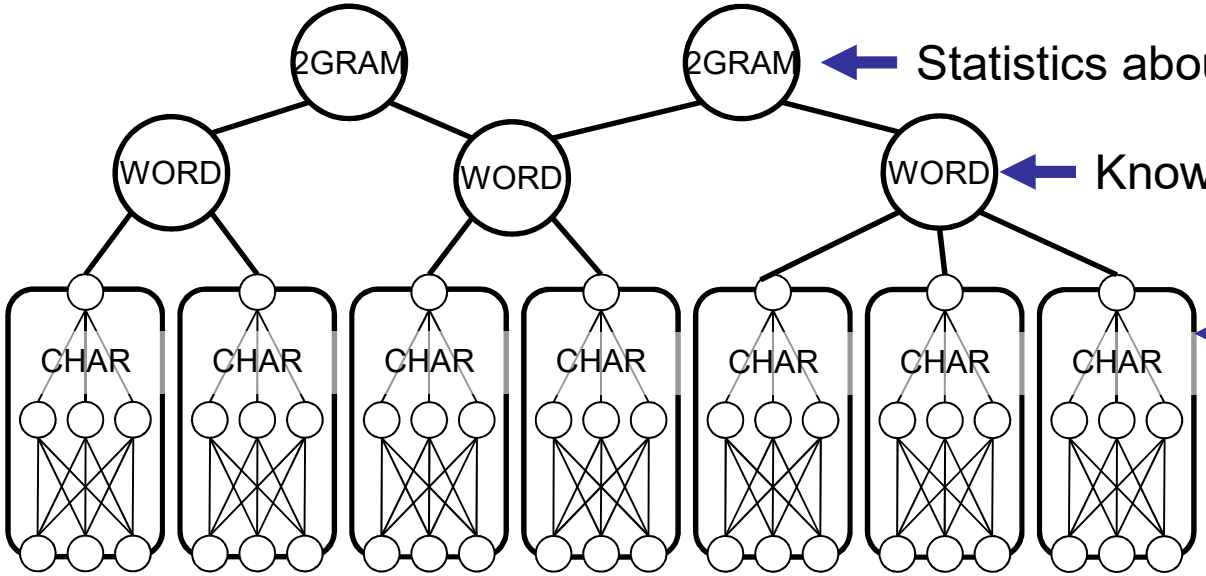


BESOM can be used as if it were a *bidirectional* Deep neural network.



Robust character recognition utilizing context information

[Nakada, Ichisugi 2017]



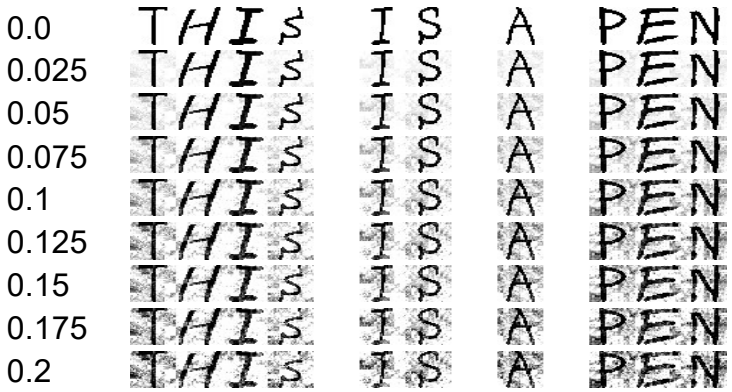
Statistics about word bigram

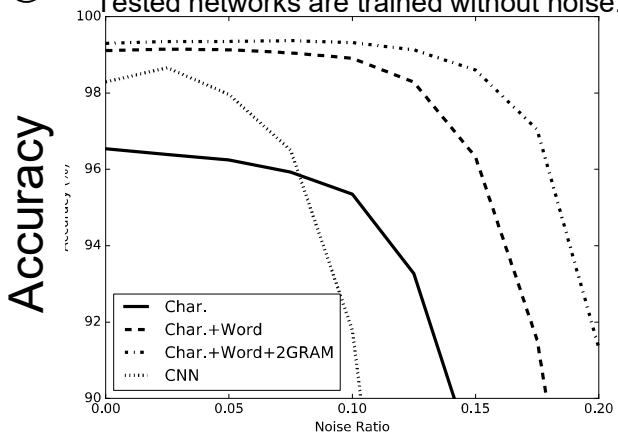
Knowledge about words

Char recognizers

Hidemoto NAKADA and Yuuji ICHISUGI,
Toward Context-Dependent Robust Character Recognition using Large-scale Restricted Bayesian Network,
TECHNICAL REPORT OF IEICE, 2017. (In Japanese)

Tested networks are trained without noise.

