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Enhancement of the transverse non-reciprocal magneto-optical effect

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The methods to enhance the transverse non-reciprocal magneto-optical (nMO) effect have been studied. The transverse nMO effect occurs in the case when light propagates perpendicularly to the magnetic field. It was demonstrated that light can experience the transverse nMO effect only when it propagates in the vicinity of a boundary between two materials and the optical field at least in one material is evanescent. The magnitude of the transverse nMO effect is comparable to or greater than the magnitude of the longitudinal nMO effect. In the case of surface plasmons propagating at a boundary between the transition metal and the dielectric it is possible to magnify the transverse nMO effect and the magneto-optical figure-of-merit may increase from a few percents to above 100%. *© 2012 American Institute of Physics*. [doi:10.1063/1.3677942]

I. INTRODUCTION

The magneto-optical (MO) effect is important for a variety of applications. The effect is utilized to read data in a MO disk driver,[1] to switch an optical beam in optical switches[2] and to modulate light intensity in spacial light modulators.[3] It is a powerful scientific tool to determine the local magnetization of a material, to study the bandgap structure and spin-orbit interaction in a solid.[4] A unique feature of the MO effect is non-reciprocity. The optical properties of non-reciprocal devices are different for two opposite directions of light propagation. The non-reciprocal effect can occur only in a MO material and the important optical non-reciprocal devices such as an optical isolator and an optical circulator can only be fabricated by utilizing MO materials.

The longitudinal non-reciprocal MO (nMO) effect occurs when light propagates along a magnetic field. The origin of the longitudinal nMO effect is well understood and can be explained as follows. When the magnetic field is applied to a material, the electrons with spin directed along and opposite to the magnetic field have different energies. Since the electrons of one spin direction interact with light either of left or right circular polarization,[5–7] the transition energy for left- and right-circularly polarized light are different and light of left and right circular polarizations experiences different refraction and absorption. When light is transmitted through a MO material, there is a difference of refractive indexes (Faraday effect) and optical absorption (magnetic circular dichroism effect) for left and right circularly polarized light. When light is reflected from a MO material, the reflectivity of the right and left circular polarized light is different (polar and longitudinal Kerr effects). Since the origin and properties of the longitudinal nMO effect are well understood, the longitudinal nMO effect can be optimized and significantly enhanced. For example, utilizing an optical cavity the Faraday rotation can be increased tenfold.[8,9]

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The transverse nMO effect occurs when light propagates perpendicularly to the magnetic field. The transverse nMO effect has several important practical applications. For example, presently there is a strong industrial demand for an optical isolator, which could be integrated into photonic integrated circuits (PIC). The isolators made of a Si wire waveguide with a bonded magnetic garnet[10] and the isolators made of a semiconductor waveguide covered by a ferromagnetic metal[11–14] are two most promising designs for the integrated isolator. Both these designs utilize the transverse MO effect. The sizes of the isolator for PIC should be as small as possible. Therefore, it is important to magnify the transverse nMO effect. At present, it is still unclear whether it is possible to enhance the transverse nMO effect and whether the optimization methods, which were successfully utilized for the longitudinal nMO effect, can be applied for the transverse nMO effect.

The transverse nMO effect was well studied both theoretically and experimentally in the cases of garnet-made optical waveguide,[15] an optical waveguide covered by a magnetic garnet[10] or ferromagnetic metal,[11–14] a garnet/gold plasmon waveguide,[16,17] ferromagnetic-metal plasmon waveguide,[16,18] the light transmission through a thin ferromagnetic-metal film,[19] and the light reflection from a ferromagnetic film.[20] However, several facts about the transverse nMO effect still remain unexplained. The first unexplained fact is that the transverse nMO effect has been observed only in layered structures (for example, an optical waveguide or a plasmonic waveguide). It has never been observed in a bulk material. The significant difference between properties of the transverse and the longitudinal nMO effects is the second unexplained fact about the transverse nMO effect. There are several such differences. Firstly, the best-known feature of the longitudinal nMO effect, the rotation of the polarization plane is not a feature of the transverse nMO effect. The transverse nMO effect does not cause any polarization rotation. Due to the transverse nMO effect the absorption and refractivity of light changes when the magnetic field is reversed. Secondly, the longitudinal nMO effect influences both orthogonal polarizations. The absorption and refraction of both left and right circularly polarized waves change when the magnetic field is reversed. By contrast, only one polarization is affected by the transverse nMO effect and the orthogonal polarization is not sensitive to MO. Thirdly, the magnitude of the longitudinal nMO effect decreases in the case of light propagation along a multilayer structure, because of "TE-TM mode mismatch" problem.[16] Higher magnitude of the longitudinal MO effect has been achieved in the structures with graded interfaces.[21] By contrast, to achieve a substantial transverse nMO effect sharp interfaces are required.[12]

