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PERSISTENCE OF FANO AND AHARONOV-BOHM PHASES IN AN INTERFEROMETER WITH A QUANTUM DOT

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Using a realistic model of an Aharonov-Bohm ring with a quantum dot in one arm and a control gate in the other, we demonstrate phase persistence in the Fano and Aharonov-Bohm effects as has been observed in experiments. The Fano effects in the conductance is examined by changing the states in the Aharonov-Bohm ring by the control gate.

Keywords: Fano Effect; Aharonov-Bohm Effect; Quantum Dot.

1. Introduction

In an Aharonov-Bohm (AB) ring containing a quantum dot, a series of consecutive conductance peaks with a Fano type interference with similar asymmetry and phase of an AB oscillation have been observed.[1,2] The purpose of this work is to theoretically study Fano resonances in a realistic AB ring with a quantum dot and to understand these interesting experimental findings.

Theoretical calculations were made in a one-dimensional (1D) model,[3,4] but unsuccessful in explaining the phase persistence of AB and Fano effects observed experimentally. A successful demonstration was made more recently in a realistic model, in which the AB ring contains several conducting channels and a quantum dot with dimensions comparable to those in the experiments.[5] The coexistence of a small number of strongly coupled states and many weakly coupled states in the dot with finite width was shown to be responsible for the observed phase persistence.

2. Model and Method

We use a model of the AB ring with radius a, straight up and down arms with length L, a quantum dot with length L_D in the down arm separated by wall barriers with length L_W, and a control gate with length L_W as shown in Fig. 1. The model is the same as that used and fully described in Ref. 5. The strength of a magnetic field

applied perpendicular to the AB ring is characterized by ϕ/ϕ_0, where ϕ is magnetic flux passing through the stadium with area $aL+\pi a^2/4$ and ϕ_0 is the magnetic flux quantum given by $\phi_0 = ch/e$.

For a realistic quantum dot embedded in the AB ring, the adiabatic conditions are satisfied, i.e., $|dD(x)/dx| \ll 1$ and $|D(x)d^2D(x)/dx^2| \ll 1$ with $D(x)$ being the width of the wave-guide at the Fermi energy E_F. Therefore, we choose $L_W/\lambda_F = 5$ with Fermi wave length λ_F. Calculations for the single wall with height $W \lesssim E_F$ reveal that essentially electrons in the lowest 1D subband with the highest velocity in the incident direction can get over the wall and there is very little mixing between different 1D subbands or channels. In order to simulate actual situations, further, we introduce a weak random potential in the dot (see Sec. 4 for more discussion). The amount of the disorder corresponds to a mean free path of $10 \times \lambda_F$ or level broadening of $0.015 \times E_F$ in the two-dimensional system.

We use the following parameters, $W/E_F = 1.03$, the ring radius $a/\lambda_F = 6$, the arm length $L/\lambda_F = 20.8$, and the width of arms and leads $1.8 \times \lambda_F$ at the Fermi energy. There are three sets of the traveling modes in the arms and leads, which can describe the actual feature of the experiment in which there are several channels. Conductance is calculated by multi-channel Landauer's formula from numerically calculated transmission probabilities with the use of a recursive Green's function technique.[6,7] Further, we shall consider the magnetic flux around $\phi/\phi_0 = 82$ corresponding to 1.3 T, which is typical magnetic field in the experiments. We assume zero temperature to see the asymmetry of resonances clearly.

3. Results

We begin by considering the case where the up arm is nearly pinched off by the control gate. In this case the situation is close to that of so-called double slit experiments, because the transmission probability of an electron passing through the up arm is small and not so much different from that through the down arm and multiple scattering in the AB ring is less important.

Figure 2 shows an example of the calculated conductance as a function of the gate potential for the control gate $V_c/E_F = 1$. Many peaks appear in the conductance, but they can be classified into two groups, small numbers of wide peaks with large broadening and large numbers of narrow peaks. In this example, the wide peaks are at $V_g/E_F = -0.18$, -0.06, 0.07, and 0.18 indicated by arrows in the figures. All resonance peaks in the conductance are asymmetric with a dip in the right or left side. This asymmetry is largely due to interference of the waves passing through the up arm and transmitted resonantly through the dot in the down arm, i.e., the so-called Fano interference.