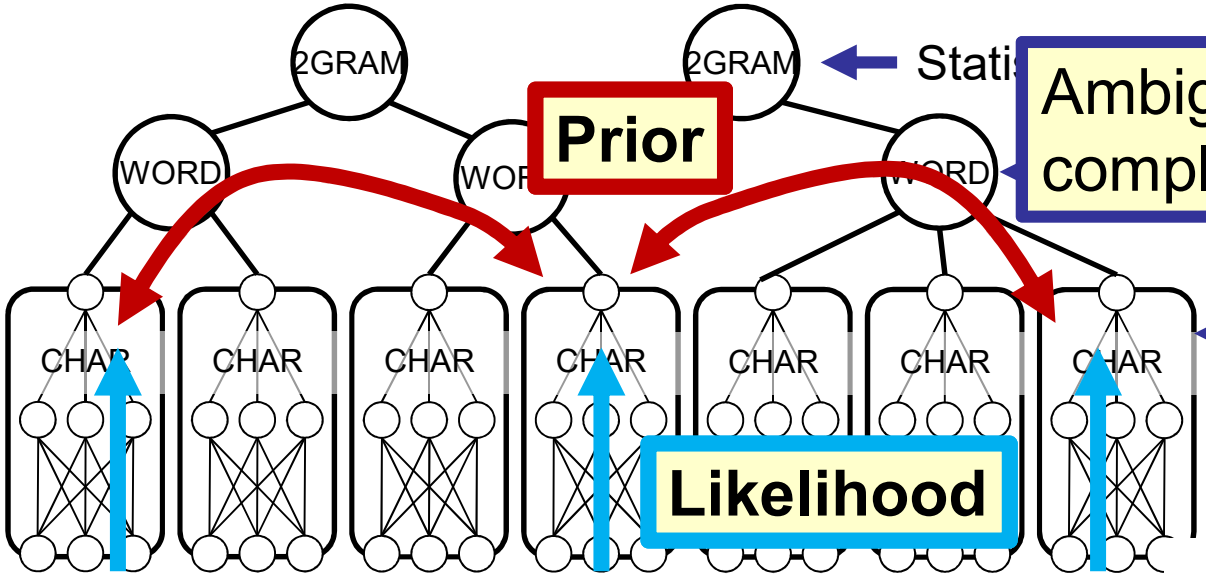


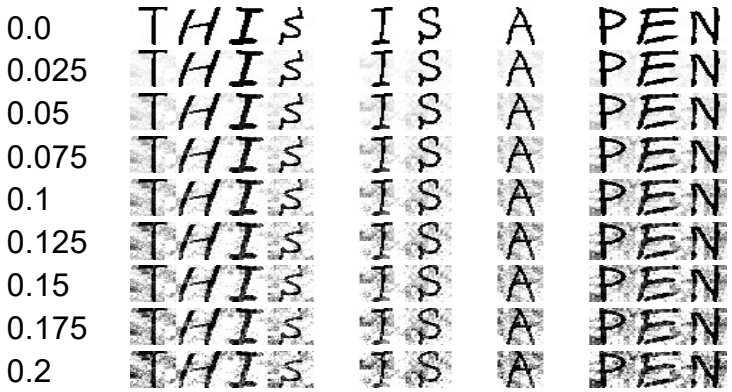


Noise Ratio

Robust character recognition utilizing context information

[Nakada, Ichisugi 2017]





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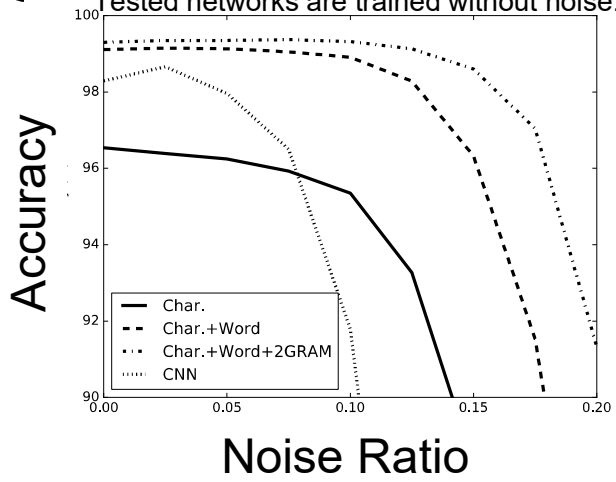

