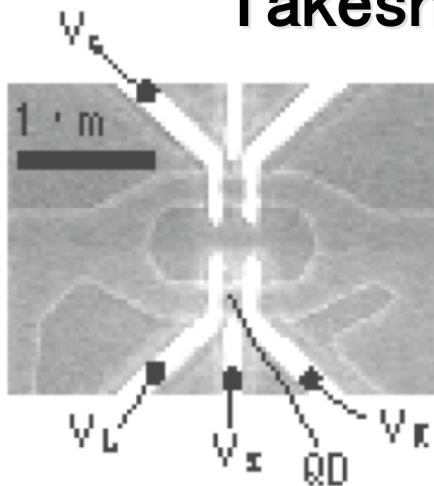


# Title: Persistence of Fano and Aharonov–Bohm phases in an interferometer with a quantum dot

Takeshi Nakanishi, Kiyoyuki Terakura (AIST)  
and Tsuneya Ando (TIT)



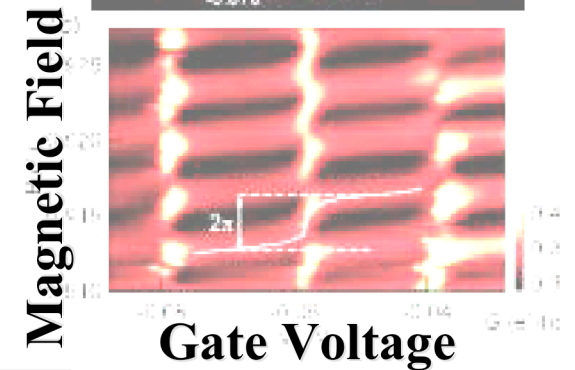
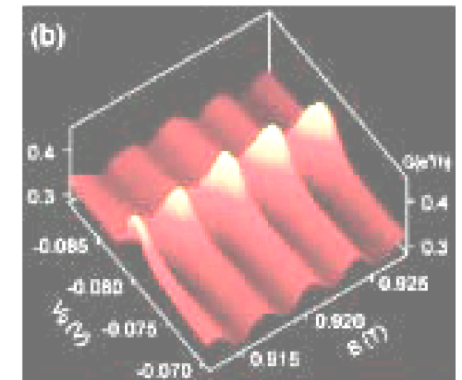
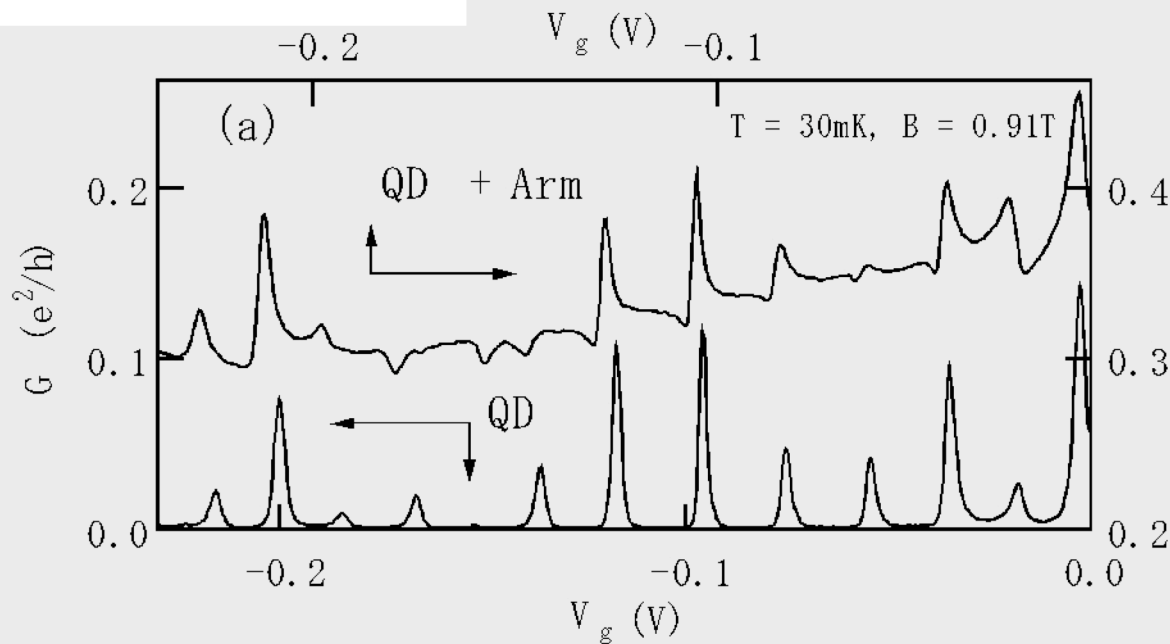
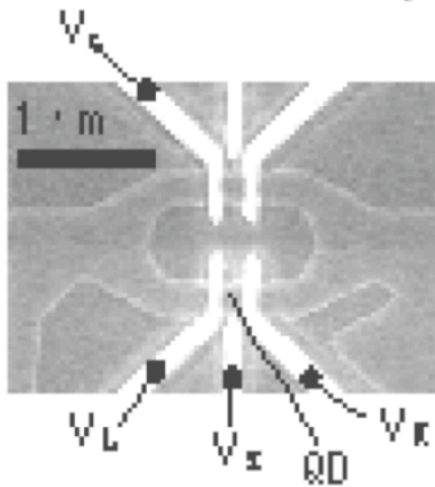
PRB 69, 115307 (2004)

# Experiment

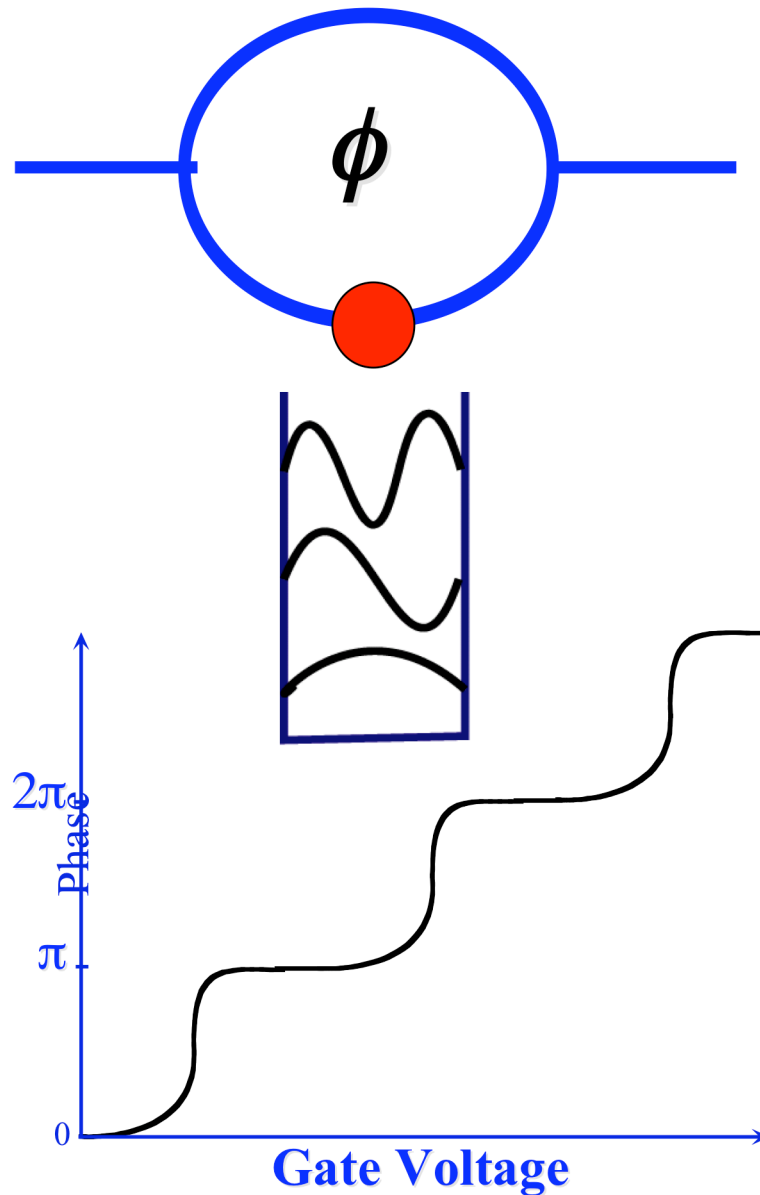
Kobayashi, Aikawa, Katsumoto, Iye, PRL 88 (2002) 256806.

◆ **Fano** (Asymmetric) line-shape

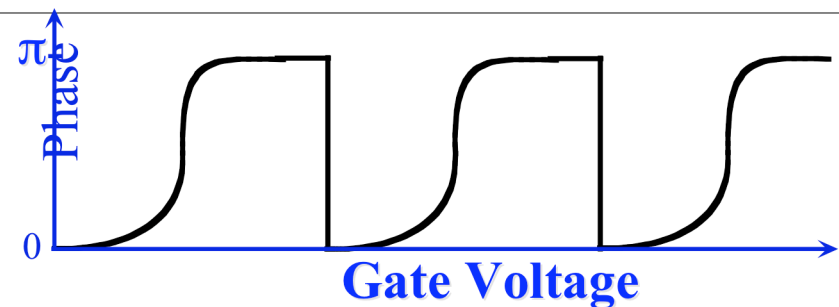
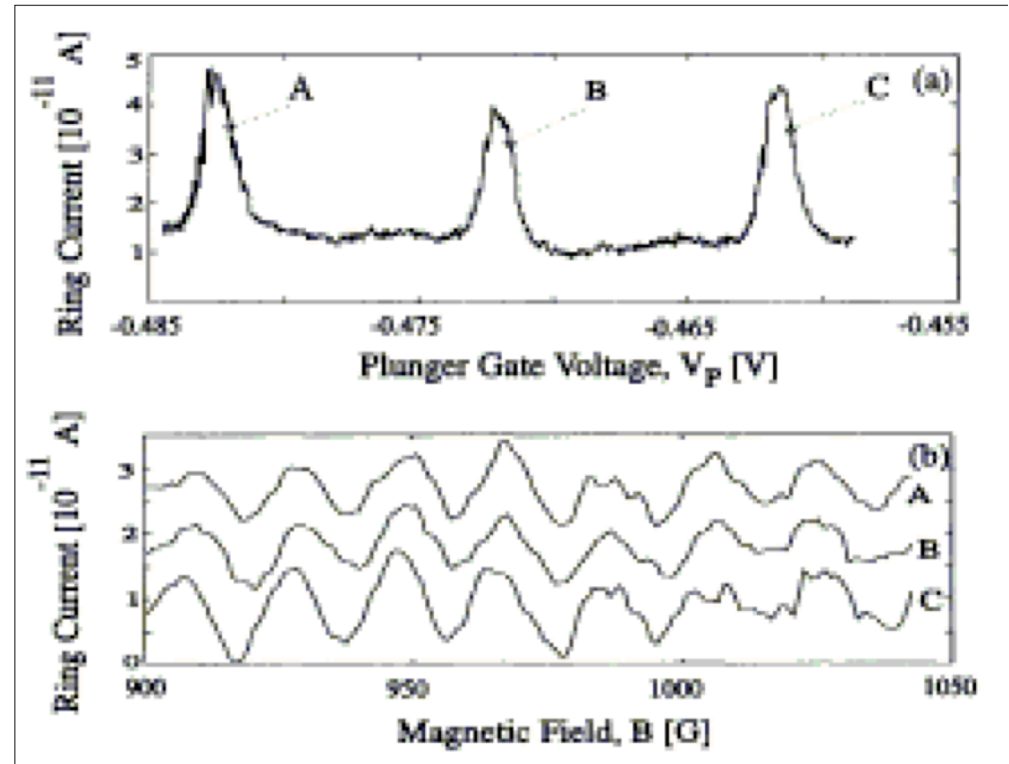
◆  **$2\pi$  shift in AB Oscillation**



# AB Phase



Yacoby *et al.* 1995, Schuster *et al.* 1997



# Fano effects (U. Fano PR 124 (1961) 1866)

## Transmission coefficients through QD

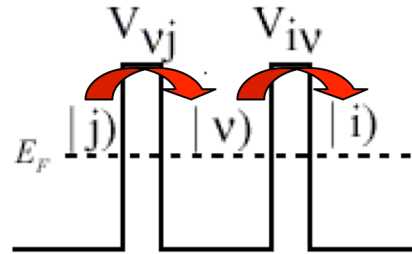
$$t_{ij}^d = \frac{\alpha_{ij}}{\epsilon + i}$$

(Breit-Wigner) with

$$\alpha_{ij} = -2\pi i V_{iv}(E) V_{vj}(E) D(E) / \Gamma,$$

$$\epsilon = (E - E_v - F) / \Gamma,$$

Level shift  $F$  and width  $\Gamma$



Phase information may be gotten from interference effects; AB and Fano effect.

## Double-slit condition

$$t_{ij} = t_{ij}^0 + t_{ij}^d,$$

$t_{ij}^0$ : continuum (constant)

$$\frac{|t_{ij}|^2}{|t_{ij}^0|^2} = \frac{|\epsilon + q_{ij}|^2}{\epsilon^2 + 1},$$

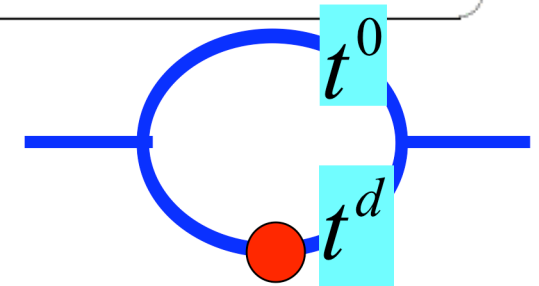
with

$$q_{ij} = \frac{\alpha_{ij}}{t_{ij}^0} + i.$$

## Conductance

$$G = \frac{e^2}{\pi \hbar} T_0 \frac{|\epsilon + q|^2}{\epsilon^2 + 1},$$

$$q = q' + iq''$$



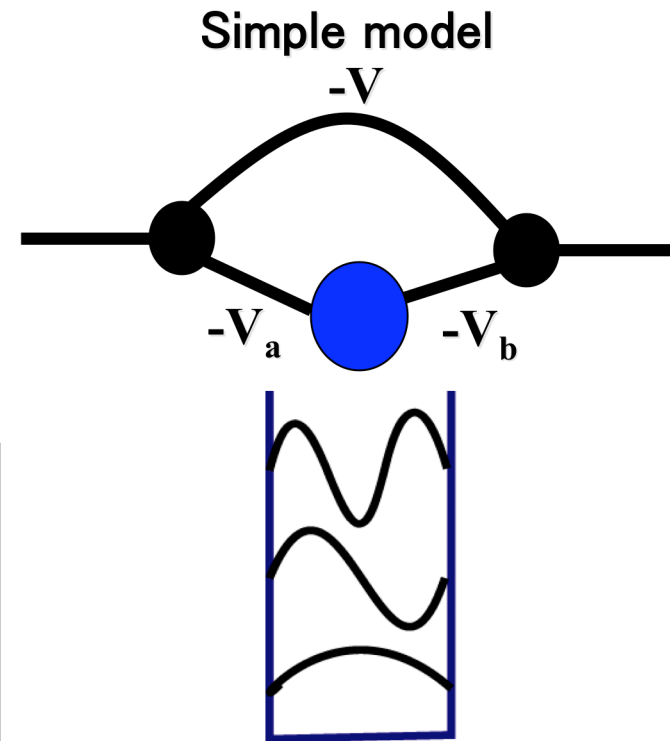
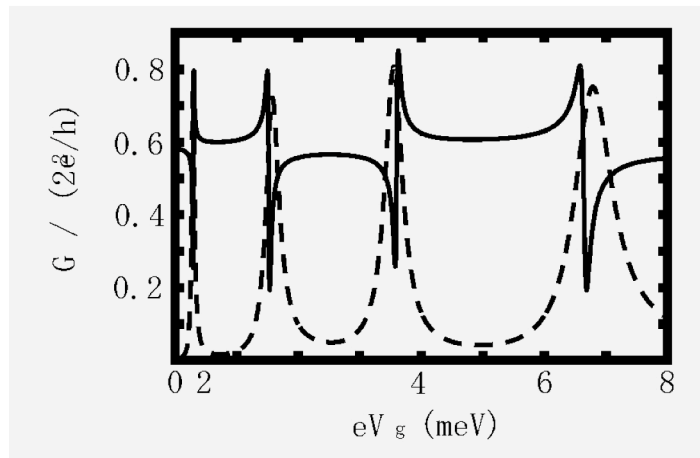