Here we will explain the origin of the transverse nMO effect and the conditions, at which light may experience the transverse nMO effect. We will demonstrate that in contrast to the longitudinal nMO effect the transverse nMO effect has two contributions and by optimizing these contributions the effect may be significantly magnified. We will demonstrate that in the case of the surface plasmons, the magneto-optical figure-of-merit may increase from a few percents to above 100%. We will derive the scalar dispersion relation, which describes the transverse MO effect in cases of waveguide modes and surface plasmons propagating in a multilayer MO slab.

II. ORIGIN OF THE TRANSVERSE nMO EFFECT

At first, we explain why the transverse MO effect cannot be observed when light propagate in a bulk material. In the case when the magnetic field is applied perpendicularly to the light propagation direction, the electrons, which spins are directed along and opposite to this direction, have different energy. When light propagates in a bulk material, its polarization is always in a plane perpendicular to the light propagation direction and light does not have any polarization component, which may rotate around the spin direction. Since there is no polarization rotation around the spin direction, the interaction of photons with electrons of opposite spins does not depend on the polarization of light. Therefore, in this geometry the probability to excite electrons with opposite spins is equal for left and right circularly polarized light and light is not sensitive to the energy difference of electrons with opposite spins. There is no nMO effect in this case. It should be noticed that in the case of the transverse magnetization, light experiences birefringence, which is proportional to the square of the intensity of the magnetic field. It is called the Cotton-Mouton effect (in the case of gases it is called the Voigt effect). It is a reciprocal effect and may be simply considered as a magnetically-induced anisotropy. The Cotton-Mouton effect will not be discussed here.

In the following, we will describe the required conditions at which the transverse MO effect may occur. Let us consider light propagating along the z-axis in a MO media and the magnetic field applied along the y-axis. In this case the electrons with opposite spins along the y-axis have different energy. As was mentioned above, in the case when light is polarized perpendicularly to its propagation direction in xy-plane, the probabilities to excite electrons with the opposite spins are equal and light will not experience any nMO effect [Fig. 1 (a)]. In order for the nMO effect to occur, the polarization of light should be rotating around the axis, which is along the magnetic field and perpendicular to the light propagation direction [Fig. 1(b)]. Only in this case light interacts differently with electrons of opposite spins. The fact that the polarization rotation should be around axis perpendicular to the light propagation direction means that light should have a polarization component along the propagation direction. Since an electromagnetic wave is transverse, it seems that such a polarization is forbidden. However, there is one special case when such a polarization is possible. It is the case when a wave has an evanescent component along the direction perpendicular to the wave propagation direction. For example, if the wave propagates along the z-direction, but it has an evanescent component along the x-direction. In this case the x-component of the wave vector has only an imaginary part and the z-component has only a real part. So the wave is described as

$$\vec{E},\vec{H} \sim e^{\frac{2\pi}{\lambda}i\cdot(-c\cdot t+k_z z+i\cdot k_x x)}. \tag{1}$$

The polarization of this wave may be either along the y-axis ($E_x, E_z = 0, E_y \neq 0$) or in the xz-plane ($E_x, E_z \neq 0, E_y = 0$). In the case of the polarization in the xz-plane, the condition for the wave to be transverse is $\vec{E}\cdot\vec{k} = E_z\cdot k_z - i\cdot k_x\cdot E_x = 0$ or

$$\frac{E_z}{E_x} = i\cdot\frac{k_x}{k_z}. \tag{2}$$

If $k_x/k_z = 1$, the polarization is circular and the axis of polarization rotation is perpendicular to the wave propagation direction and along the magnetic field. This wave will interact only with electrons of one spin direction and will experience the transverse nMO effect. The wave (1), (2) is transverse, but it has a polarization component along the