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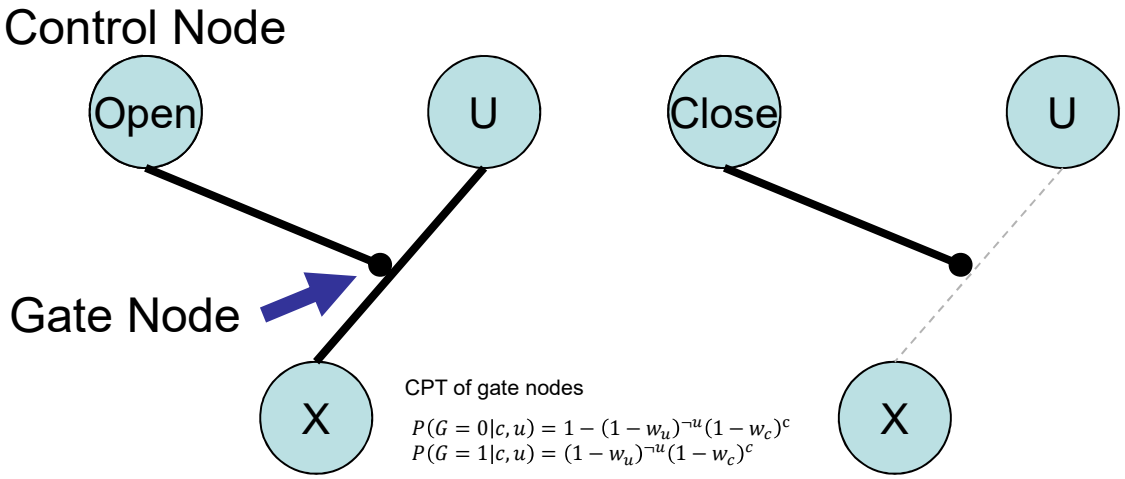
Problem of BESOM Ver.3

- Recognition and Learning is fast
  but accuracy is not good enough, probably because conditional probability tables are too restricted.

$$P(x|u_1, \ldots, u_m) = \frac{1}{m}\Sigma_{k=1}^{m} P(x|u_k)$$

- We are now investigating new conditional probability table models (BESOM Ver.4):
  - Noisy-OR model [Pearl 1988] and **Gate-nodes**.
  - More expressive and fast enough.

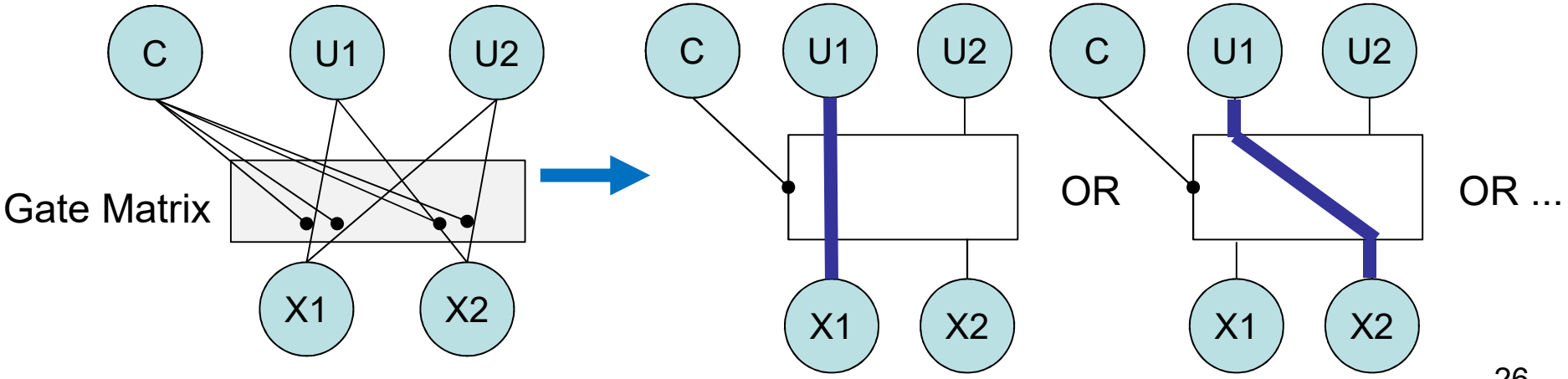
Gate Nodes



If Open,
U and X are connected.
If Close, disconnected.

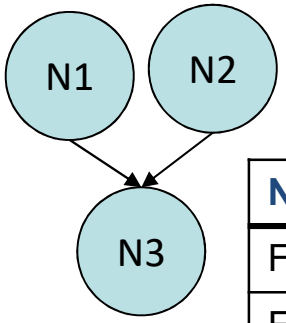
Like inhibitory connections on dendrites.