# Fano & AB in 1D Model

## Alternative Fano $q$ & $\pi$ shift in AB

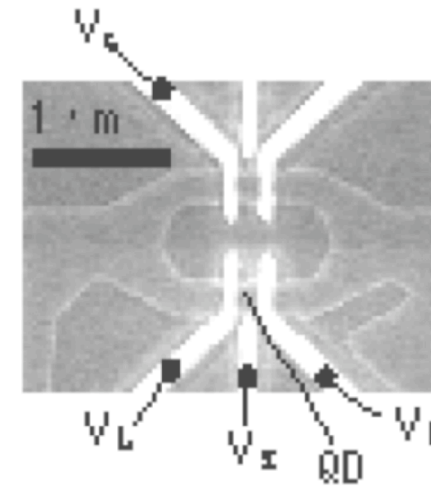
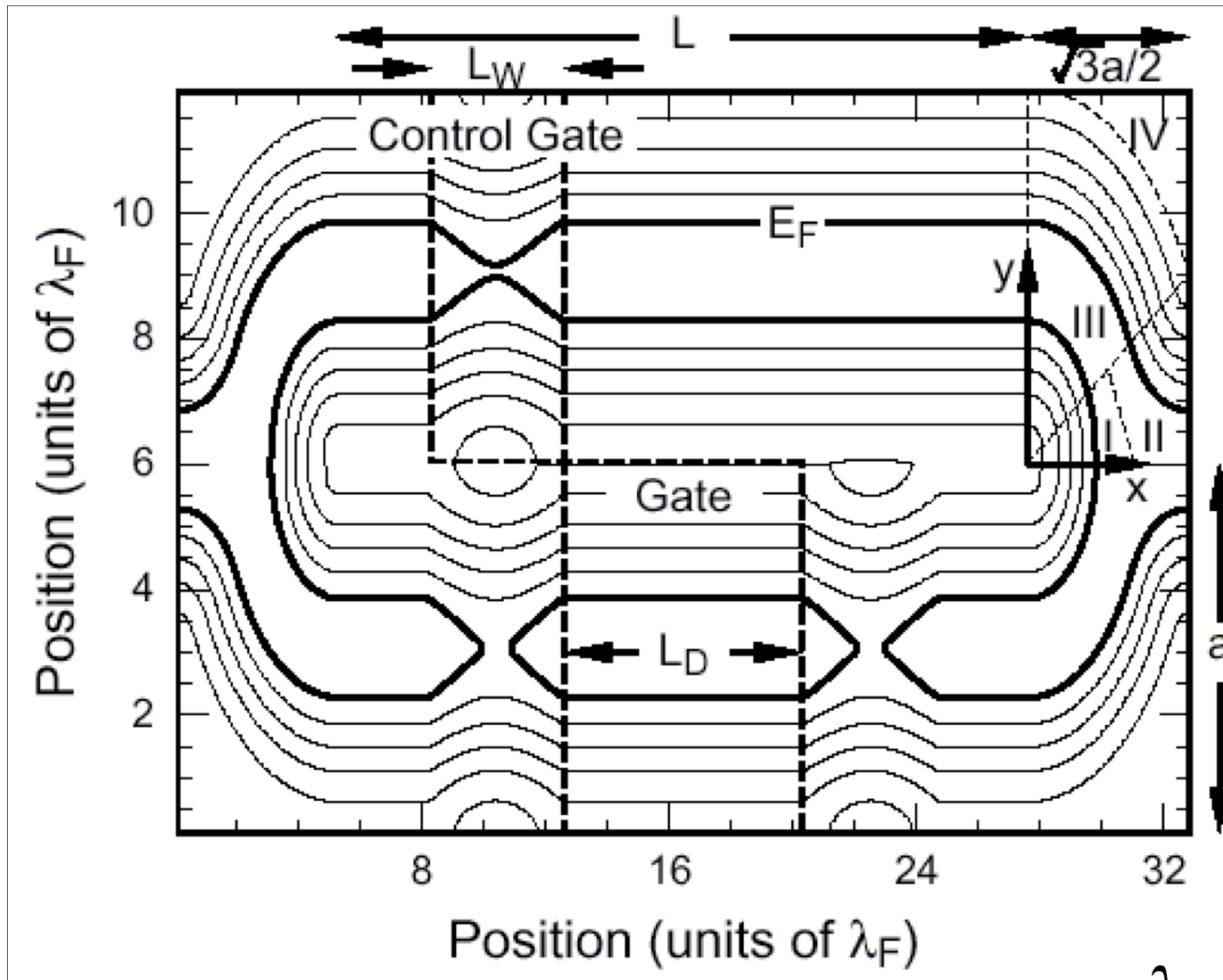
Entin-Wohlman, *et al.*, J. Low. Temp. Phys. 126 (2002) 1251

Ueda, *et al.*, J. Phys. Soc. Jpn 72 Suppl. A (2003) 157



	Fano $q$	AB
1D Model	+--+	$\pi$
Experiment	++++	$2\pi$

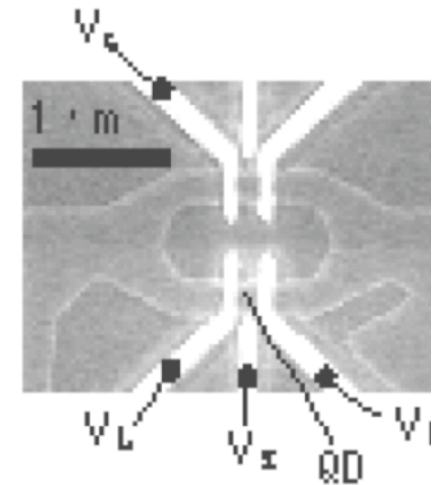
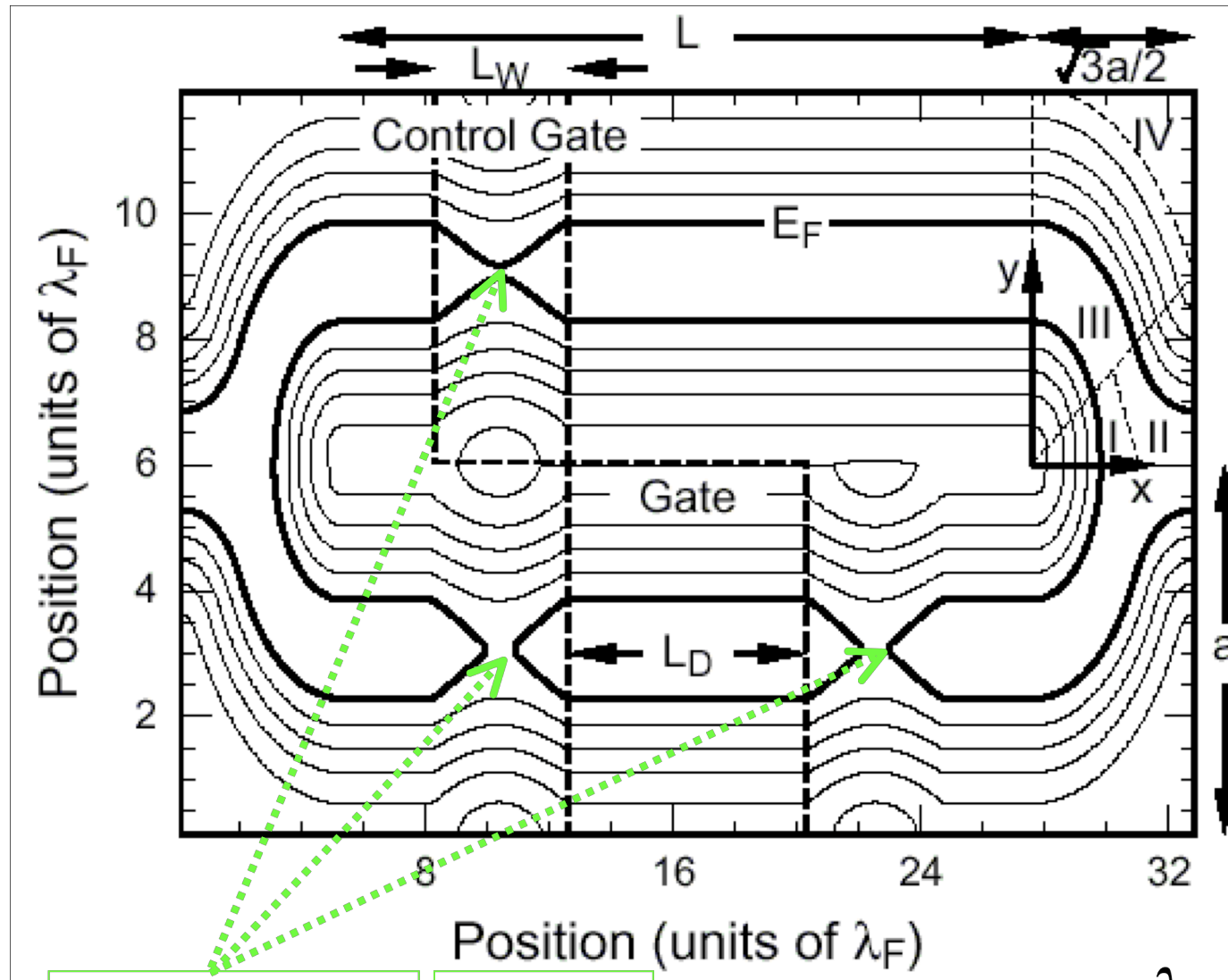
## Realistic Model (Equi-potential lines)



**3 channels  
in arms  
and leads**

$\lambda_F$  Fermi wave length

## Realistic Model (Equi-potential lines)



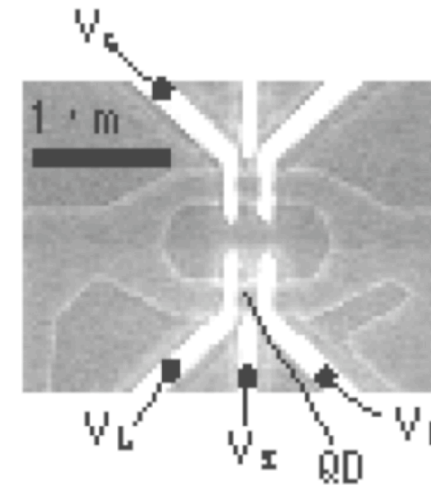
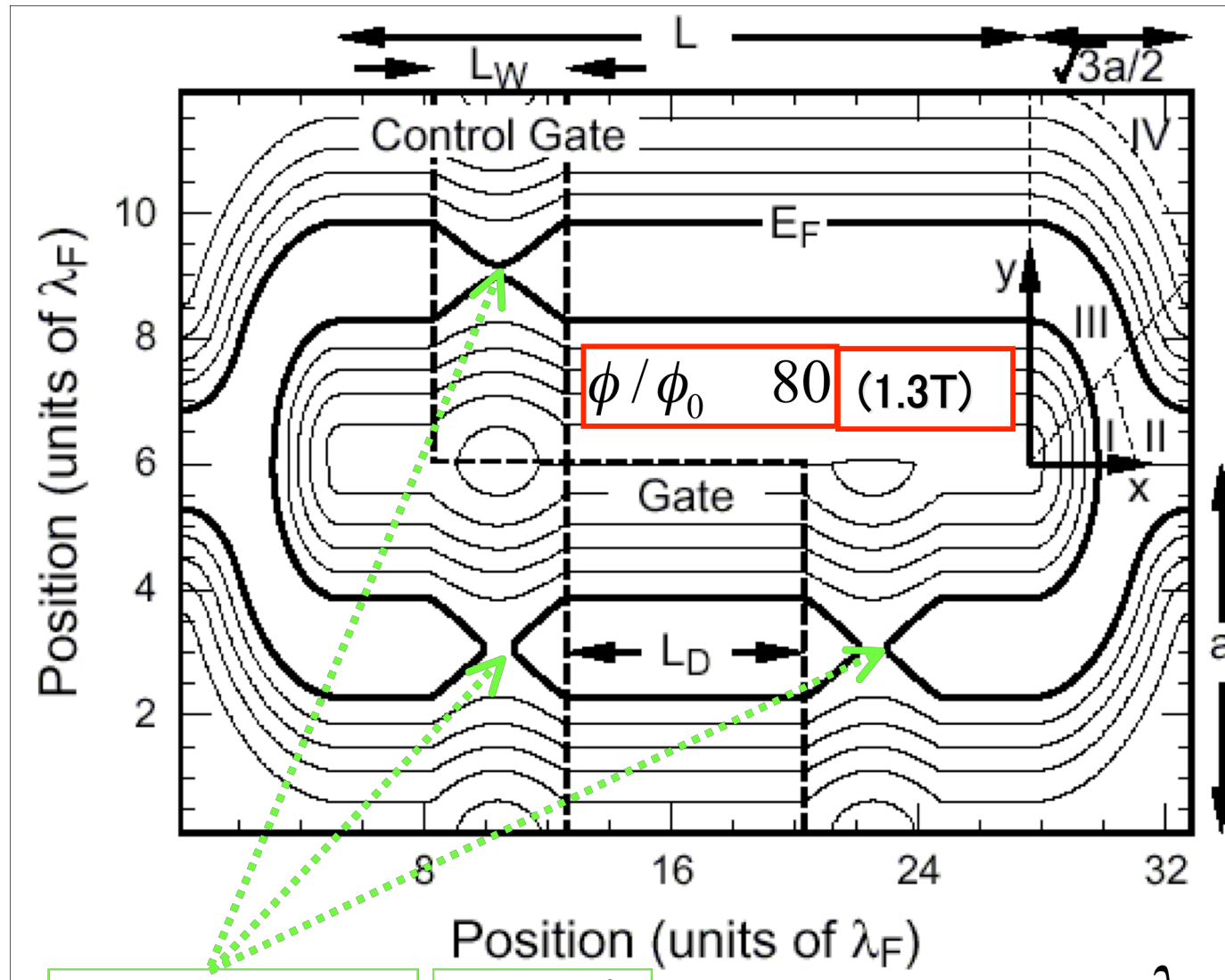
**3 channels  
in arms  
and leads**