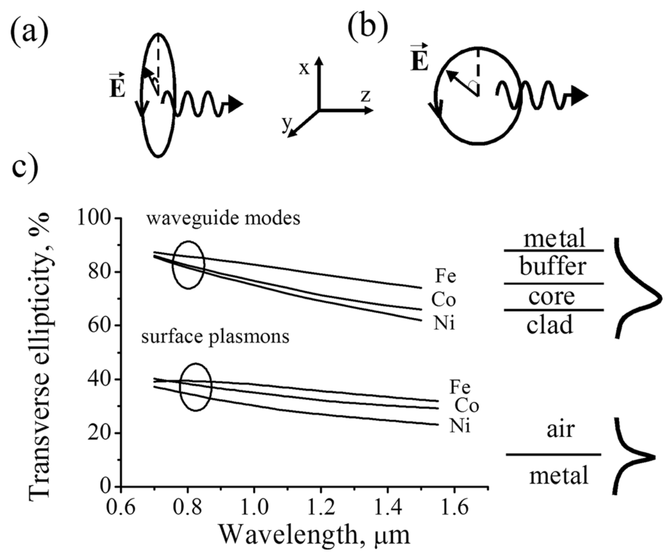

FIG. 1. (a) Conventional ellipticity of light. Light propagates in z-direction. Polarization rotates in xy-plane. (b) Transverse ellipticity of light. Polarization rotates in xz-plane. (c) Transverse ellipticity of waveguide mode propagating in metal/AlGaAs waveguide and single-surface plasmon propagating at metal/air interface. 100% corresponds to a circular polarization. The upper and lower insets show the optical field distribution of a waveguide mode and a single-surface plasmon, respectively.

propagation direction. The seeming contradiction in the above-mentioned properties is explained as follows. As required for a transverse wave, the wave polarization is perpendicular to the direction of the wave vector $\vec{k}$, but the $\vec{k}$ direction is not the propagation direction for this wave. For an electromagnetic wave the propagation direction is determined by a direction of electromagnetic energy flow,[7] which is defined by time-averaged Poynting vector $\vec{S} = 1/2\mathrm{Re}(\vec{E} \times \vec{H}^*)$ and for the wave (2) it is along z-direction,[22] along which the wave has a polarization component.

From the above discussion, it can be concluded that only light, which has an evanescent component and is elliptically polarized in a plane of the propagation direction, may experience the transverse nMO effect. That condition explains several properties of the transverse nMO effect. Light may have an evanescent component only in the vicinity of the interface. This explains why the transverse nMO effect occurs only in layered structures. Also, only one of the two orthogonal polarizations of light (2) may experience the transverse MO effect. The polarization $(E_x, E_z = 0, E_y \neq 0)$ is not rotating and light of such polarization will not experience any nMO effect.

It should be noticed that having a polarization component along propagation direction is not sufficient for light to experience the transverse MO effect. The transverse ellipticity is essential for light to experience the transverse nMO effect. The transverse ellipticity χ is calculated as

$$\chi = 2 \cdot \mathrm{Im}\left(\frac{E_z}{E_x}\right) \Big/ \left(1 + \left|\frac{E_z}{E_x}\right|^2\right). \tag{3}$$

In the case of the circular polarization the transverse nMO effect will be strongest. Figure 1(c) shows calculated transverse ellipticity in cases of a waveguide mode propagating in Fe/AlGaAs, Co/AlGaAs, Ni/AlGaAs waveguides (buffer $Al_{0.5}Ga_{0.5}As$(300 nm)/core $Al_{0.3}Ga_{0.7}As$(1200 nm)/clad $Al_{0.5}Ga_{0.5}As$) and in cases of a single-surface plasmon propagating along Fe/air, Co/air, Ni/air interfaces. In all cases the traverse ellipticity is significant. The ellipticity is close to circular for the waveguiding mode. That is the reason why the experimentally observed transverse nMO effect in waveguides is substantial.[12–14] The data were calculated by solving Maxwell's equations (Appendix A) and utilizing known optical and magneto-optical constants of transition metals[23,24] and semiconductors.[25]

It is interesting to compare the strength of the longitudinal and transverse nMO effects. Figure 2 shows the calculated MO Figure of Merit (FoM) for the longitudinal nMO effect in Fe, FoM of transverse nMO effect for a single-surface plasmon[26] propagating at Fe/air interface and FoM of the transverse nMO effect for a waveguide mode propagating in Fe/$Al_{0.5}Ga_{0.5}As$ (300 nm)/$Al_{0.3}Ga_{0.7}As$ (1200 nm)/$Al_{0.5}Ga_{0.5}As$ waveguides. FoM is a ratio of the magnetization dependent optical loss to the average loss. The nMO effect in the case of the transverse magnetization is 2–3 times stronger than in the case of the longitudinal magnetization.