Using matrix of gates, Control node can control connections between nodes.



Prototyping by Quasi Bayesian Networks

- We are designing prototype models of the visual areas and the language areas using ***Quasi Bayesian Networks***, which are simplified Bayesian networks that only makes a distinction between zero and non-zero of probabilities.
  - Parameter learning is not supported.
  - Solutions are found by SAT solver.



N3	N1	N2	P(N3|N1,N2)
False	False	False	0.2
False	True	False	0.3
False	False	True	0
False	True	True	0.9
True	False	False	0.8
...	...	...	...

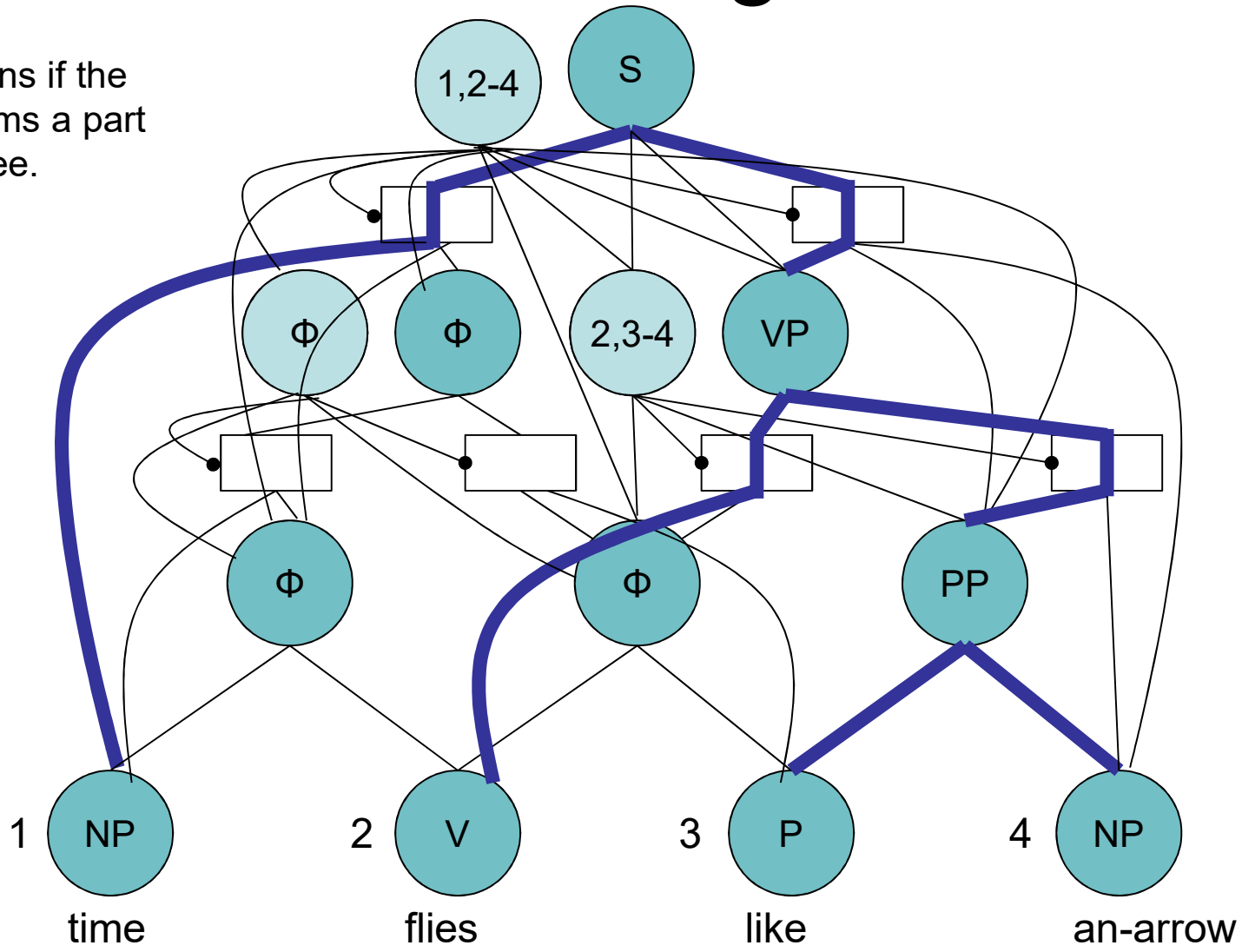
True CPT

N3	N1	N2	P(N3|N1,N2)
False	False	False	non-zero
False	True	False	non-zero
False	False	True	zero
False	True	True	non-zero
True	False	False	non-zero
...	...	...	...