Adiabatic Walls

$$L_w = 5\lambda_F$$

$\lambda_F$  Fermi wave length

## Realistic Model (Equi-potential lines)



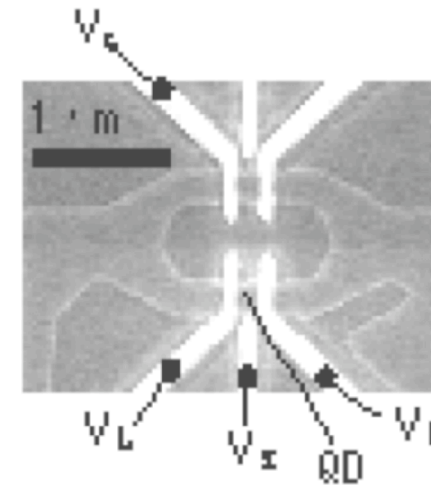
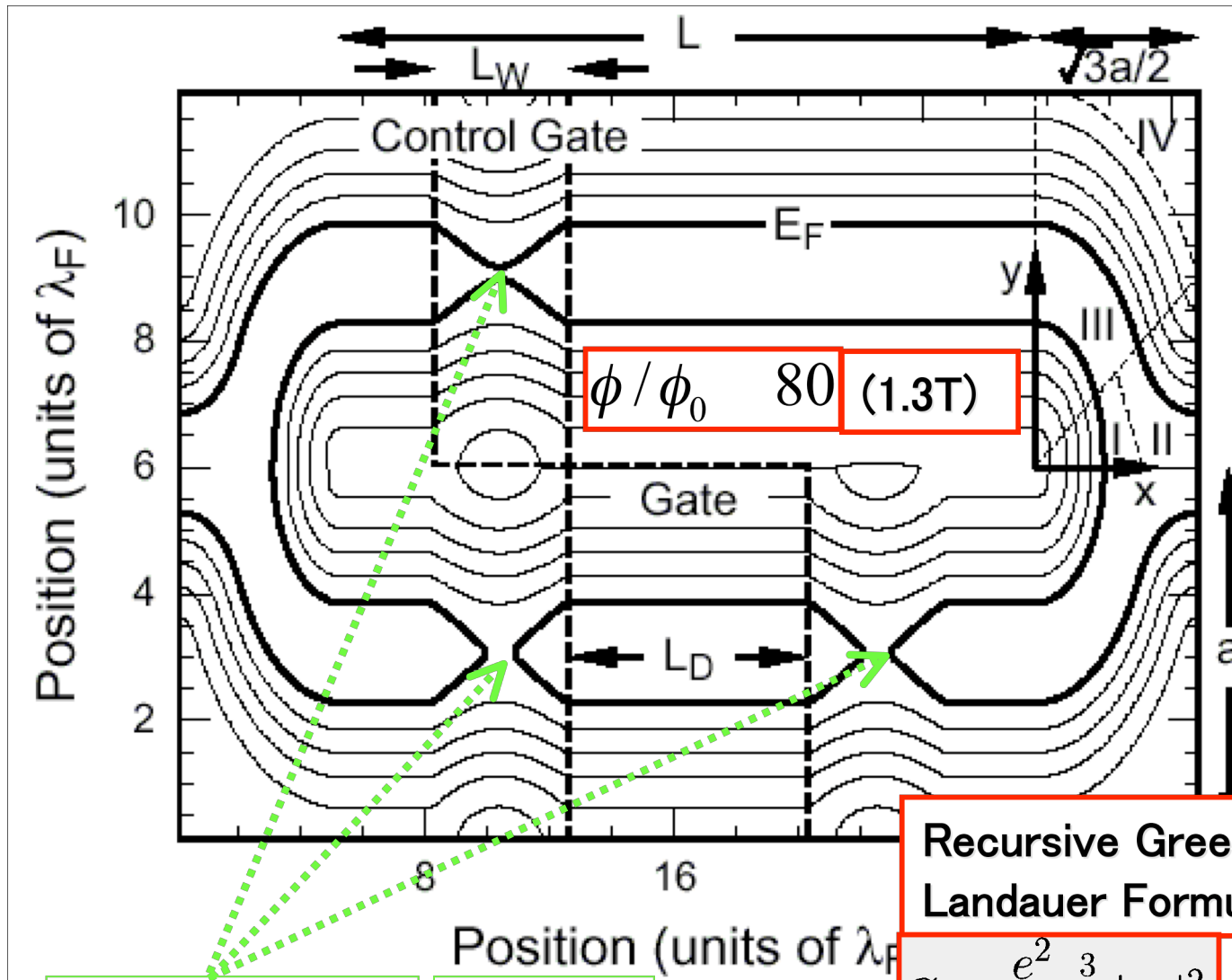
**3 channels  
in arms  
and leads**

**Adiabatic Walls**

$$L_w = 5\lambda_F$$

$\lambda_F$  Fermi wave length

# Realistic Model (Equi-potential lines)



**3 channels  
in arms  
and leads**

Recursive Green's function method  
Landauer Formula

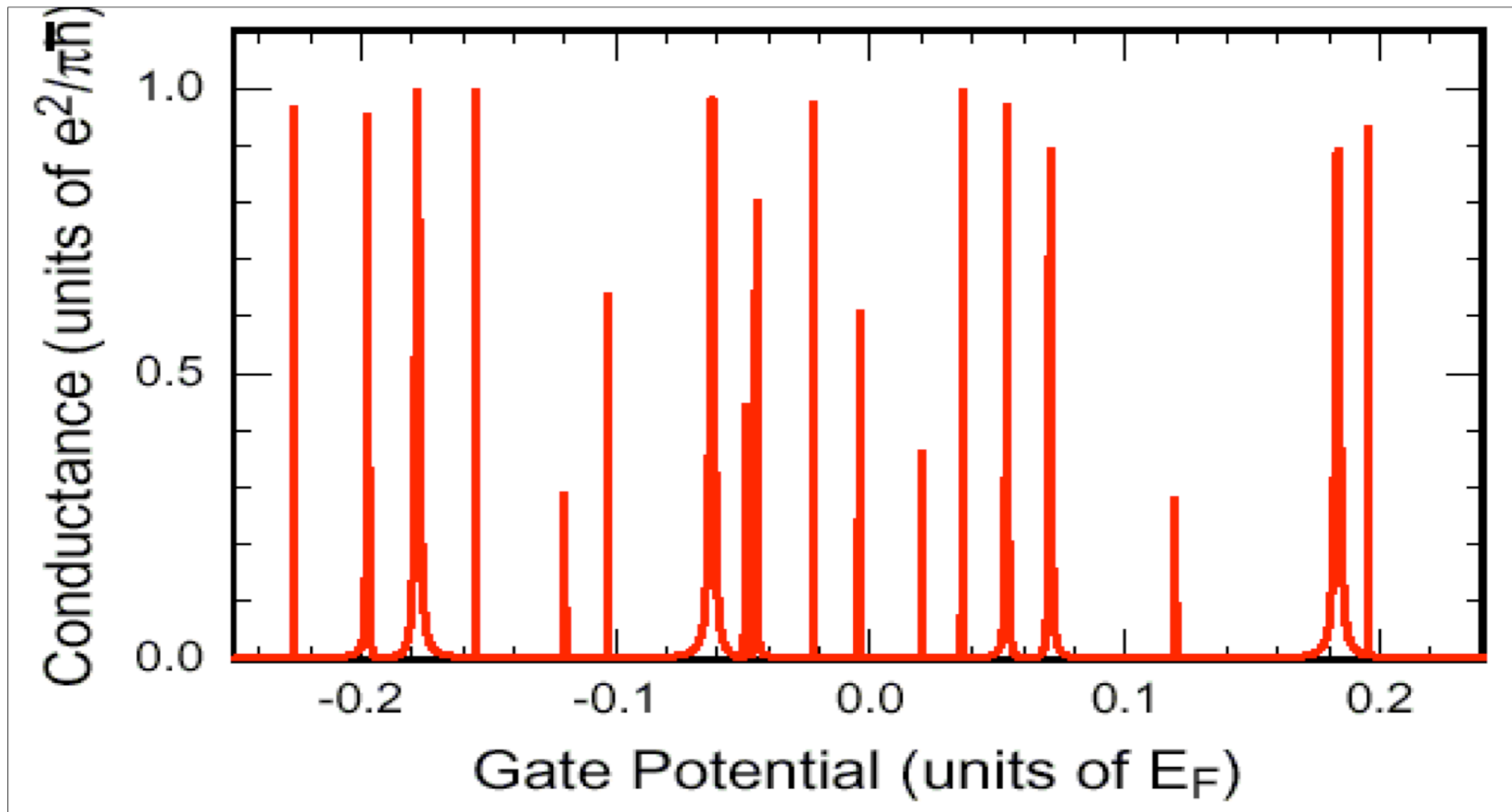
$$G = \frac{e^2}{\pi \hbar} \sum_{ij}^3 |t_{ij}|^2$$