When the double-slit condition is valid, the transmission through the AB ring is given by $t^0 + t^d$ where t^0 is a transmission coefficient for the up arm, essentially independent of energy. A transmission coefficient t^d through a quantum dot is given

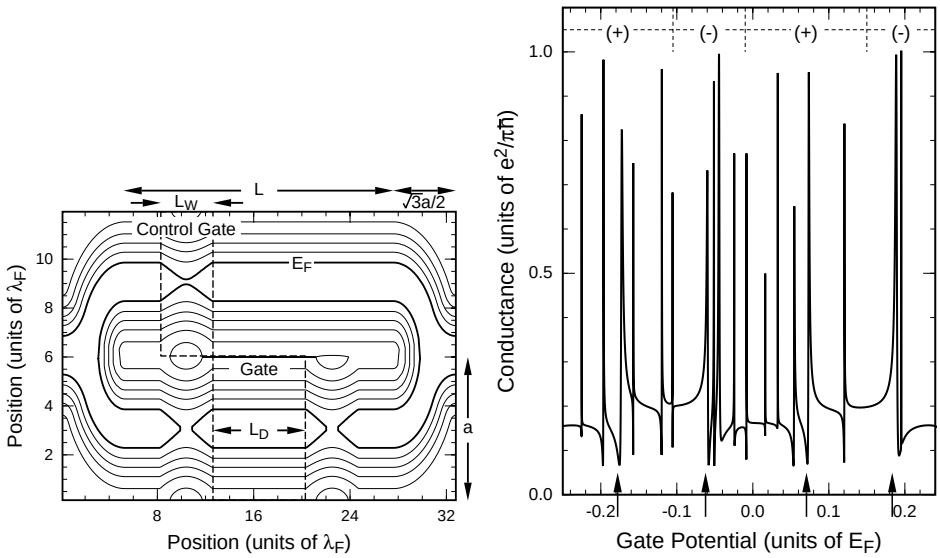

Fig. 1. [Left] Equi-potential lines of the model AB ring with a dot, plotted with energy interval of Fermi energy E_F. The thick lines correspond to the Fermi energy. The gate potential V_g and the control gate potential V_c are applied in the dashed rectangular regions with width L_W and L_D, respectively. In this example $V_g/E_F = 0$, $V_c/E_F = 0.95$, and $W/E_F = 1.03$.

Fig. 2. [Right] Calculated conductance for flux $\phi/\phi_0 = 82$. The arrows indicate the position of the wide conductance peaks when the up arm is pinched off. The region for positive and negative values of the asymmetry parameter q' are shown by $(+)$ and $(-)$, respectively.

by the conventional Breit-Wigner form

$$t^d = \frac{\alpha}{\epsilon + i}, \qquad \alpha = -2\pi i V_{j\nu}(E) V_{\nu j'}(E) D(E) \Gamma_\nu^{-1}, \tag{1}$$

with $\epsilon = (E_F - \tilde{E}_\nu)\Gamma_\nu^{-1}$, where $V_{j\nu}(E)$ and $V_{\nu j'}(E)$ are the matrix elements of transitions from the dot state to the out-going states and from the incident to the dot state, respectively, $D(E)$ is the density of states in each wave-guide, and Γ_ν is a peak width and $\tilde{E}_\nu$ the dot level. As a result, the conductance is given by

$$G = \frac{e^2}{\pi\hbar} T_0 \frac{|\epsilon + q|^2}{\epsilon^2 + 1}, \qquad q = \frac{\alpha}{t^0} + i = q' + iq''. \tag{2}$$

The calculated conductance is well fitted with this Fano line shape.

Figure 2 shows also that q' and correspondingly the phase of the AB oscillation of the wide peaks change sign alternately when the gate potential crosses them. For the narrow peaks, on the other hand, the sign of q' does not show such an alternate change from peak to peak but follows the sign of the nearest wide peaks. In fact, four narrow peaks in the range $-0.11 < V_g/E_F < -0.01$ have a dip in the right side of peaks, in agreement with the behavior of the wide peak at -0.06. Further, five narrow peaks in $-0.01 < V_g/E_F < 0.13$ have a dip in the left side of the peak again following the nearest wide peak at 0.07.