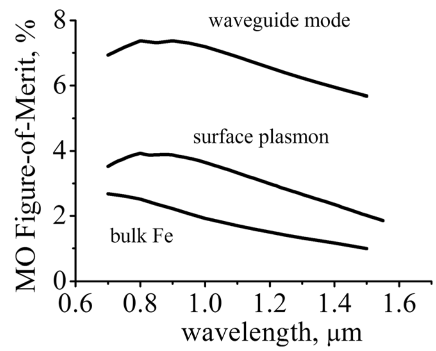

FIG. 2. MO Figure-of-Merit (FoM) of the longitudinal nMO effect in Fe and FoM of transverse nMO effect for waveguide mode propagating in Fe/AlGaAs waveguide and for surface plasmons propagating at Fe/air interface.

III. TWO CONTRIBUTIONS TO TRANSVERSE nMO EFFECT AND ENHANCEMENT OF TRANSVERSE nMO IN THE CASE OF SINGLE-SURFACE PLASMONS

In the following, we will describe two major contributions to the transverse nMO effect. Since the transverse nMO effect may occur only in the case when a wave has an evanescent component, the effect is pronounced near an interface. The first contribution comes from a bulk of MO material and the second contribution comes from the interface. These two contributions are often of opposite sign and about the same magnitude. Often they compensate each other and this causes a weak transverse nMO effect. However, in the case when these two contributions are optimized (for example by optimizing the device structure), the transverse nMO may be significantly magnified.

To demonstrate this, let's consider light propagating along a ferromagnetic-metal/dielectric interface (for example, a waveguiding mode or a surface plasmon). The magnetization of the metal is perpendicular to the light propagation direction. Light is absorbed by the metal and the flux of light energy dissipates along the propagation direction. Utilizing Poynting theorem (see Appendix B), the absorption coefficient $\mathrm{Im}(k_z)$ for light is calculated as

$$\mathrm{Im}(k_z) = \frac{c}{8}[\mathrm{Im}(\varepsilon_d) + \mathrm{Im}(\gamma) \cdot \chi] \frac{\iint_{metal} |\vec{E}|^2 dx \cdot dy}{\iint_{metal+dielectric} S_z \cdot dx \cdot dy}, \tag{4}$$

where

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_d & 0 & -i \cdot \gamma \\ 0 & \varepsilon_d & 0 \\ i \cdot \gamma & 0 & \varepsilon_d \end{pmatrix} \tag{5}$$

is the permittivity tensor of the ferromagnetic metal, S_z is z-component of Poynting vector, χ is the transverse ellipticity (3) and c is the speed of light.

When the magnetic field is reversed, the absorption of light changes due to the transverse nMO effect. Assuming that the distribution of an optical field does not depend on the applied magnetic field, the ratio of integrals at the left part of (4) would be the same for both directions. That assumption means that the redistribution of the optical field due to the influence of the interface is ignored. Taking into consideration that the sign of γ changes when the magnetic field is reversed, FoM of the bulk contribution to transverse nMO effect is derived from (4) as

$$FoM_{bulk} = \frac{\mathrm{Im}\left(k_z^{\uparrow}\right) - \mathrm{Im}\left(k_z^{\downarrow}\right)}{\mathrm{Im}\left(k_z^{\uparrow}\right) + \mathrm{Im}\left(k_z^{\downarrow}\right)} = 2\chi\frac{\mathrm{Im}(\gamma)}{\mathrm{Im}(\varepsilon_d)}, \qquad (6)$$

where $\mathrm{Im}\left(k_z^{\uparrow}\right), \mathrm{Im}\left(k_z^{\downarrow}\right)$ is the absorption coefficient for two opposite directions of the magnetic field.