Fermi wave length

Adiabatic Walls

$$L_w = 5\lambda_F$$

## Asymmetry in sequence

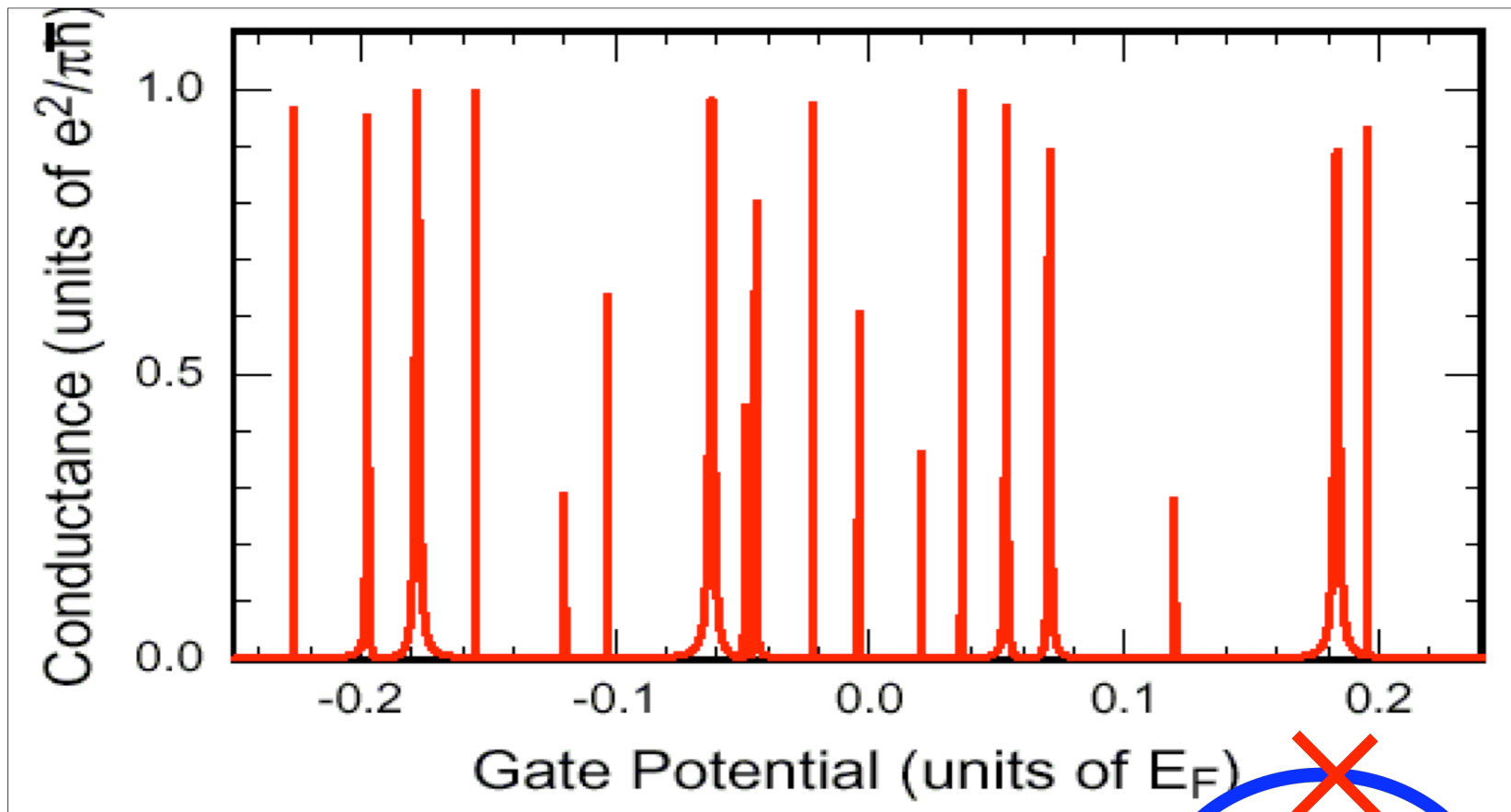


AB ring



Pinched Off

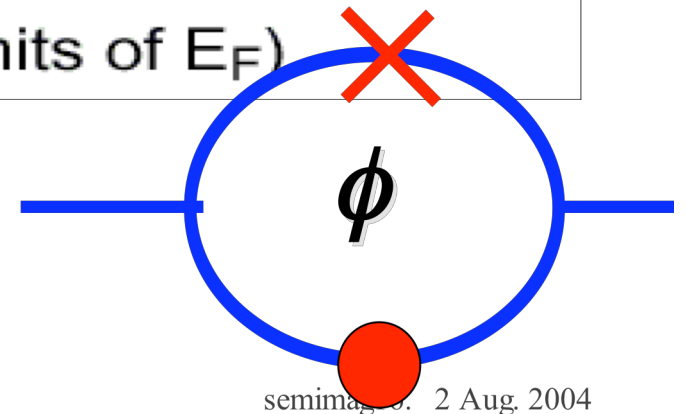
## Asymmetry in sequence



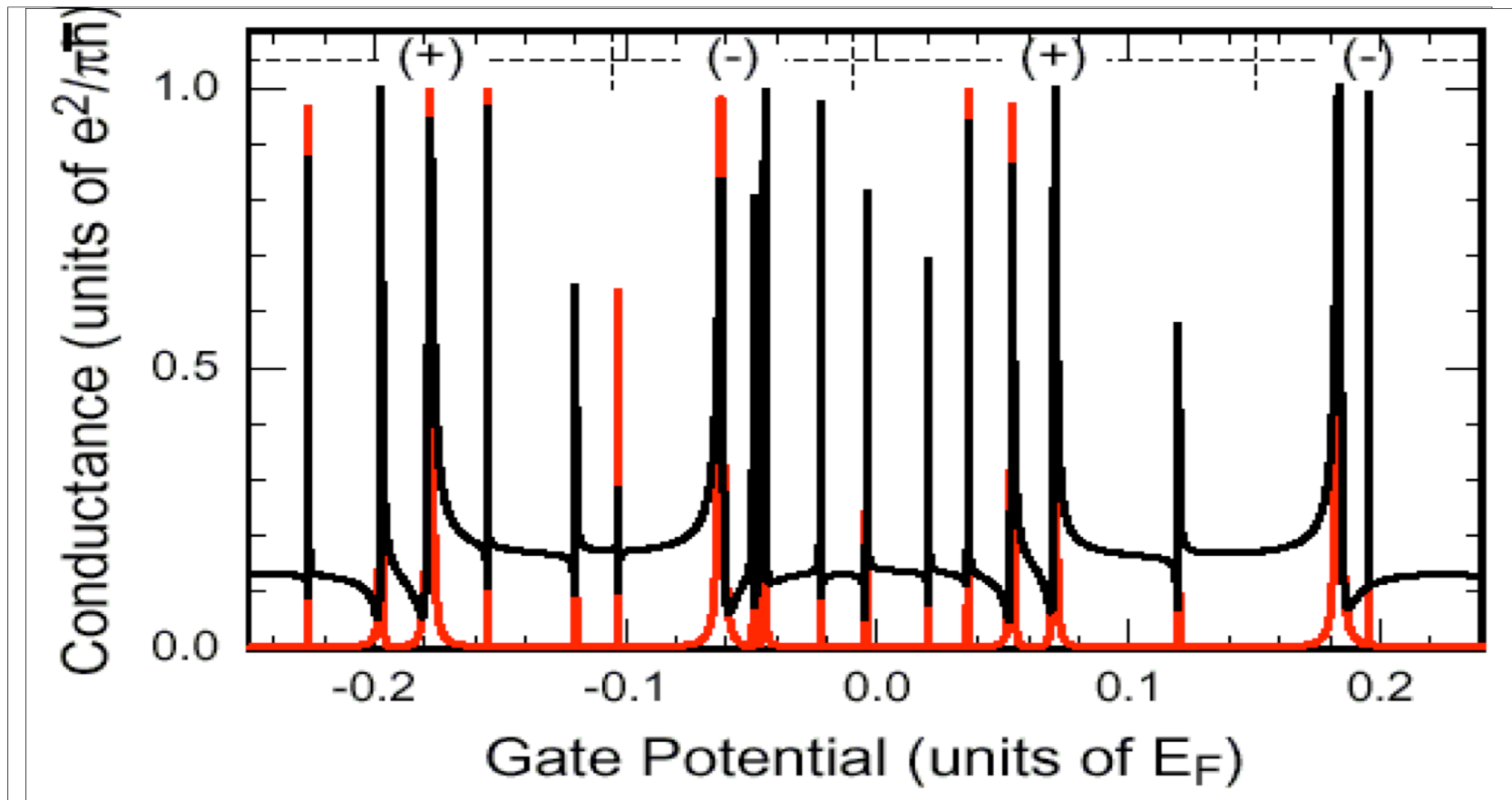
AB ring



Pinched Off



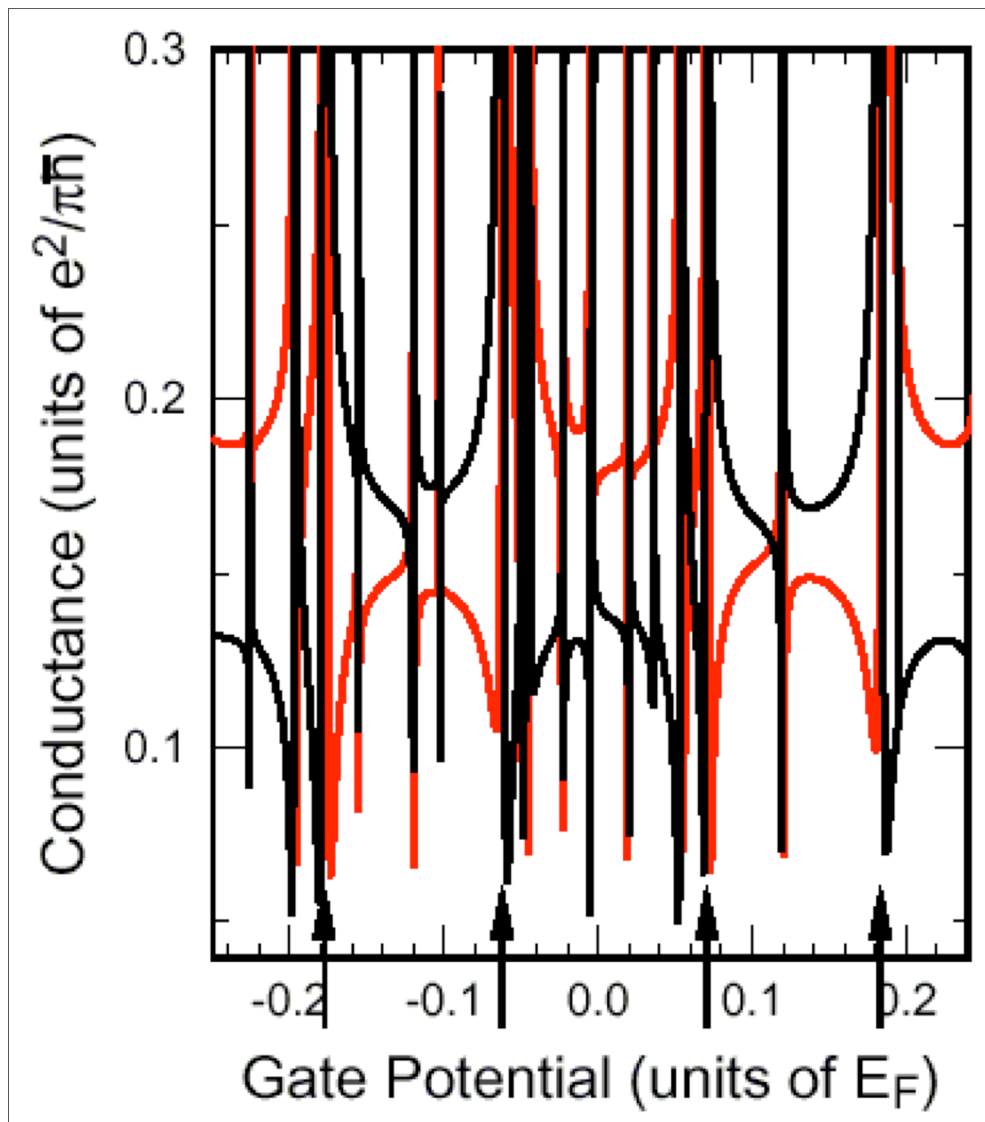
## Asymmetry in sequence



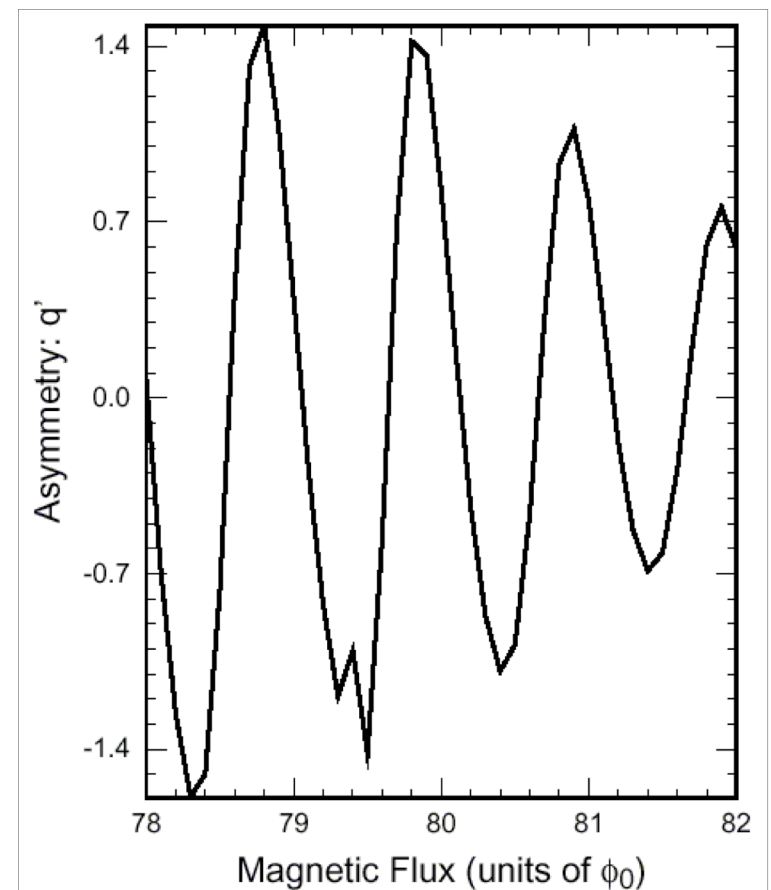
- ◆ — AB ring  $V_c = E_F$  ◆ (+)  $q > 0$  dip in left
- ◆ — Pinched Off  $V_c = 2E_F$  ◆ (-)  $q < 0$  dip in right



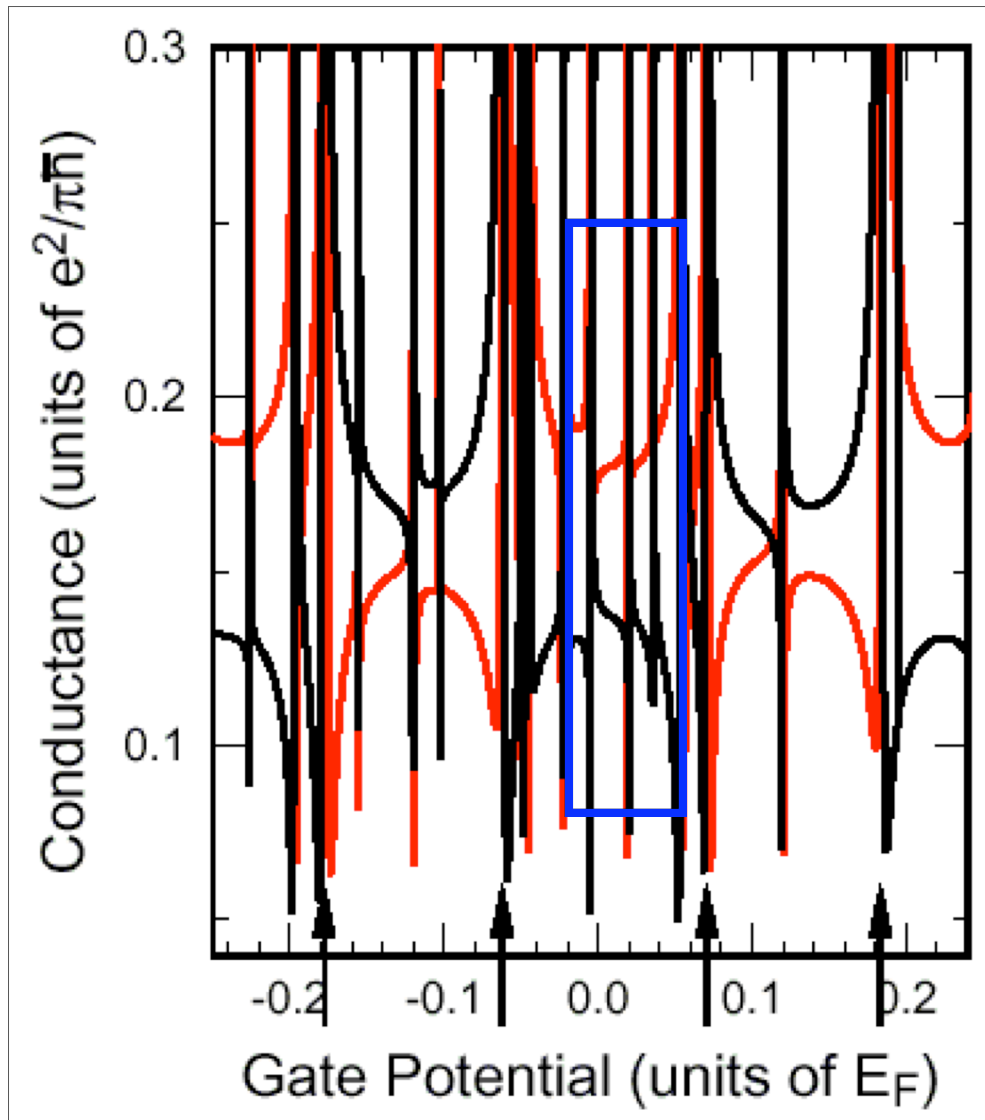
# AB oscillation



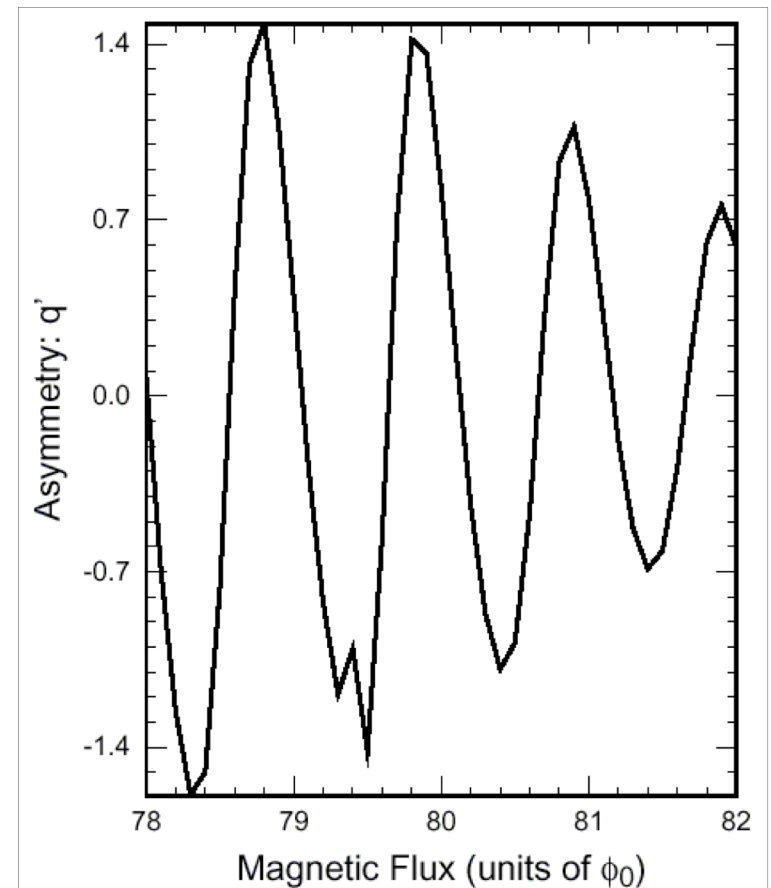
—  $\phi / \phi_0 = 79.9$   
—  $\phi / \phi_0 = 80.5$