In the adiabatic limit, where the confinement potential varies slowly in the scale of the Fermi wave length, each one-dimensional channel has its own effective potential and mixing between different channels are small. Therefore, the transmissions through dot states with the same 1D subband index are possible and in particular those associated with the lowest subband having the largest kinetic energy in the wave-guide direction contribute to transmissions because of the lowest effective tunneling barrier. The wide resonances shown in Fig. 2 (a) actually correspond to such states.

This selection rule is slightly violated in the realistic confinement potential and also by the presence of unavoidable disorder. Let $\hat{H}'$ be the Hamiltonian describing effects of such deviation from the adiabatic limit, ψ_n^0 be a dot state uncoupled to wave-guide states in the absence of $\hat{H}'$, and ψ_N^0 be the nearest dot state coupled to wave-guide states even in the absence of $\hat{H}'$. Then, apart from energy shift, the state ψ_n associated with ψ_n^0 now contains a contribution of ψ_N^0, i.e.,

$$\psi_n \approx \psi_n^0 + \psi_N^0 \frac{(N|\hat{H}'|n)}{E_n - E_N}, \tag{3}$$

where the lowest order energy shift has been taken into account already in energies E_n and E_N. Then, in the vicinity of E_n, the matrix element for the transmission through the dot becomes

$$V_{jn}V_{nj'} \approx V_{jN}V_{Nj'} \frac{|(N|\hat{H}'|n)|^2}{(E_n - E_N)^2}. \tag{4}$$

This shows that the phase of $V_{jn}V_{nj'}$ is given by that of $V_{jN}V_{Nj'}$ of the nearest wide peak, explaining the essential feature of the numerical result that the asymmetry of the Fano interference of narrow peaks follows that of a neighboring wide peak. (See Eqs. (1) and (2))

Numerical results show that the asymmetry of the Fano lineshape varies as a function of the magnetic flux, i.e., the Fano parameter shows a sinusoidal oscillation with period ϕ_0. This AB oscillation of q' is qualitatively in good agreement with the behavior observed experimentally. One important finding is that the crossing of narrow peaks does not change the phase of the AB oscillation and that the phase changes by π only when the gate potential crosses the wide peak.[5] This is because the corresponding levels in the dot do not contribute to the conductance except in the extreme vicinity of the resonance and the conductance under off-resonant conditions is determined mainly by coupling to levels responsible for broad resonances.

With the decrease of the control gate, the channels are opened one by one, leading to conductance steps with a small oscillation due to interference in the AB ring.[5] Many asymmetric peaks due to the interference with states in the dot are superposed on this off-resonance conductance. As a function of the control gate the asymmetry of the peaks changes almost in an irregular manner. However, the rule that asymmetry of narrow peaks follows that of a nearest wide peak is always valid and the asymmetry of a wide peak is the same as that of next-nearest neighbor wide peak, as expected from the parity of dot states corresponding to the wide peaks.

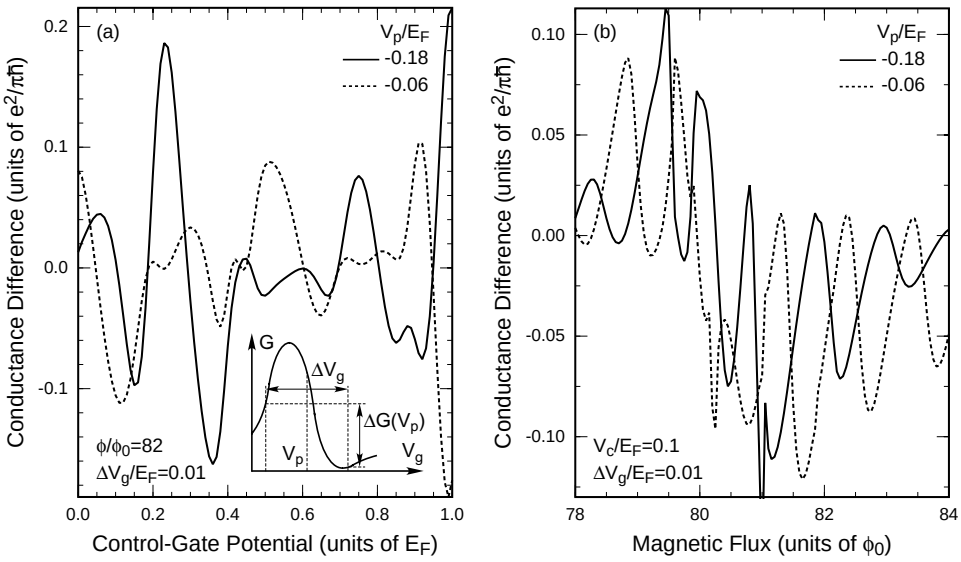

Fig. 3. Examples of the conductance difference as a function of (a) the control-gate potential and (b) the magnetic flux. The solid line is for the wide peak at $V_p/E_F = -0.18$ and dotted line for -0.06. The inset in (a) shows the definition of the conductance difference $\Delta G(V_p)$.