The *FoM* of the bulk contribution is linearly proportional to the transverse ellipticity. It supports above explanation of the origin of the transverse MO effect. It should be considered that the optical field distribution near the interface depends on the refractive index of both the metal and the dielectric. In the case of non-zero transverse ellipticity the effective refractive index of the metal changes with reversal of the magnetic field due to the transverse nMO effect. That leads to the redistribution of the optical field near the interface and the ratio of integrals at the left part of (4) will be different for opposite directions of the magnetic field. That defines the interface contribution to the transverse nMO effect. The interface contribution is explained as follows. The MO change of the effective refractive index of the metal redistributes relative amounts of light energy between the dielectric and the metal. Since the absorption by the metal is proportional to the amount of light inside it and this amount changes with reversal of the magnetic field, the optical loss of the wave will also change with the reversal of the magnetic field. For the transverse nMO effect, both the bulk and interface contributions are substantial, often have opposite signs and should be carefully considered. For example, it was observed experimentally[12] that in the case of Co:AlGaAs waveguide the sign of the transverse nMO effect changes when an AlGaAs buffer layer between waveguide core and cobalt is replaced with a SiO_2 buffer layer. This is because the bulk contribution is the same in both cases, but interface contribution is of opposite signs and it is negligible in the case of Co:AlGaAs interface and is about two times greater than the bulk contribution in the case of a Co:SiO_2 interface. Another example, which demonstrates the importance of the interface contribution, is the single-surface plasmons[26] propagation at a Fe:MgO: AlGaAs interface. Figure 3 shows calculated plasmon's 1/e propagation distance and MO FoM as a function of MgO thickness. The structure does not support the plasmons when MgO thickness is between 14 and 110 nm. Near "cutoff" thicknesses of 14 and 110 nm, the MO FoM significantly increases and the plasmon's propagation distance becomes longer. The reason for the reduction of plasmon's optical loss near the cutoff is following. When MgO thickness approaches a "cutoff" thickness, the penetration depth of plasmon's optical field into AlGaAs significantly increases, but the penetration depth into the iron remains unchanged. That means that the amount of the optical field decreases inside the metal and increasing inside the AlGaAs. Therefore, the optical field is "pushed out" from the metal. That causes a significant decrease of optical loss and increase of the plasmon's propagation distance. The following explains the increase of MO FoM near the cutoff thickness. A change of the refractive index of the metal changes the value of the "cutoff" thickness. The plasmon's optical loss sharply decreases when the MgO thickness approaches the "cutoff" thickness. Therefore, in the case of the MgO thickness close to the "cutoff" thickness, a small MO change of the refractivity of the metal causes a significant MO change of plasmon's optical loss. That causes a substantial transverse nMO effect. As seen from Fig. 3, the MO FoM may reach 100%. This is a significant enhancement of nMO effect compared to 2%–3% of the MO FoM, which is the most that can be achieved for the longitudinal nMO effect in Fe (Fig. 2).

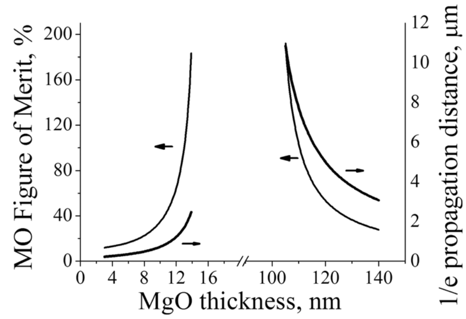

FIG. 3. MO FoM and 1/e propagation distance for surface plasmons propagating at Fe/MgO/$Al_{0.5}Ga_{0.5}As$ interface ($\lambda = 800$ nm).

IV. CONCLUSION AND REMARKS

In the following, we will describe general rules, which should be applied in order to optimize and enhance the transverse MO effect. Firstly, the materials and device structure should be chosen to maximize the transverse ellipticity (3). The polarization of light in MO material should be close to the transverse-circular polarization. As was described above, the transverse MO effect linearly proportional to the transverse ellipticity. Secondly and most importantly, the bulk and surface contributions to transverse MO effect should be well optimized. The sign of the bulk and surface contributions are often opposite. The bulk contribution depends mainly on MO constants of a MO material, but it is influenced only weakly by a device structure. In contrast, the interface contribution significantly depends on the device structure. Therefore, the device structure must be optimized so that the interface contribution is either significantly smaller or significantly greater than the bulk contribution. As was showed above in the case of the single-surface plasmons the transverse nMO effect becomes significant near the cutoff due to the surface contribution. It is a general tendency. In the cases of waveguide modes and plasmons, the transverse MO effect is significant near the cutoff. Also, the surface contribution may be significantly increased in a structure, which has interfaces with a significant step of refractive index.[12]