# AB oscillation

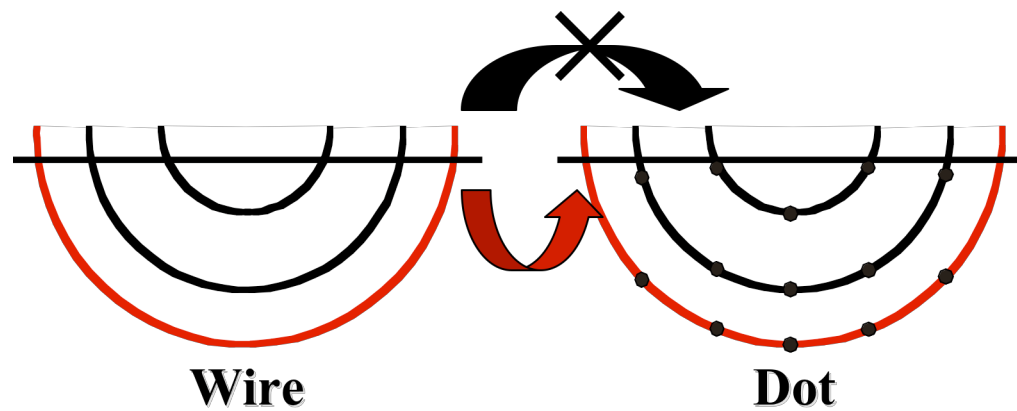


—  $\phi / \phi_0 = 79.9$   
—  $\phi / \phi_0 = 80.5$



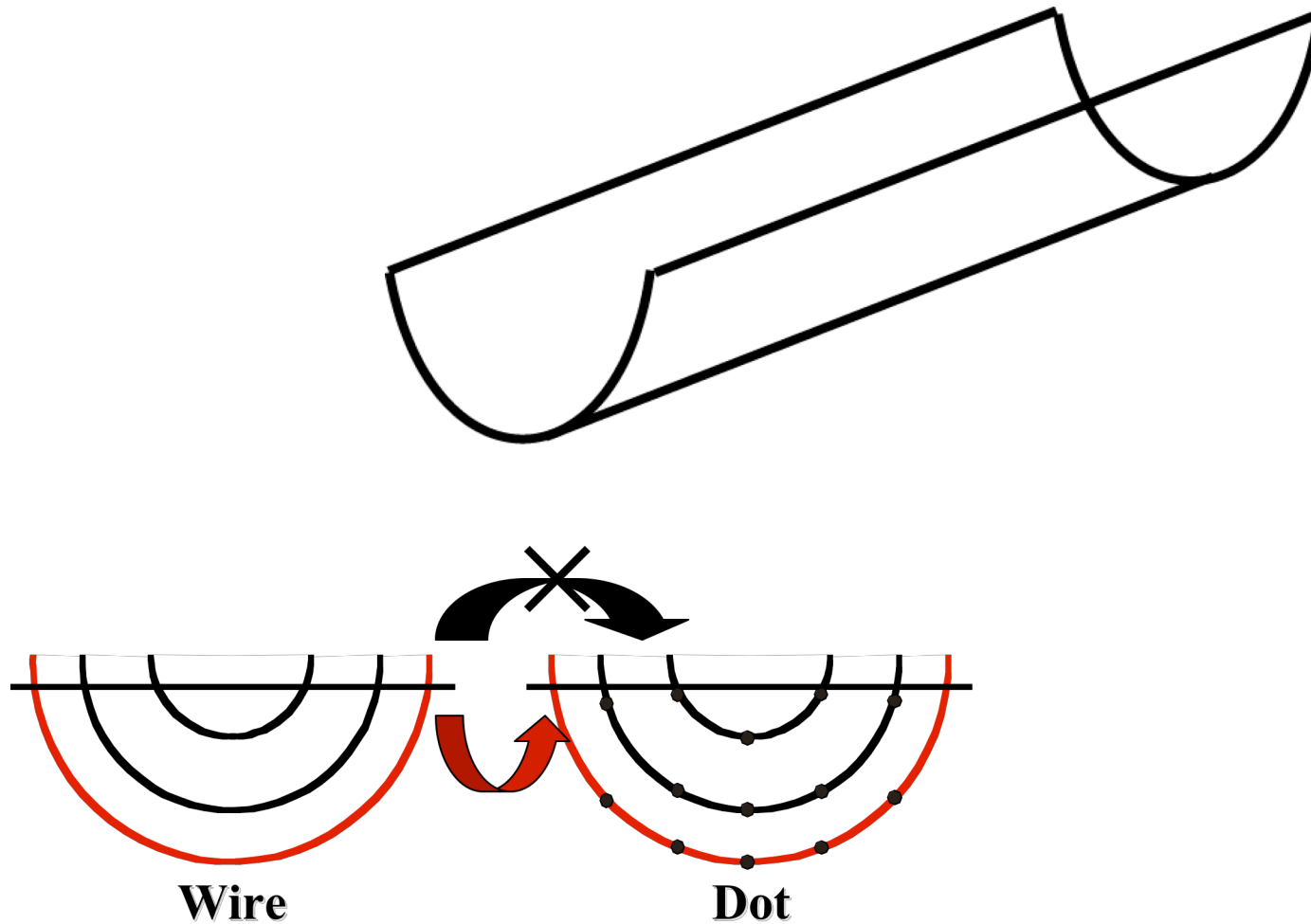
# Broad and Narrow Peaks

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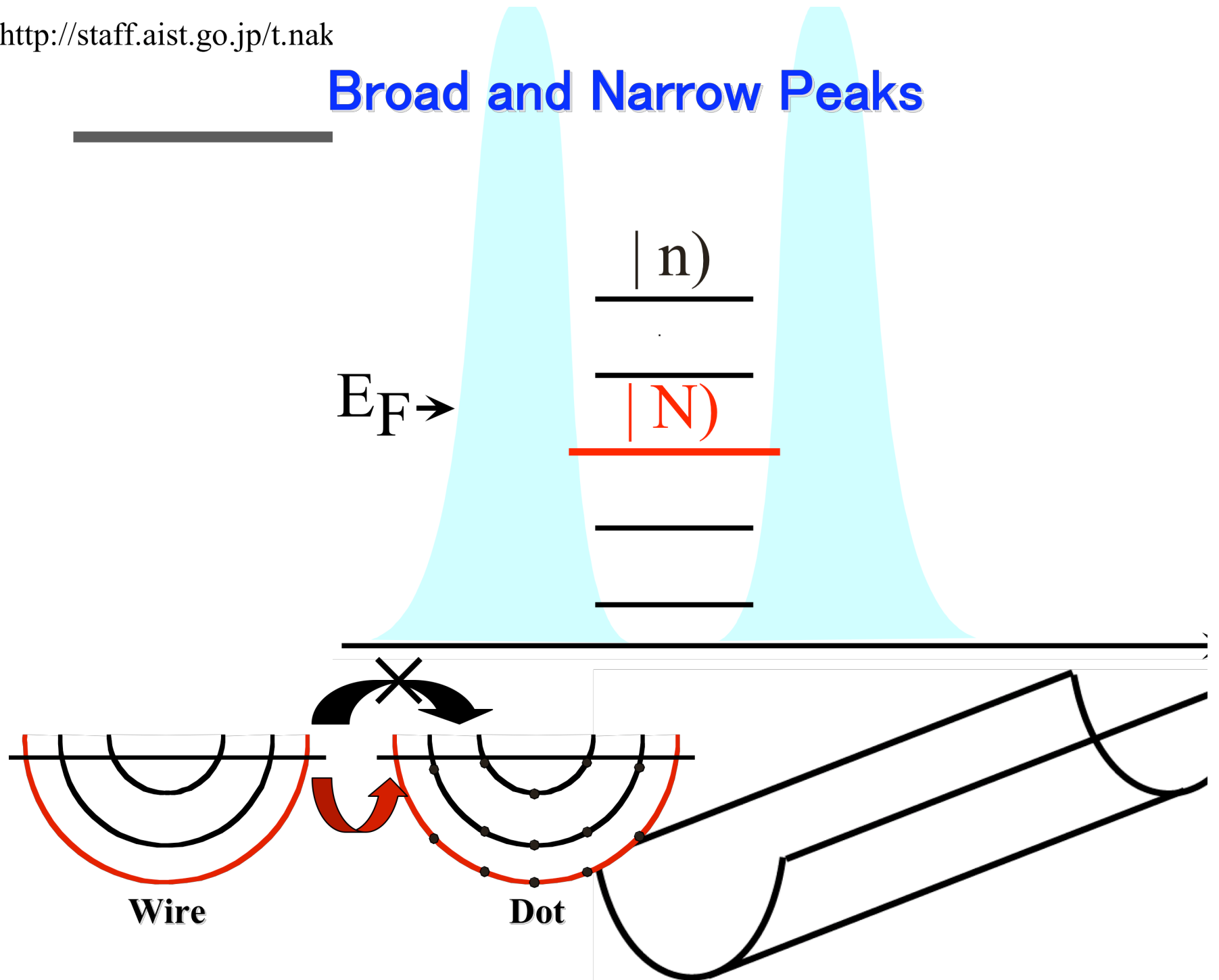


# Broad and Narrow Peaks

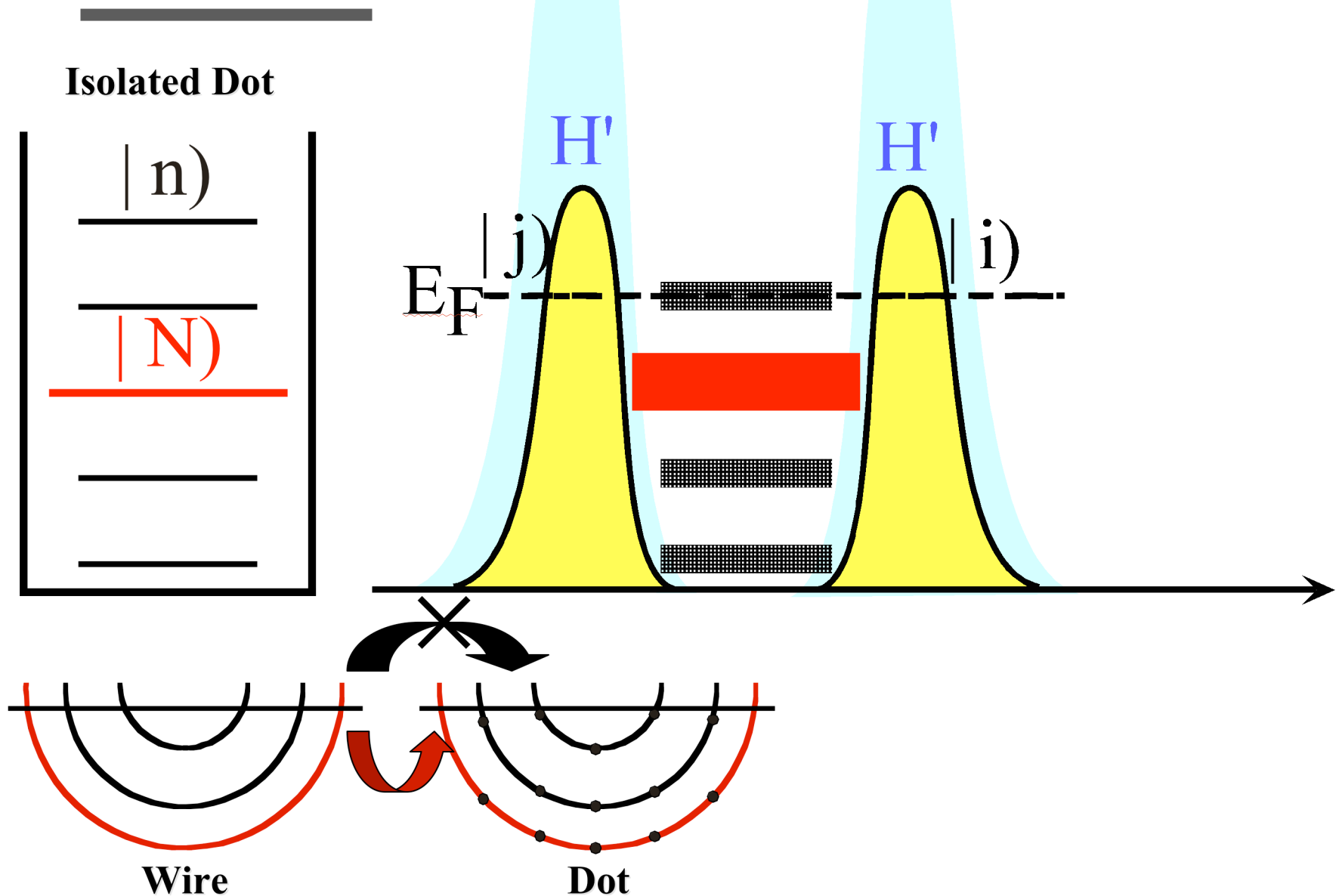
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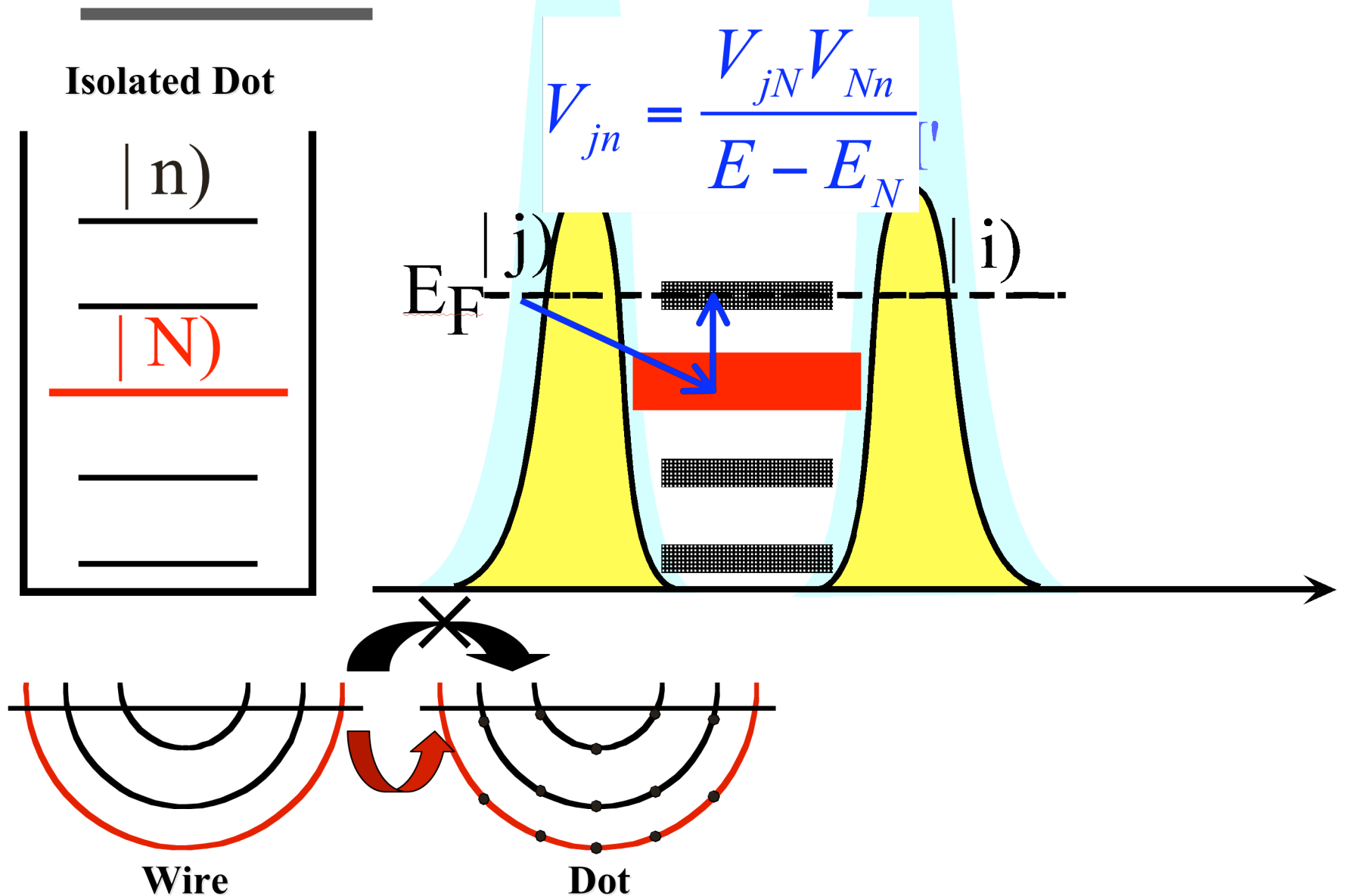
# Broad and Narrow Peaks