The asymmetry of the peaks can be obtained simply by calculating the difference of the conductance at the right and left side of the peak, i.e., $\Delta G(V_p) \equiv G(V_p + \Delta V_g/2) - G(V_p - \Delta V_g/2)$, where V_p is the peak gate potential and ΔV_g is chosen to be slightly larger than the peak width. Figure 3 (a) shows $\Delta G(V_p)$ with $\Delta V_g/E_F = 0.01$ at two adjacent wide peaks $V_p/E_F = -0.18$ (solid line) and -0.06 (dotted line) for $V_c/E_F = 0.1$. As a function of the control gate the asymmetry of the peaks changes continuously but in a complicated manner due to strong interferences inside the AB ring. There seems to be no correlation in the asymmetry of the adjacent peaks. This feature is true of all nearest-neighbor wide peaks.

Figure 3 (b) shows $\Delta G(V_p)$ as a function of the magnetic flux. It exhibits an AB oscillation with period ϕ_0 superposed on a background which changes gradually with the magnetic field. It is interesting that the phase of the oscillation of $\Delta G(V_p)$ is opposite between the two adjacent wide peaks.

4. Discussion

In actual experiments, the charging energy of a dot is dominant, causes a Coulomb blockade effect, and determines a typical scale of the gate potential. Most of dot states contributing to the Coulomb oscillation are those of narrow peaks. This means that the asymmetry of the Fano resonance stays the same for several consecutive narrow conductance peaks as long as they are connected with the same wide peak (Eq. (3)) and also the phase of the AB oscillation does not change among such

peaks. This feature is essentially independent of the parameters characterizing the system as long as the confinement potentials satisfy near-adiabatic conditions and the AB ring contains several channels.

Random scatterers have been introduced in the dot region. In the absence of randomness, the result is qualitatively same with a few exceptions on the asymmetry of the Fano resonance of narrow peaks. Without randomness, the present dot is symmetric and therefore the symmetry of the wave function can sometimes be important in mixing between dot levels, causing such exceptions.

5. Conclusion

We have numerically calculated the conductance using a realistic model of an AB ring with a quantum dot. Many peaks appear in the conductance, but they can be classified into two groups, small numbers of wide peaks with large broadening and large numbers of narrow peaks. The asymmetry of the Fano-type interference of narrow peaks is almost always same as that of a nearest wide peak.

When the up arm is nearly pinched off, the situation is close to that of double-slit experiments, and therefore the asymmetry parameter changes at the middle of neighboring wide peaks and the phase of the AB oscillation changes by $\sim \pi$ only when the gate potential crosses the wide peak. With the decrease of the control gate, interference effects in the AB ring become important and the asymmetry of the wide peaks changes in an irregular manner. The rule that asymmetry of narrow peaks follows that of a nearest wide peak remains always valid. These behaviors can account for the most of the features of the experimental results.[1]

Acknowledgments

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References

1. K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, Phys. Rev. Lett. **88**, 256806 (2002).
2. K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, Phys. Rev. B **68**, 235304 (2003).
3. O. Entin-Wohlman, A. Aharony, Y. Imry, Y. Levinson, and A. Schiller, Phys. Rev. Lett. **88**, 166801 (2002).
4. A. Ueda, I. Baba, K. Suzuki, and M. Eto, Suppl. A to J. Phys. Soc. Jpn. **72**, 157 (2003).
5. T. Nakanishi, K. Terakura, and T. Ando, Phys. Rev. B **69**, 115307 (2004).
6. R. Landauer, IBM J. Res. Dev. **1**, 223 (1957); Phil. Mag. **21**, 863 (1970).
7. T. Ando, Phys. Rev. B **44**, 8017 (1991).