Another interesting effect can be predicted from understanding of the origin of the transverse nMO effect. Recently, several new designs of photonic devices have been proposed, in which spin-polarized electrons are excited by circular-polarized light.[27,28] However, it is difficult to use optical waveguides in such devices, because of the difficulties in achieving circular polarization of a waveguiding light due to "TE-TM mode mismatch problem." By contrast, utilizing the transverse nMO effect, the spin-polarized electrons may be efficiently excited in waveguiding devices. For example, let us consider an optical waveguide, in which the transverse magnetic mode (TM mode) propagates. If the waveguide contains a semiconductor layer and the photon energy of light is slightly above the semiconductor bandgap, light will experience absorption and will excite electrons in this layer. As was explained above, if the optical field in this layer has an evanescent component, the polarization of light will be transverse elliptical. Therefore, the photo-exited electrons will be spin-polarized and the direction of the spin will be perpendicular to the light propagation direction. This effect does not require any mode phase matching and it could be utilized for efficient and reliable excitation of spin polarized electrons in future spintronics and spin-photonics devices.

In conclusion, the origin and the properties of the transverse nMO effect have been investigated. The transverse nMO effect occurs because the polarization of light, which has an evanescent component, rotates around an axis, which is parallel to the magnetic field and perpendicular to the light propagation direction. The transverse nMO effect is comparable to and often greater than the longitudinal nMO effect. In contrast to the longitudinal nMO effect, the transverse nMO effect can be significantly magnified by optimizing the device structure. We have demonstrated that in the case of surface plasmons the magneto-optical figure-of-merit may increase from few percents to above 100%. Because of this unique property, the transverse nMO effect has an advantage for usage in a variety of applications where only the longitudinal nMO effect is currently utilized.

APPENDIX A

In this Appendix, we will derive the dispersion relation, which describes the transverse nMO effect in cases of waveguide modes and surface plasmons propagating in a multilayer nMO slab. Since the permittivity tensor of a MO material has non-zero off-diagonal components, the wave propagation in a MO structure is conventionally described by a dispersion relation, which is a combination of (4×4) matrixes.[16,29] Because of the complexity of this dispersion relation, the approximation of small off-diagonal components is often utilized.[29] However, in the case of the transition metals, the off-diagonal components have the same order of magnitude as the diagonal components and that approximation is not always valid. In the following, without usage of any approximations we will derive a scalar dispersion relation, which described the transverse nMO effect in a multilayer MO slab. The availability of the simple scalar dispersion relation significantly simplifies analysis and predictions for the transverse nMO effect.

Let's consider a multilayer MO slab, where $\hat{\varepsilon}_j$ and t_j the permittivity tensor (5) and the thickness of each j-layer, respectively. The layers of the slab are infinite in the xy-plane and the wave propagation direction is along z-direction. For a plain wave

$$E_x^j, E_z^j, H_y^j \sim e^{\frac{2\pi}{\lambda}i\left(-c\cdot t+k_{xj}x+k_z z\right)}, \tag{A1}$$

a solution for Maxwell's equations

$$rot(\vec{H}_j)=\frac{1}{c}\frac{\partial}{\partial t}\left(\hat{\varepsilon}_j\cdot\vec{E}_j\right);\quad rot(\vec{E}_j)=-\frac{1}{c}\frac{\partial}{\partial t}\left(\vec{H}_j\right) \tag{A2}$$

will be $k_{xj}^2=\varepsilon_{dj}-k_z^2-\gamma_j^2\big/\varepsilon_{dj}$,

$$\begin{bmatrix}E_x^j\\H_y^j\end{bmatrix}=\begin{bmatrix}\dfrac{i\cdot\gamma_j-k_zk_{xj}}{\varepsilon_{dj}-k_z^2}\\[2ex]\dfrac{i\cdot\gamma_j\cdot k_z-\varepsilon_{dj}\cdot k_{xj}}{\varepsilon_{dj}-k_z^2}\end{bmatrix}E_z^j. \tag{A3}$$