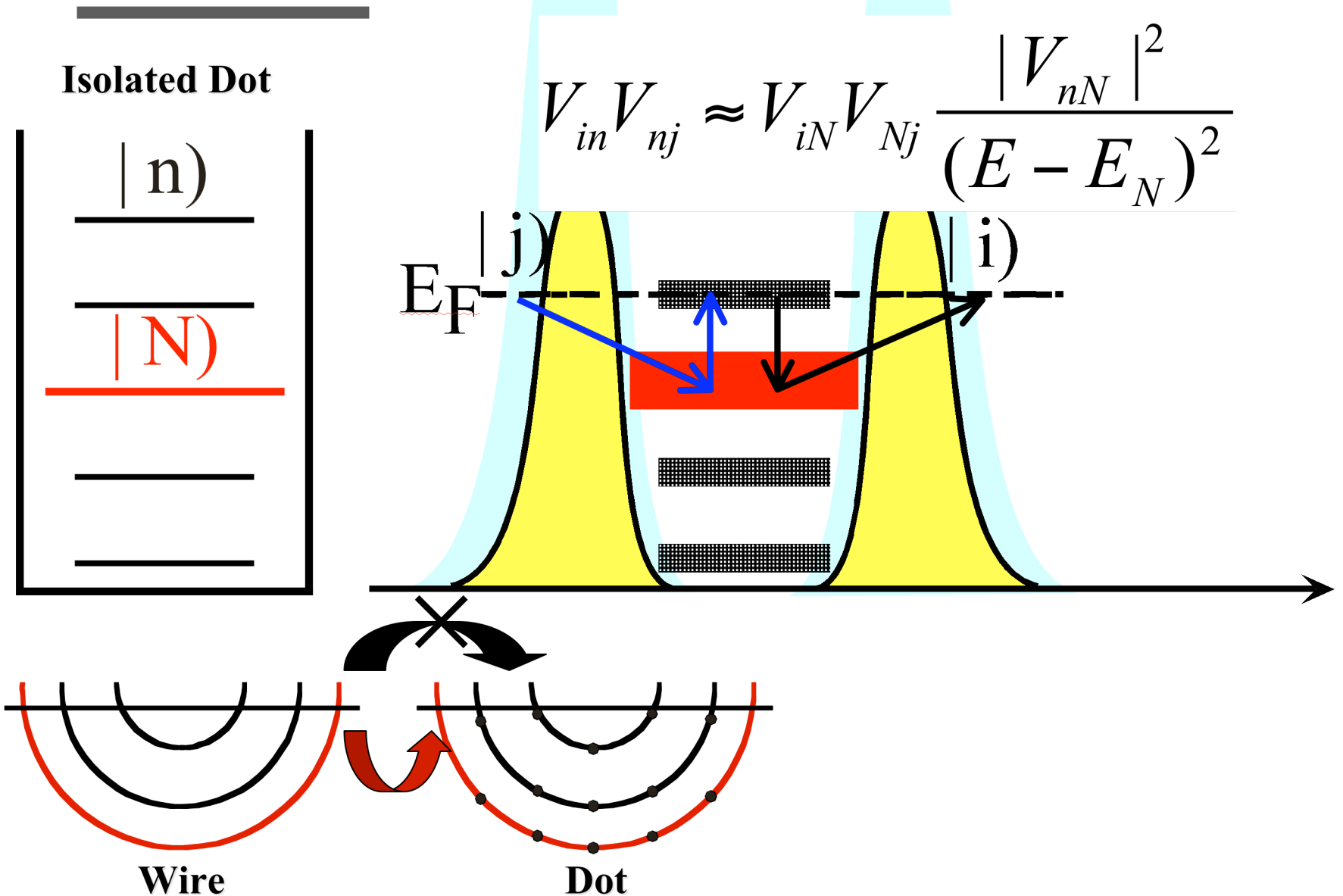
## Broad and Narrow Peaks



## Broad and Narrow Peaks

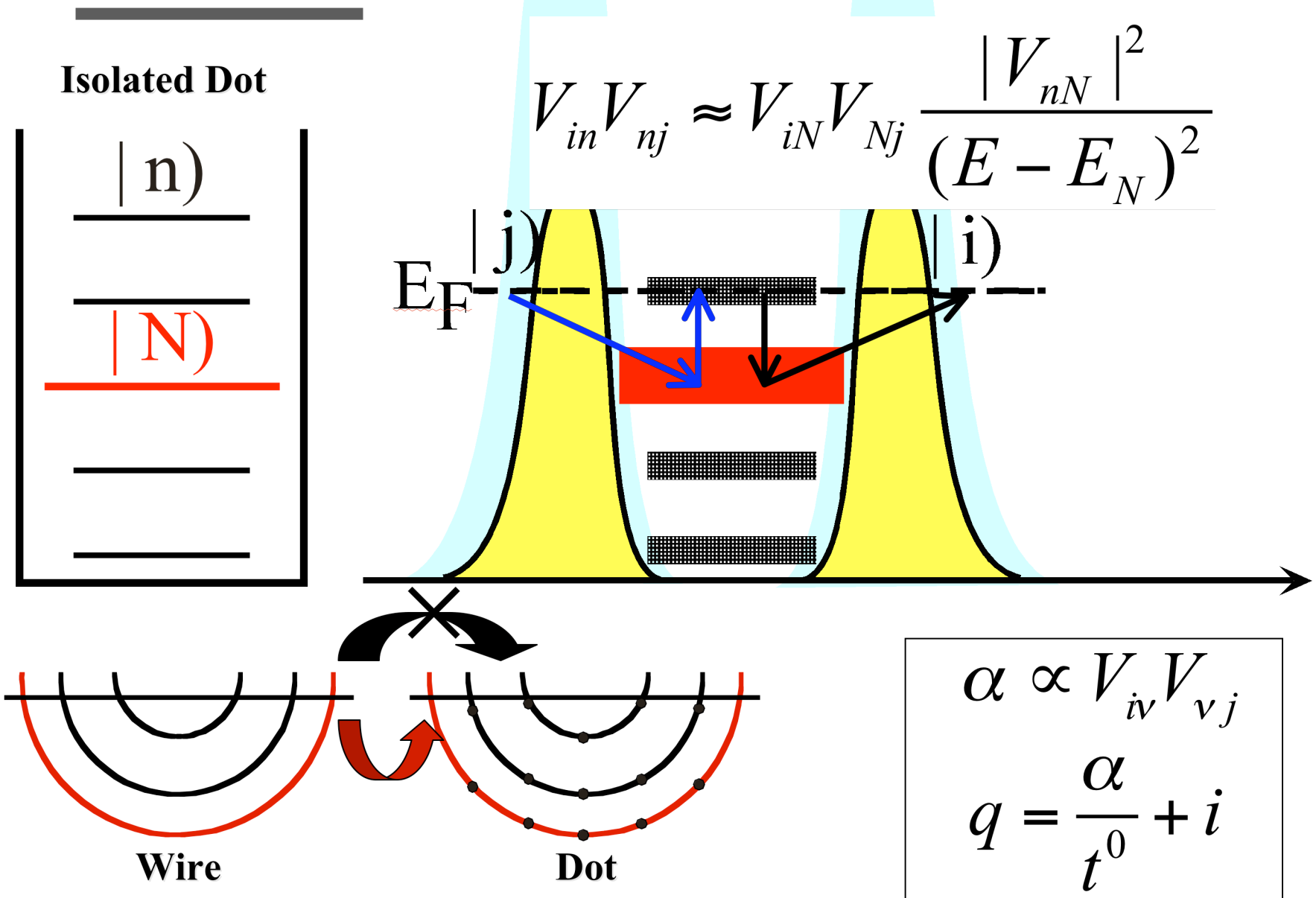


## Broad and Narrow Peaks

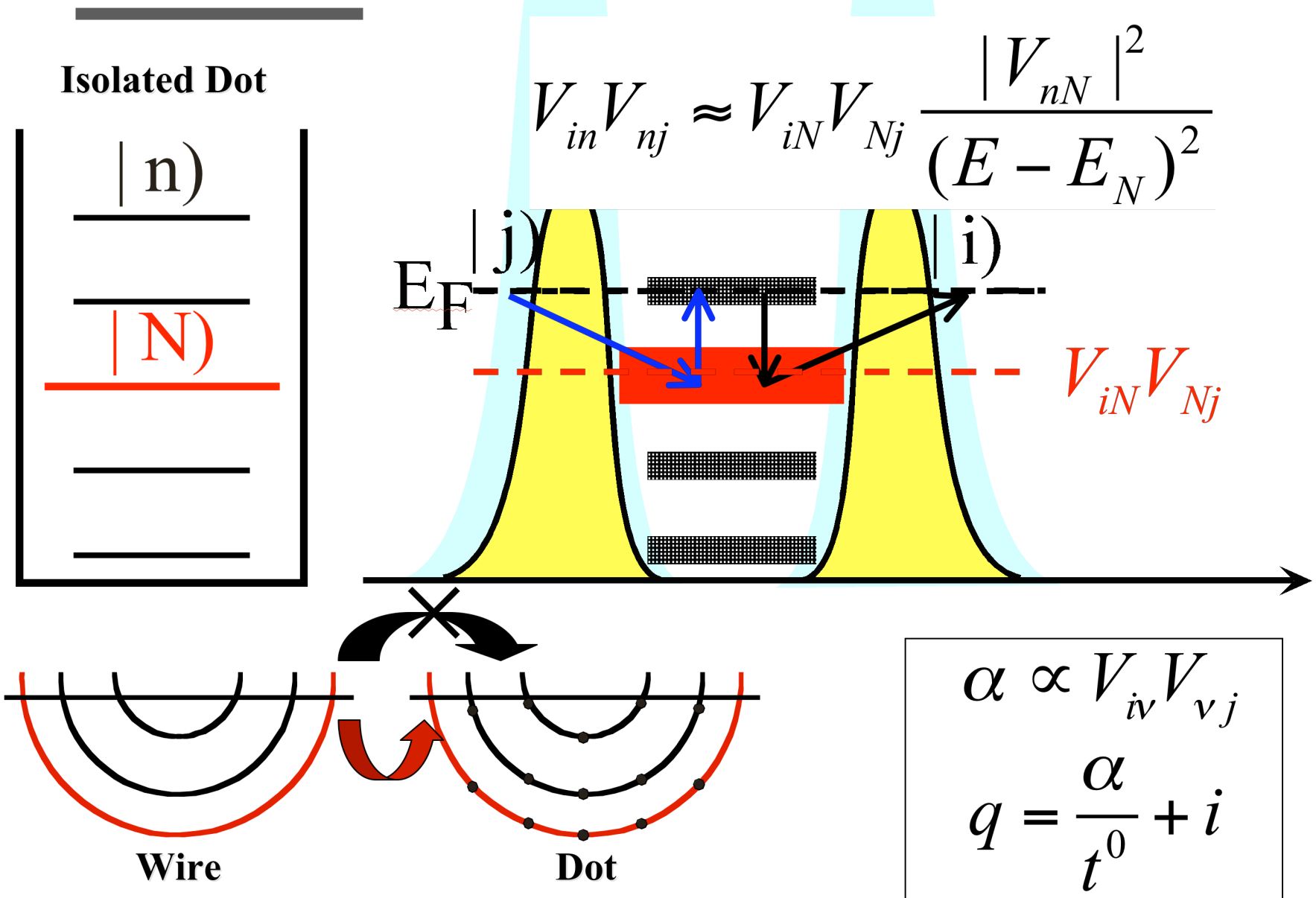




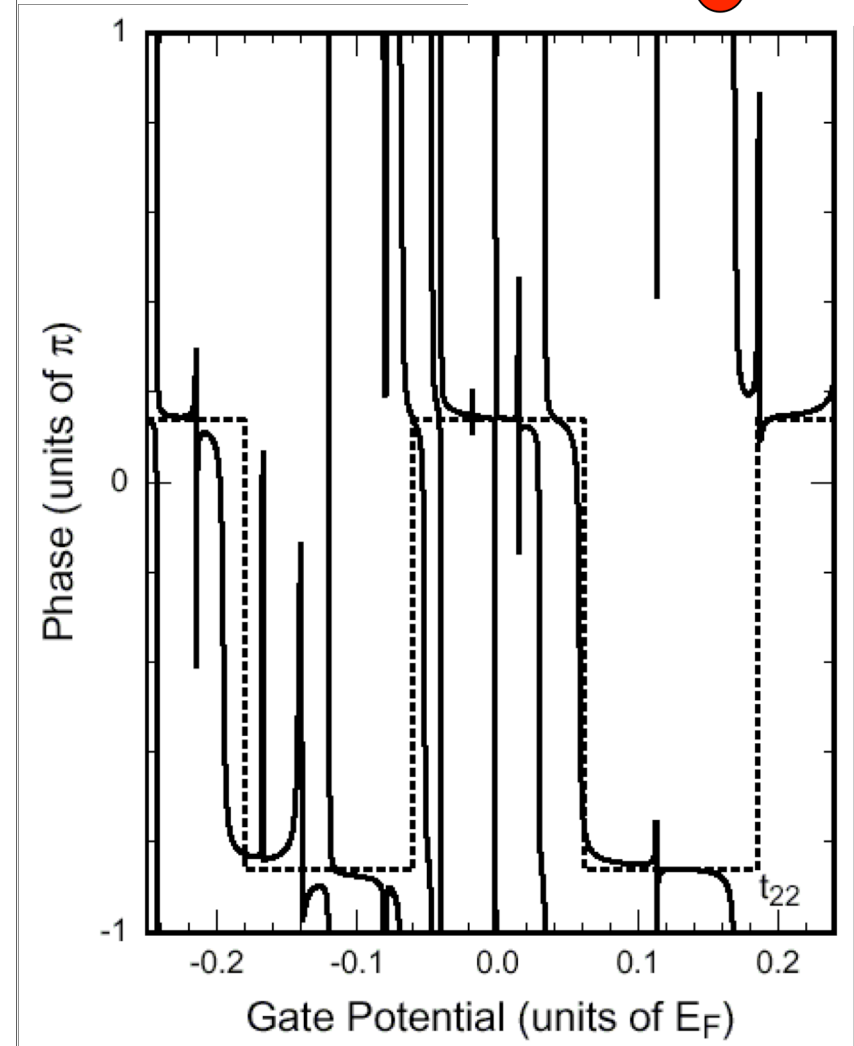
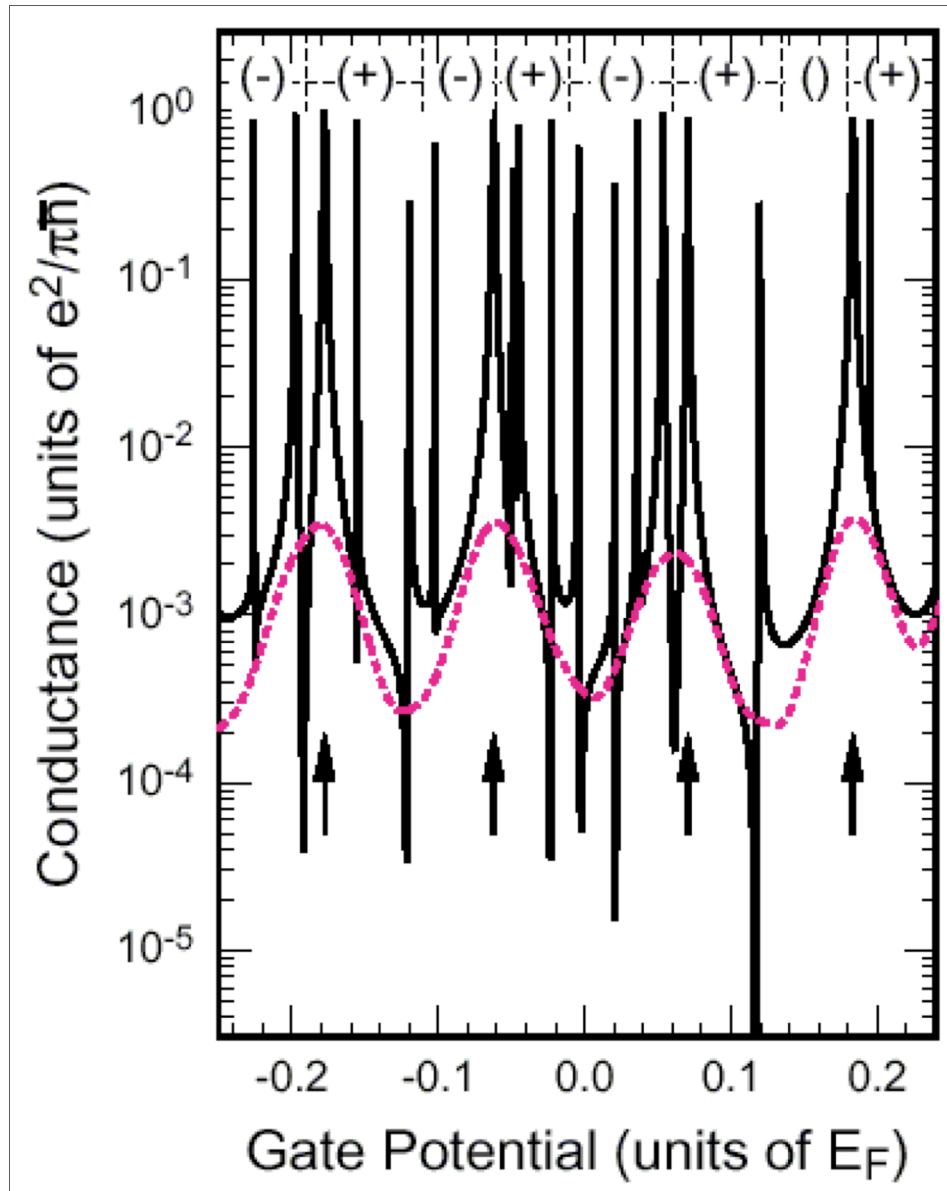
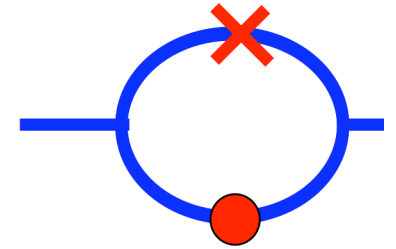
## Broad and Narrow Peaks



