Thermoelectric power in coherent transport as a tool for transmission-phase measurement

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Abstract. Thermoelectric power of a quantum dot is studied in a coherent region. Pronounced peaks are shown in the thermoelectric power, corresponding to a transmission zero in the conductance. Phase information of wavefunction in the quantum dot can be extracted from peak-and-dip structures of the thermoelectric power without the use of a magnetic field.

Keywords: thermoelectric power, quantum dot, conductance, Landauer formula, Mott formula, Fano effects
PACS: 73.23.Hk, 73.50.Lw, 72.15.Jf

A series of systematic experiments for a quantum dot (QD) embedded in an Aharonov-Bohm (AB) ring [1, 2] has shown that the phase of AB oscillations changes by π across a resonance peak as expected by coherent transmission of electrons through the QD. A surprising and unexpected finding is that the phase becomes the same between adjacent peaks by the presence of a phase lapse by π between those peaks. Recent observation of Fano-type asymmetric conductance peaks has also revealed that the adjacent peaks tend to have the same phase [3, 4]. In order to understand the phase lapse, the existence of 'zero transmission' coefficient has been pointed out theoretically [5].

In this paper, we show that measurement of a thermoelectric power (TEP) is suitable for detection of zero transmission in coherent transport through a QD. We also show that, by observation of zero transmission points, the phase information of the wavefunction in the QD can be obtained without the use of a magnetic field.

The conductance and TEP for quasi-one-dimensional systems is obtained from the Landauer formula as

$$G(\mu,T)=\frac{e}{\pi\hbar}\int d\varepsilon T(\varepsilon)\left[-\frac{\partial f}{\partial\varepsilon}\right], \tag{1}$$

$$S(\mu,T)=-\frac{k_B\beta}{e}\frac{\int d\varepsilon T(\varepsilon)(\varepsilon-\mu)[-\partial f/\partial\varepsilon]}{\int d\varepsilon T(\varepsilon)[-\partial f/\partial\varepsilon]}, \tag{2}$$

with transmission probability $T(\varepsilon)$, the Fermi distribution function f, and the Fermi energy μ in leads. The Sommerfeld expansion with taking up to the first order to T gives the Mott formula

$$S_M(\mu,T)=-\frac{\pi^2}{3}\frac{k_B^2T}{e}\frac{1}{G(\mu,T)}\frac{\partial G(\mu,T)}{\partial\mu}. \tag{3}$$

From this expression, we can expect that transmission zeros induce a singular behavior in energy dependences of TEP due to suppression of conductance in the denominator.

We first consider transmission through a quantum dot by the Hamiltonian

$$H=\sum\varepsilon_{k,\alpha}C^\dagger_{k,\alpha}C_{k,\alpha}+\sum_j\varepsilon_jd^\dagger_jd_j+\sum_{k,\alpha,j}[V_{\alpha,j}C^\dagger_{k,\alpha}d_j+H.c.], \tag{4}$$

where the operators $c_{k,\alpha}$ refer to electronic states in the left and right leads $(\alpha=L,R)$ and the operators d_j describe QD levels. Following to the derivation by Silva *et al.* [6], one obtains the transmission coefficient and calculates TEP as well as conductance for a non-interacting electron model [7]. Transmission probabilities exhibit resonance peaks with the conventional Breit-Wigner line-shape with a width Γ. Transmission probability vanishes (transmission zero) with the phase lapse between adjacent transmission peaks j and $j+1$ for the case that the relative coupling sign, $\sigma\equiv\mathrm{sign}(V_{L,j}V_{R,j}V_{L,j+1}V_{R,j+1})$, equals $+1$, while no phase lapse occurs between them for $\sigma=-1$. Corresponding to the transmission zeros, we have shown a significant enhancement of TEP and then jump from positive to negative. The peak and the dip of the TEP around the transmission zero are $\pm(\pi/\sqrt{3})(k_B/e)\sim\pm1.81(k_B/e)$ for temperatures lower than a level spacing Δ. The prominent structure shows clearly the transmission zero, while it is hard to see it in the tail of conductance. For high temperatures $k_BT\gg\Gamma$ we have observed a sawtooth shape of TEP with amplitude of $\sim(k_B/e)(\Delta/2k_BT)$ as predicted in a sequential tunneling regime [8].