The optical field in j-layer can be described as

$$\begin{bmatrix}E_z^j\\H_y^j\end{bmatrix}=\left\{A_f^je^{i\frac{2\pi}{\lambda}k_{xj}(x-t_j)}\begin{bmatrix}1\\\frac{i\cdot\gamma_j\cdot k_z-\varepsilon_{dj}\cdot k_{xj}}{\varepsilon_{dj}-k_z^2}\end{bmatrix}+A_b^je^{-i\frac{2\pi}{\lambda}k_{xj}(x-t_j)}\times\begin{bmatrix}1\\\frac{i\cdot\gamma_j\cdot k_z+\varepsilon_{dj}\cdot k_{xj}}{\varepsilon_{dj}-k_z^2}\end{bmatrix}\right\}e^{\frac{2\pi}{\lambda}i(-c\cdot t+k_z z)}+\text{c.c.}, \tag{A4}$$

where A_f^j and A_b^j are unknowns and c.c. is complex conjugative. Introducing new unknowns

$$Z_j=\frac{A_f^j-A_b^j}{A_f^j+A_b^j}\quad B_j=A_f^j+A_b^j \tag{A5}$$

and applying boundary conditions $E_z=const$, $H_y=const$ at boundary between j and $j+1$ layers, we have

$$Z_j=T_j\left[F_{j,j+1}\left[Z_{j+1}\right]\right], \tag{A6}$$

where

$$T_j[Z]=\frac{Z-i\cdot tan\left(\frac{2\pi}{\lambda}k_{xj}t_j\right)}{1-i\cdot Z\cdot tan\left(\frac{2\pi}{\lambda}k_{xj}t_j\right)},$$

$$F_{j,j+1}[Z]=\left[i\cdot\gamma_j\cdot k_z-\left(i\cdot\gamma_{j+1}\cdot k_z-Z\cdot\varepsilon_{d(j+1)}\cdot k_{x(j+1)}\right)\times\frac{\varepsilon_{dj}-k_z^2}{\varepsilon_{d(j+1)}-k_z^2}\right]\bigg/\left(\varepsilon_{dj}\cdot k_{xj}\right).$$

In the case of a evanescent wave, $|\vec{E}|\to0$ when $x\to\infty$, which leads to $A_b^n=0$ $Z_n=1$. The general solution describing transverse nMO effect in the case of a plain wave propagating in a MO multilayer slab will be

$$Z_j=T_j[F_{j,j+1}[...[T_j[F_{j,j+1}[...F_{n-1,n}[1]]]]]]. \tag{A7}$$

The Eq. (A7) can be used to derive reflectivity of MO multilayer and dispersion relation for surface plasmons and waveguide modes. If the plain wave propagates in layer 1 and is reflected by a MO multilayer, the reflection coefficient will be

$$R=\frac{A_{bi}}{A_{fi}}=\frac{1-Z_1}{1+Z_1}, \tag{A8}$$

where

$$Z_1=F_{1,2}[...[T_j[F_{j,j-1}[...F^{n,n-1}[1]]]]], \tag{A9}$$

where n is the number of MO layers. The Eqs. (A8) and (A9) describe transverse Kerr effect in the case of the reflection from the multilayer.

In the case of a surface plasmon, light is confined near the metal/dielectric interface, so $|\vec{E}| \to 0$ when $x \to \pm\infty$. The dispersion relation for surface plasmons will be

$$F_{1,2}[...[T_i[F_{i,i-1}[...F^{n,n-1}[1]]]]] = -1. \quad \text{(A10)}$$

Next let's consider an optical waveguide, in which layer 1 is a core, layer 0 is a cladding layer under the core and the MO multilayer on the top covering the core. The dispersion relation for the waveguide mode is derived as

$$k_{x1}d_1 = a\tan(i \cdot Z_1) + a\tan\left(i\frac{\varepsilon_{d0}}{\varepsilon_{d1}}\frac{k_{x1}}{k_{x0}}\right) + m \cdot \pi, \quad \text{(A11)}$$

where Z_1 determined by (A9) and m is a mode number

APPENDIX B

In the following, we will derive an expression, which can be used to calculate the non-reciprocal optical loss due to the transverse nMO effect. There are several cases when this expression may be used instead of the rigorous solution obtained in Appendix A. Firstly, this expression is effective for a rough quick estimation of the non-reciprocal optical loss. Secondly, it is effective to calculate the non-reciprocal optical loss in case when light has a complex path (for example, a curved waveguide or a ring waveguide). Thirdly, this expression allows calculating magnitudes of two contributions of transverse MO effect separately. Therefore, this expression may be useful to optimize and to enhance the magnitude of the transverse MO effect.

In the following, we will calculate the magnetization-dependent loss for a very general structure, which contains a ferromagnetic metals and a transparent dielectric. We do not put any limitation on the structure. Light propagates along z-direction. The light energy flow through xy-plane is

$$s = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_z dx \cdot dy, \quad \text{(B1)}$$

where S_z is z-component of Poynting vector.