## Broad and Narrow Peaks



# Control Gate is Pinched Off

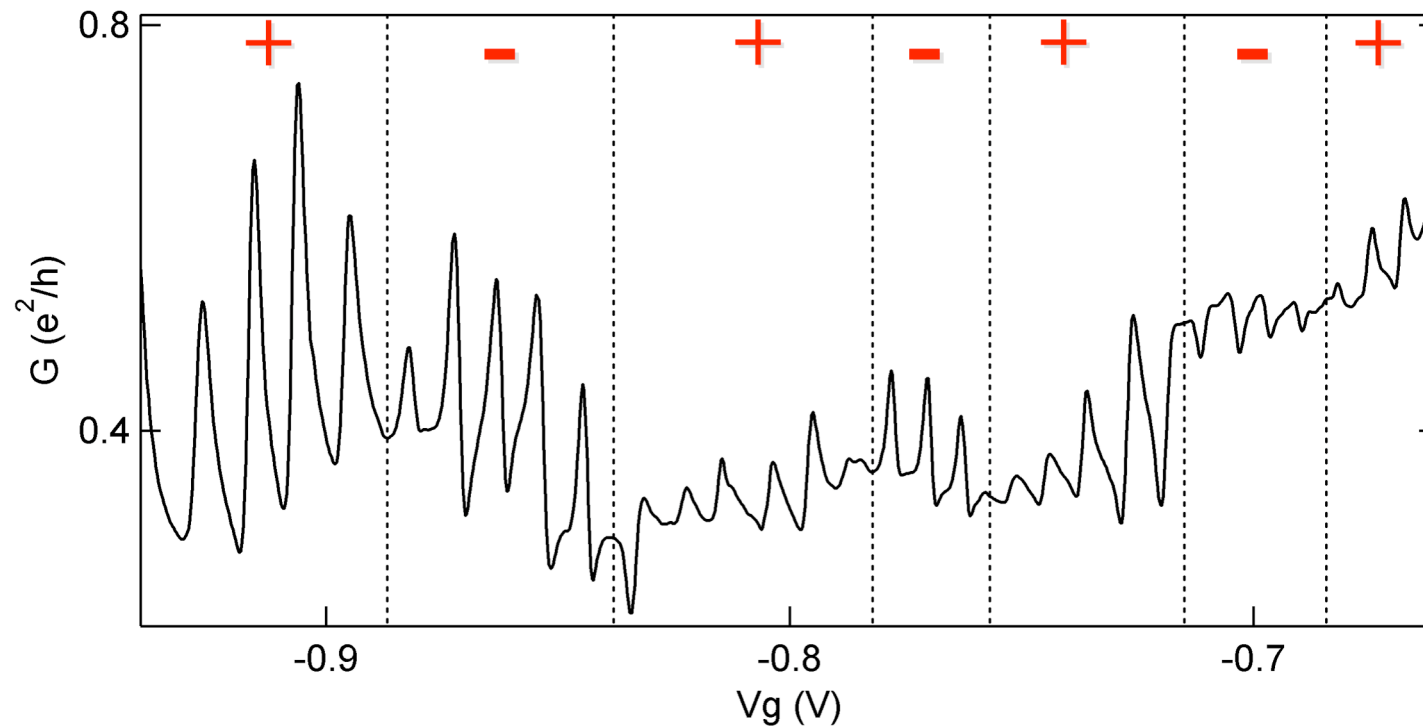
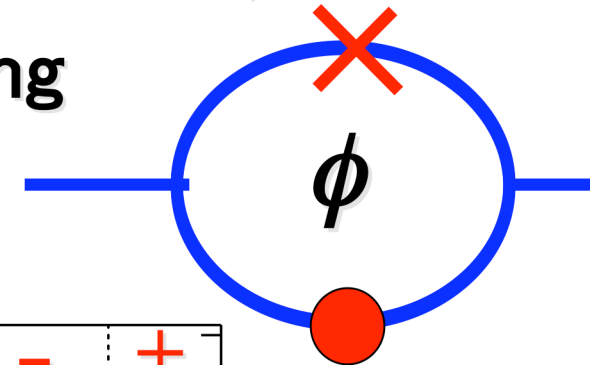


Continuum:  $|N\rangle$

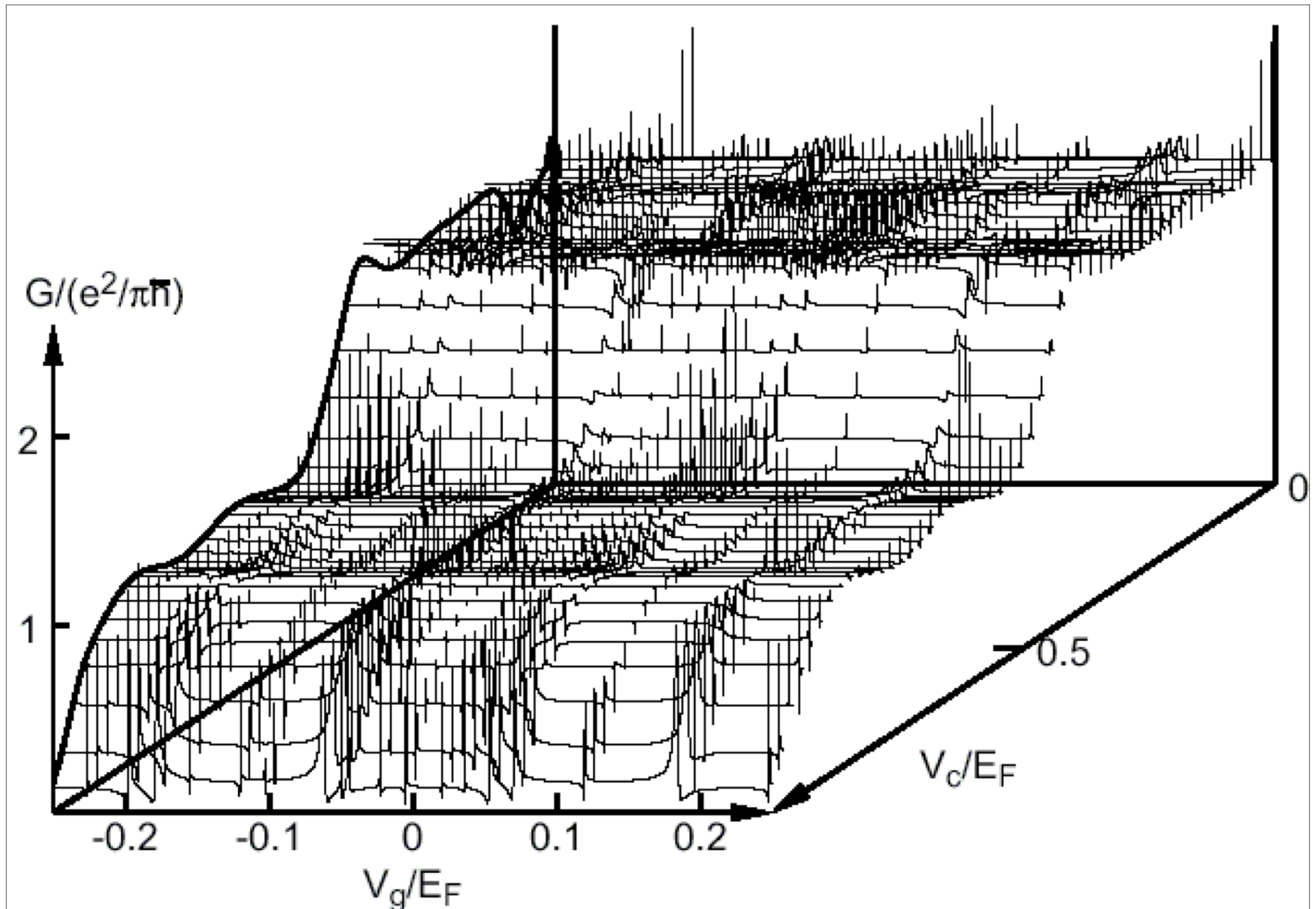
## Experiment

( H. Aikawa *et al.* cond-mat/0312431)

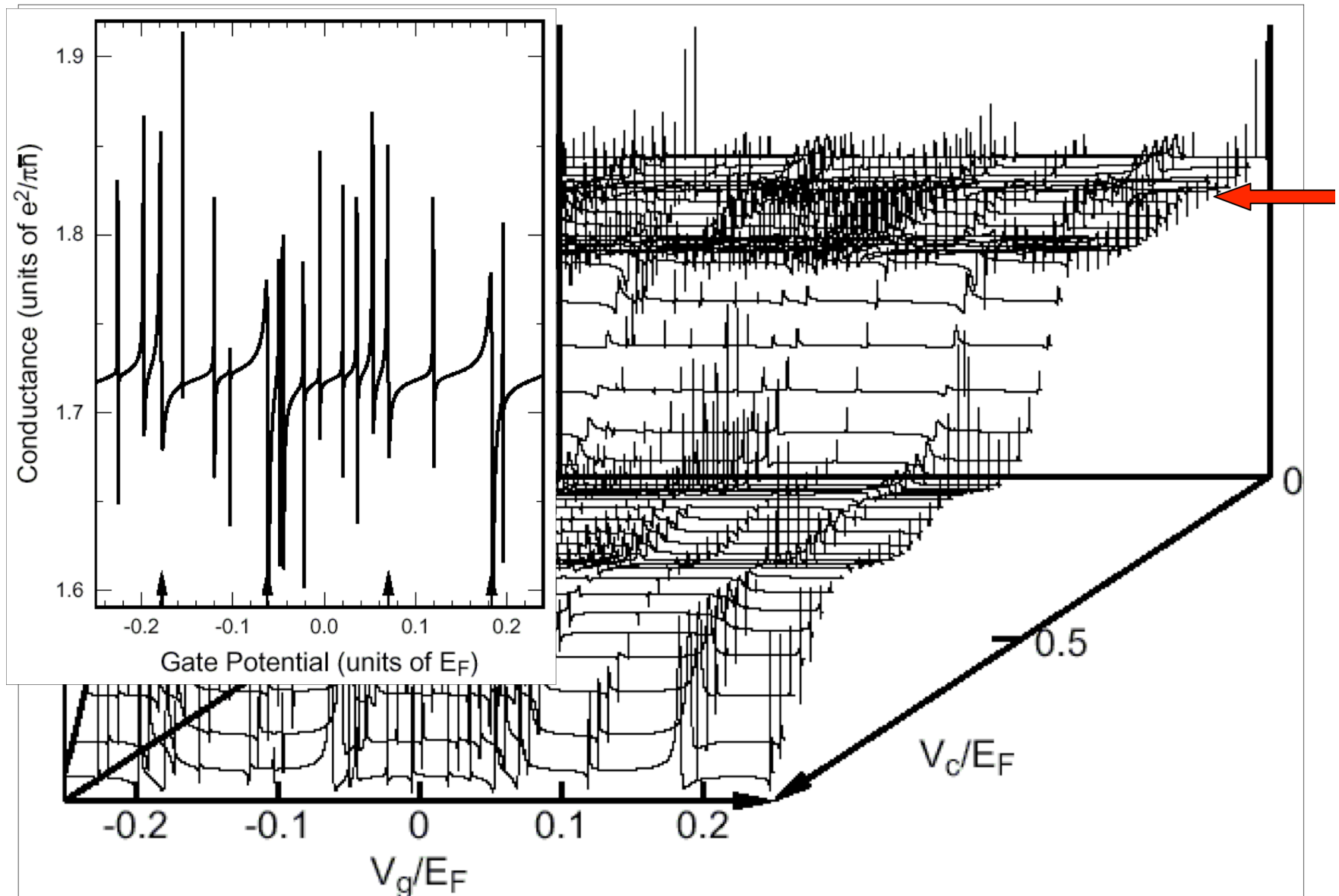
A Quantum Dot without AB ring



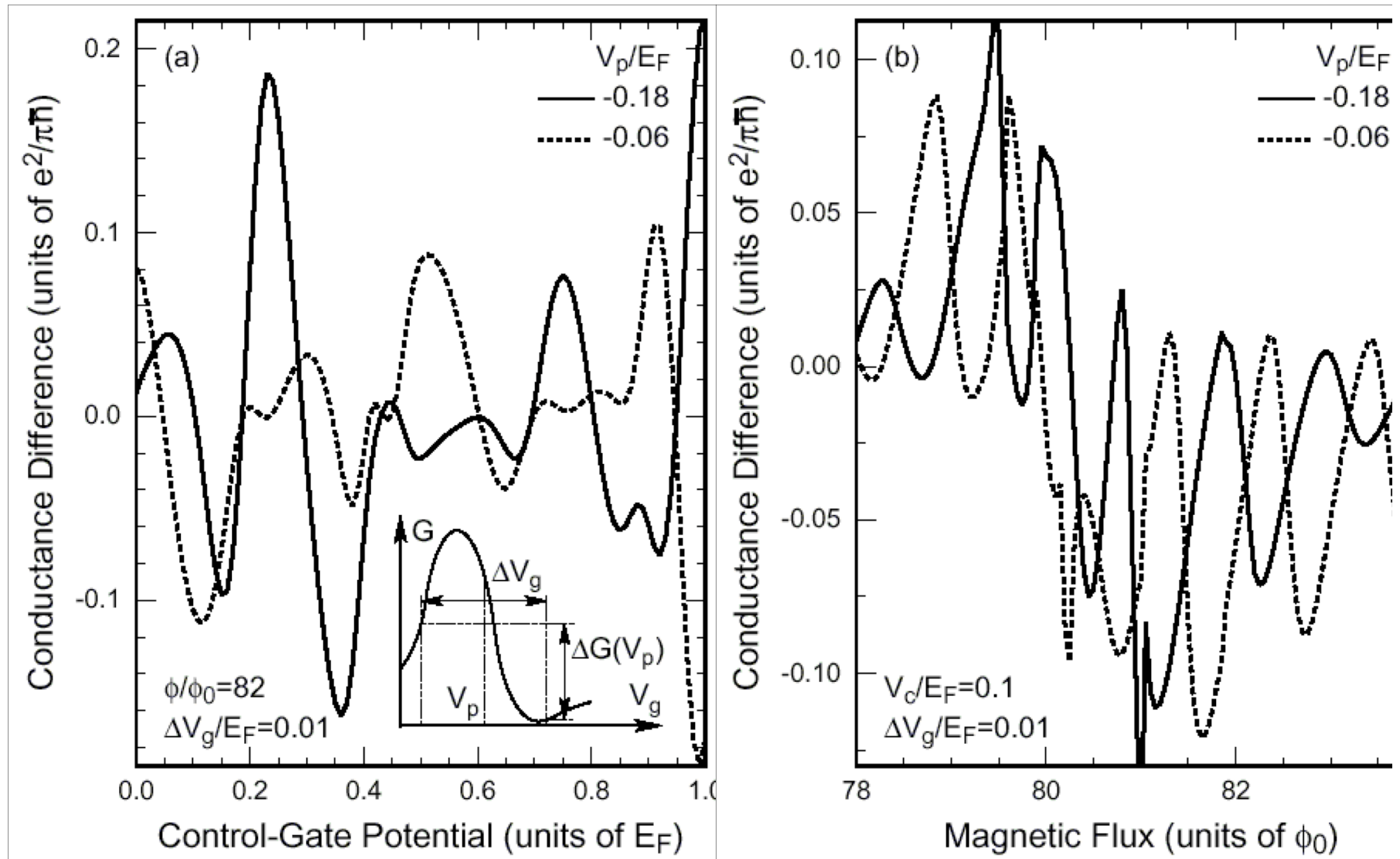
## Control gate (Several channels)