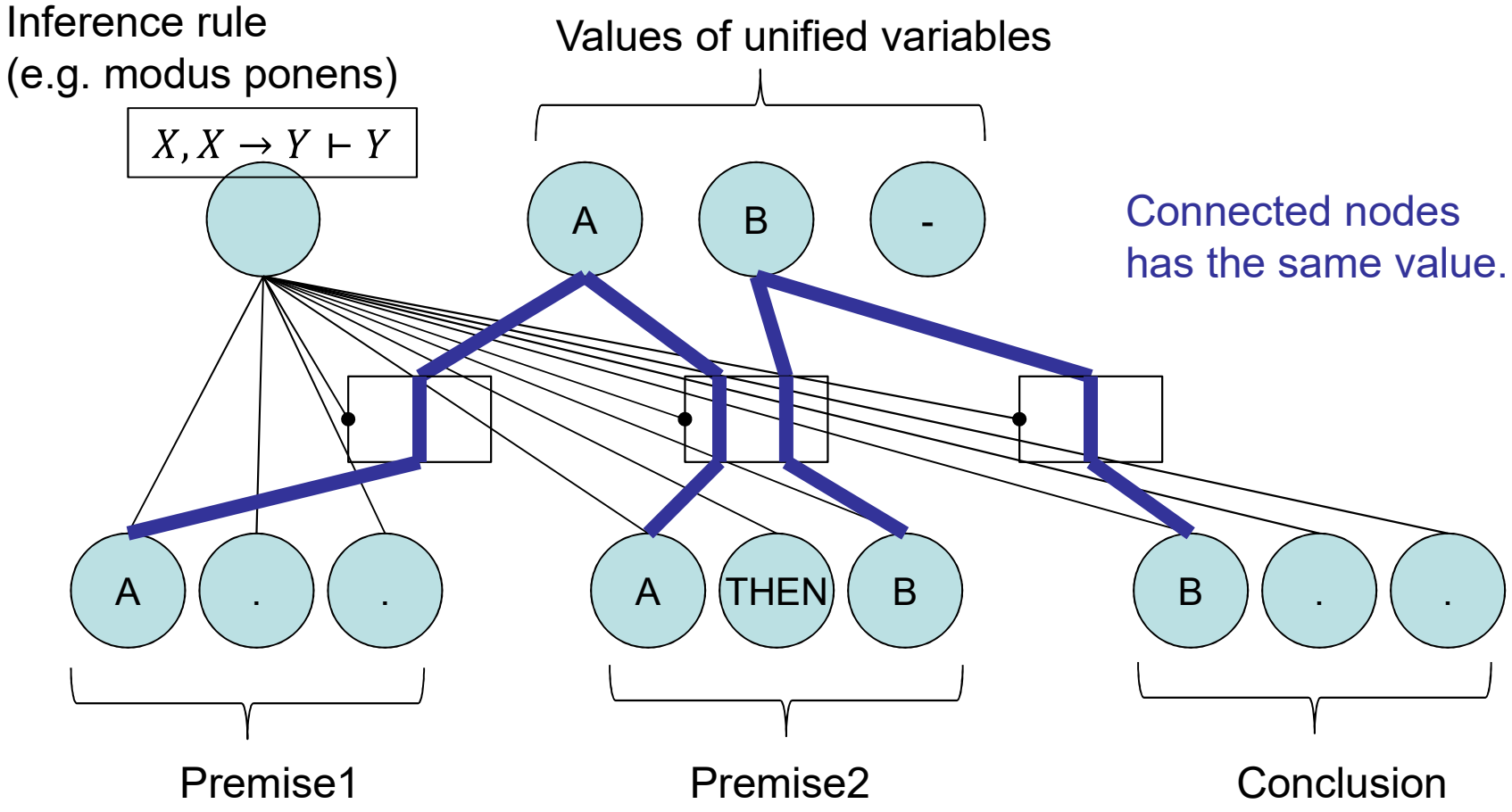
Simplified CPT of quasi Bayesian network

Prototype of chart parser for context free grammar

Each gate opens if the connection forms a part of the parse tree.



Prototype of variable unification mechanism



The network can infer not only a conclusion from the premises, but also the premises from a conclusion.
The network will be able to learn inference rules from sample data of premises and conclusions.
This mechanism will become key technique to implement unification grammar parser such as CCG.

Conclusion

- A BESOM can be used as a *bidirectional* Deep Neural Network.
  - Thanks to the restricted CPT model, recognition and learning algorithms are scalable.
- Using Gate nodes, parser and unification mechanism would be implemented. (ongoing project)
- Future work
  - Sequence learning, short-term memory
  - Large scale implementation