Since light is absorbed by the metal, the energy flow is decreasing along z axis, which can be described as

$$s = s_{z=0} \cdot e^{-2\frac{2\pi}{\lambda}\mathrm{Im}(k_z)\cdot z}. \quad \text{(B2)}$$

$\mathrm{Im}(k_z)$ is the light absorption coefficient, which needs to be calculated.

Differentiating (B2) and combing it with (B1) the absorption coefficient can be calculated as

$$\mathrm{Im}(k_z) = -\frac{1}{2}\frac{\lambda}{2\pi}\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial S_z}{\partial z}dx \cdot dy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_z \cdot dx \cdot dy}. \quad \text{(B3)}$$

Poynting theorem reads

$$\frac{\partial}{\partial t}U = -\nabla \cdot \vec{S}, \quad \text{(B4)}$$

where U is the energy density of light. Integrating (B4) over the xy-plane, applying Gauss–Ostrogradsky theorem and using the condition that there is no flux of light when $x \to \infty$ or $y \to \infty$, we obtain

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial}{\partial t}U \cdot dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial S_z}{\partial z} \cdot dxdy. \quad \text{(B5)}$$

Taking into the consideration that only metal absorbs light and substituting (B5) into (B3) give

$$\mathrm{Im}(k_z) = -\frac{1}{2}\frac{\lambda}{2\pi}\frac{\iint_{metal}\frac{\partial U}{\partial t}dx \cdot dy}{\iint_{metal+dielectric} S_z \cdot dx \cdot dy}. \quad \text{(B6)}$$

The time-average energy dissipation in the metal is calculated as

$$\begin{aligned}\frac{\partial U}{\partial t} &= \frac{1}{2}\vec{E}\frac{\partial}{\partial t}\hat{\varepsilon} \cdot \vec{E} = \frac{1}{2}\mathrm{Re}(\vec{E} \cdot e^{i\omega t})\frac{\partial}{\partial t}[\mathrm{Re}(\hat{\varepsilon} \cdot \vec{E} \cdot e^{i\omega t})] \\ &= \frac{i\omega}{8}[\vec{E}^* \cdot (\hat{\varepsilon} \cdot \vec{E}) - \vec{E} \cdot (\hat{\varepsilon} \cdot \vec{E})^* + \vec{E} \cdot (\hat{\varepsilon} \cdot \vec{E}) \cdot e^{i2\omega t} \\ &\quad -\vec{E}^* \cdot (\hat{\varepsilon} \cdot \vec{E})^* \cdot e^{-i2\omega t}]. \end{aligned} \quad \text{(B7)}$$

The time average of third and forth terms is zero, which gives

$$\frac{\partial U}{\partial t} = \frac{i\omega}{8}[\vec{E}^* \cdot (\hat{\varepsilon} \cdot \vec{E}) - \vec{E} \cdot (\hat{\varepsilon} \cdot \vec{E})^*]. \quad \text{(B8)}$$

Substituting the permittivity tensor $\hat{\varepsilon}$ of the metal (5) into (B8), we obtained

$$\frac{\partial U}{\partial t} = \frac{i\omega}{8}2i \cdot |\vec{E}|^2(\mathrm{Im}(\varepsilon_d) + \mathrm{Im}(\gamma) \cdot \chi), \quad \text{(B9)}$$

where

$$\chi = 2 \cdot \mathrm{Im}\left(\frac{E_z}{E_x}\right) \Big/ \left(1 + \left|\frac{E_z}{E_x}\right|^2 + \frac{|E_y|^2}{|E_x|^2}\right) \quad \text{(B10)}$$

is the transverse ellipticity inside the metal. Considering only TM mode $E_y = 0$, (B10) is simplified to

$$\chi = 2 \cdot \mathrm{Im}\left(\frac{E_z}{E_x}\right) \Big/ \left(1 + \left|\frac{E_z}{E_x}\right|^2\right). \quad \text{(B11)}$$

Substituting (B9) into (B6), the absorption coefficient for light is calculated as

$$\mathrm{Im}(k_z) = \frac{c}{8}[\mathrm{Im}(\varepsilon_d) + \mathrm{Im}(\gamma) \cdot \chi]\frac{\int_{metal}|\vec{E}|^2 dx \cdot dy}{\int_{metal+dielectric} S_z \cdot dx \cdot dy}, \quad \text{(B12)}$$

where c is the speed of light.

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