Next we consider another example of the zero transmission. When the double-slit condition is valid, the transmission through an AB ring with a QD on one arm and a reference path on the other is given by $t=t^0+t^d$, where t^0 is a transmission coefficient through a contin-

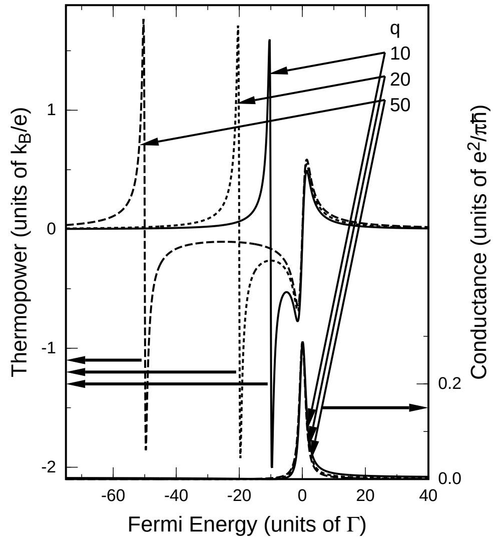

FIGURE 1. TEP and conductance for the Fano-type transmission with $q=10$ (solid line), 20 (dotted line) and 50 (dashed line) in low temperature $k_BT=0.2\Gamma$.

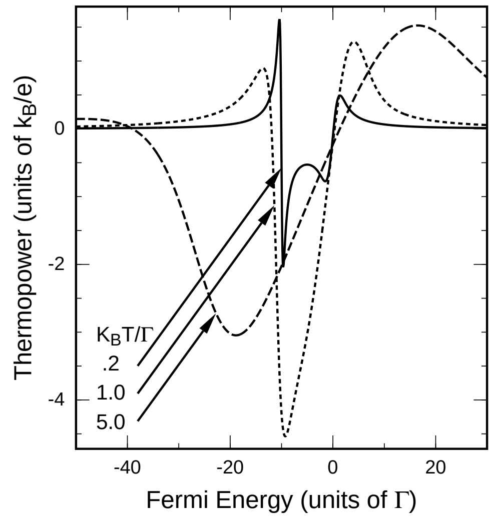

FIGURE 2. Temperature dependence of TEP for the Fano-type transmission with $q = 10$.

uum state of the reference path, essentially independent of energy [4]. The transmission $t^d = \alpha/(\varepsilon/\Gamma + i)$ through a QD shows the Breit-Wigner line-shape, where the peak energy is chosen as the origin of ε. Total transmission probability is given by

$$T(\varepsilon) = |t|^2 = |t^0|^2 \frac{|\varepsilon/\Gamma + q|^2}{(\varepsilon/\Gamma)^2 + 1}, \tag{5}$$

with a Fano parameter $q = \alpha/t^0 + i$ that relates on the phase of QD states through the constant α. Note that, we can chose real q in the presence of time-reversal symmetry. Then conductance shows the asymmetric Fano line shape. For large $q = 10, 20$ and 50, however, calculated conductance with the normalization $|t^0|^2 = 1/(|q|^2+1)$ shown in Fig. 1 is almost symmetric and difficult to distinguish. Actually the Fano line shape converges in symmetric Breit-Wigner line shape in the limit of infinite q. The Fano parameter q can be observed from the transmission zero, where transmission probability vanishes at $\varepsilon = -q\Gamma$ due to the interference of Fano effects, although it is not clearly seen in the tail of conductance. On the other hand, calculated TEP's shown in Fig. 1 exhibit a significant enhancement at $\varepsilon = -q\Gamma$, and then jump from positive to negative. At $\varepsilon = 0$, corresponding to the peak of conductance, TEP shows an additional small structure. With the increase of temperature, thermal averaging smears these structures for $k_BT > \Gamma$, as shown in Fig. 2.

Transmission zeros are expected to be sensitive to dephasing in the QD. For TEP of QD's with short dephasing time, the picture of an inelastic co-tunneling will become valid by suppression of higher-oder hopping processes, and then pronounced peaks and dips of TEP will disappear [9].

In summary, we have shown pronounced structures in the thermoelectric power that gives us phase information of wavefunction in the quantum dot. Because of its simpleness, observation of zero transmission by TEP's has advantages for detection of phase information in comparison with conductance measurement of the Aharonov-Bohm ring with a quantum dot in the magnetic field. Furthermore the structure in TEP's is clearly observed even when the transmission zero is well separated from resonance peaks. This can not be revealed from the conductance, which may be too small to accurately measure in the tail of these peaks.

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