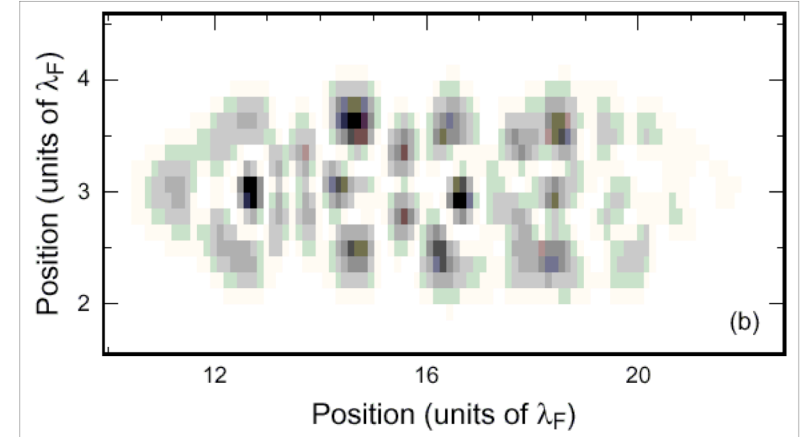
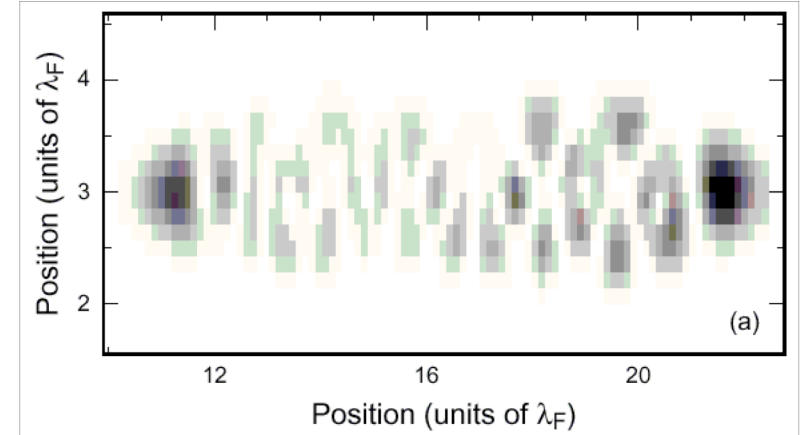
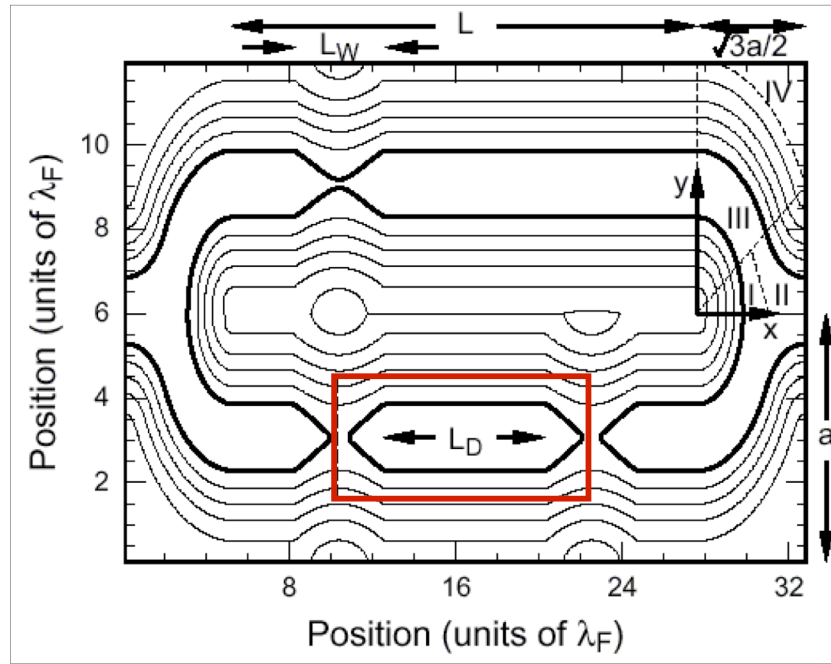
## Control gate (Several channels)



# Asymmetry $q$

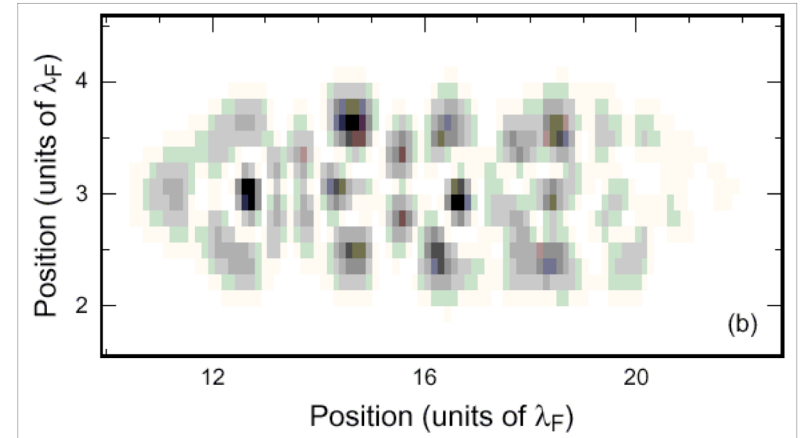
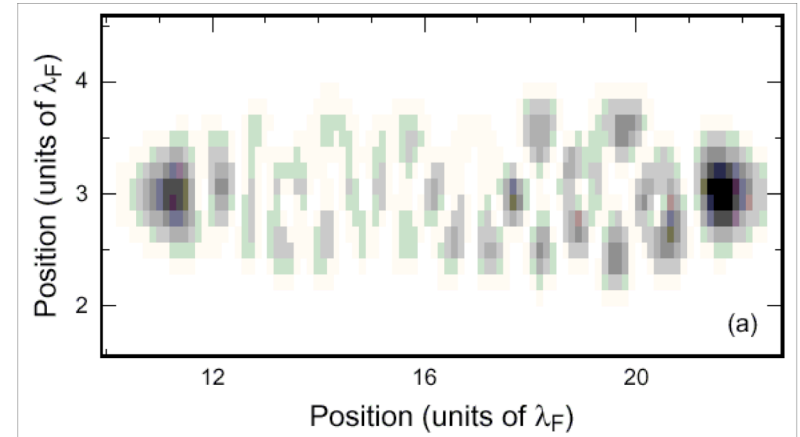
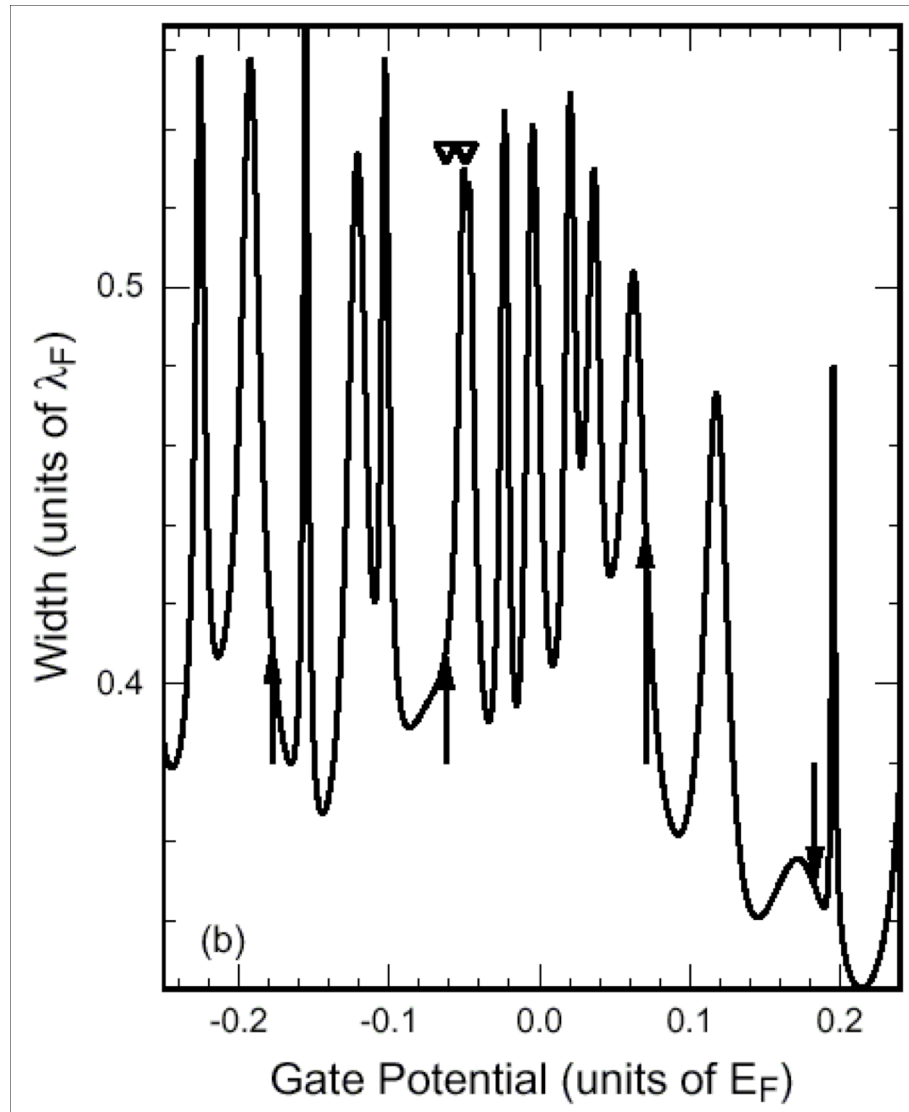


# Wave Functions

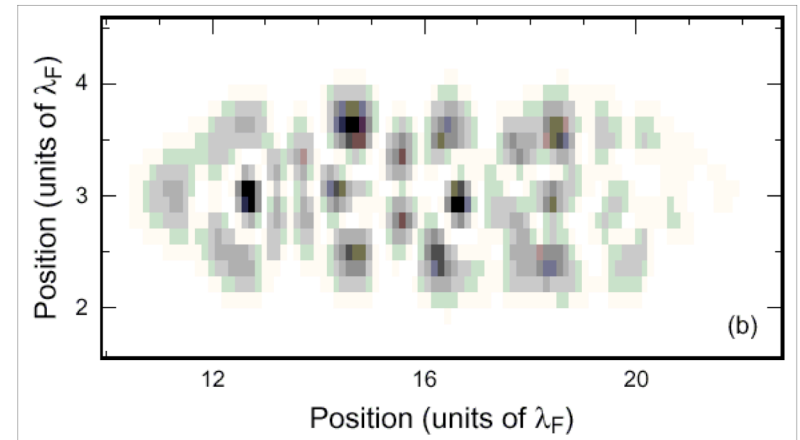
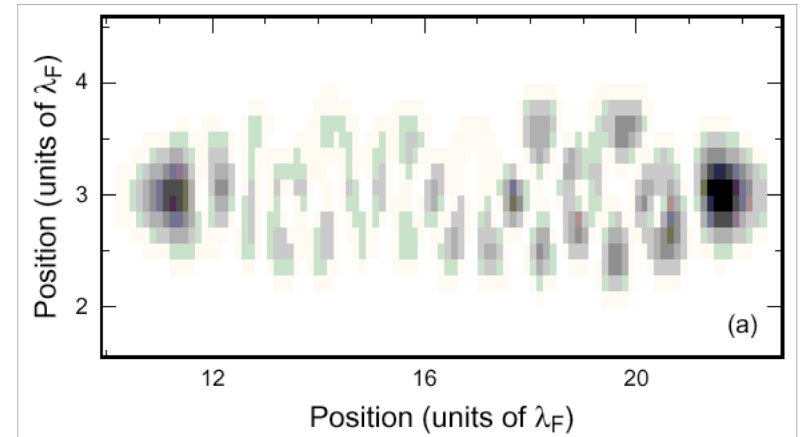
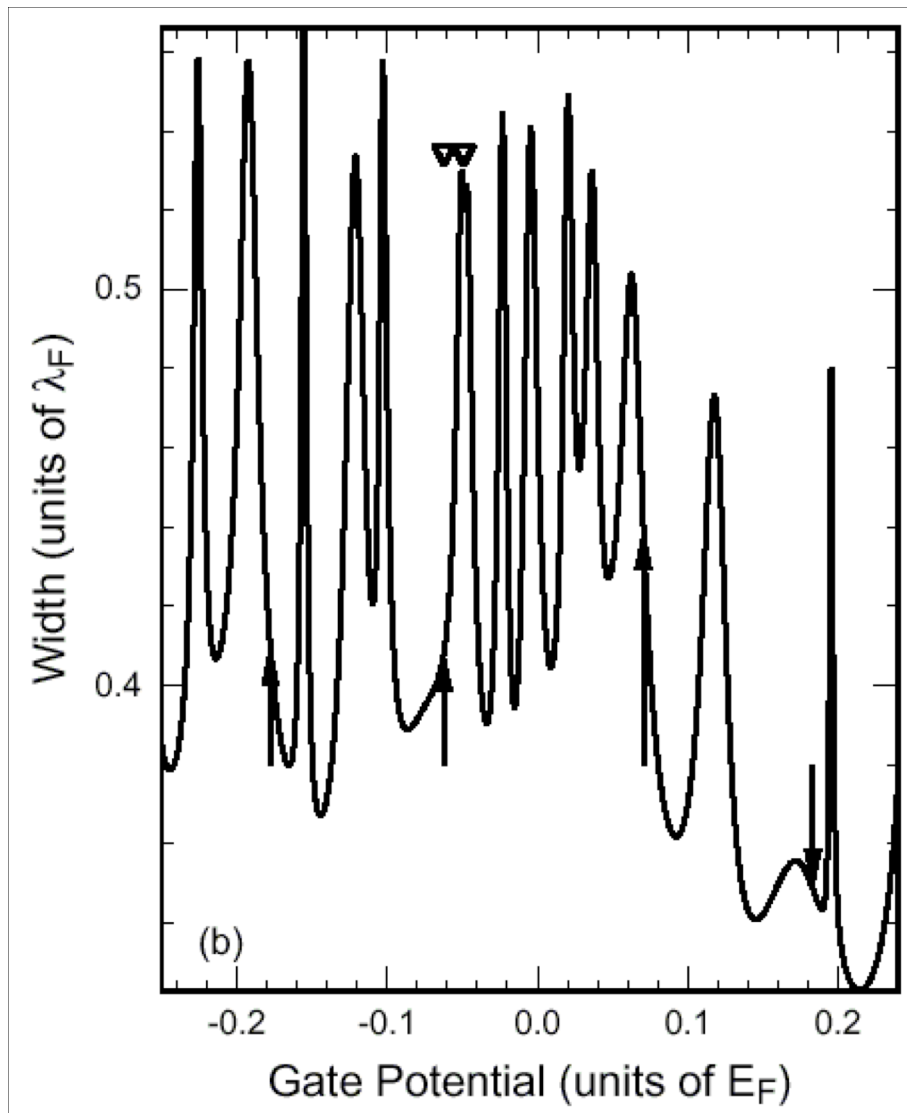




# Wave Functions



# Wave Functions



$$\Delta y = \sqrt{\langle (y - \langle y \rangle)^2 \rangle}$$

## Conclusion

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- ◆ We have shown

- ◆ Sequence of Fano line-shape with similar asymmetry and

- ◆ Phase persistence in AB effect,

which **qualitatively explains the experiment.**

- ◆ Strongly and Weakly coupled levels

- ◆ The **continuum** in Fano effect is state in

- ◆ opposite arm to QD in AB ring

- ◆ strongly coupled level in QD

- ◆ TN, Terakura, Ando, PRB 69, 115307 (2004)