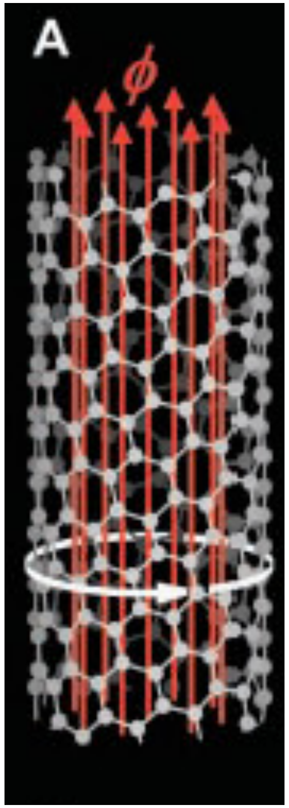
Aharonov-Bohm Effects and Conductivity in Carbon Nanotubes

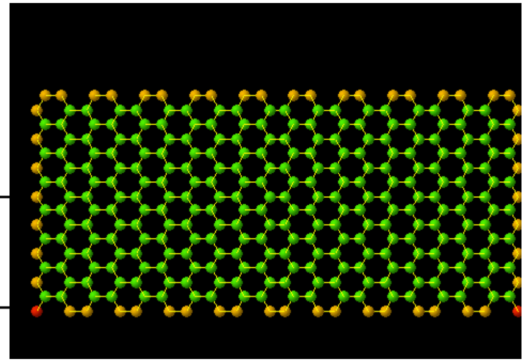
Takeshi Nakanishi (AIST,ISSP,CREST) and

Tsuneya Ando (TIT)

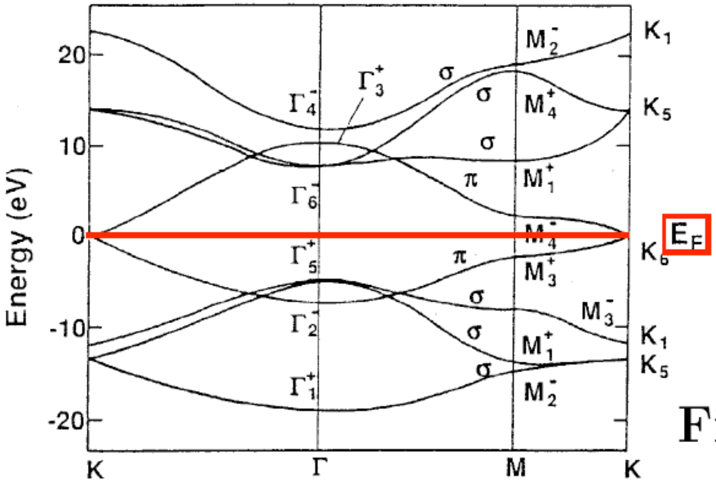


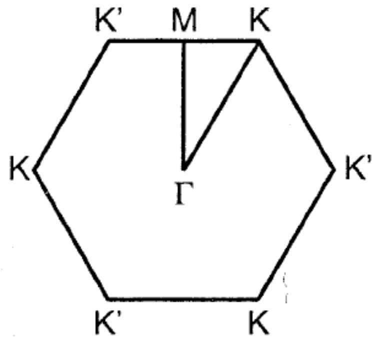
- Introduction
  - AB effect
  - Absence of back scattering
  - Effective flux for strain, curvature and stress effects
- Boltzmann conductivity in carbon nanotubes with magnetic flux in axis direction
- Impurity and electron-phonon scatterings

Graphite sheet (Graphene)

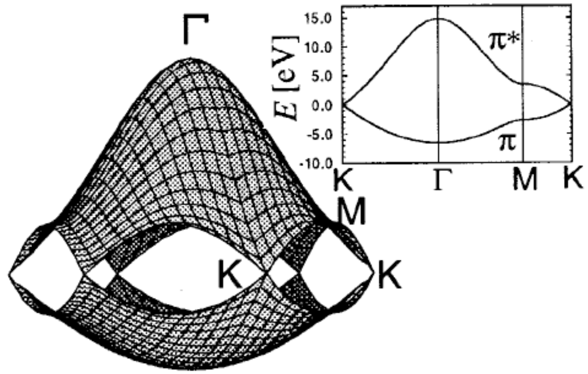


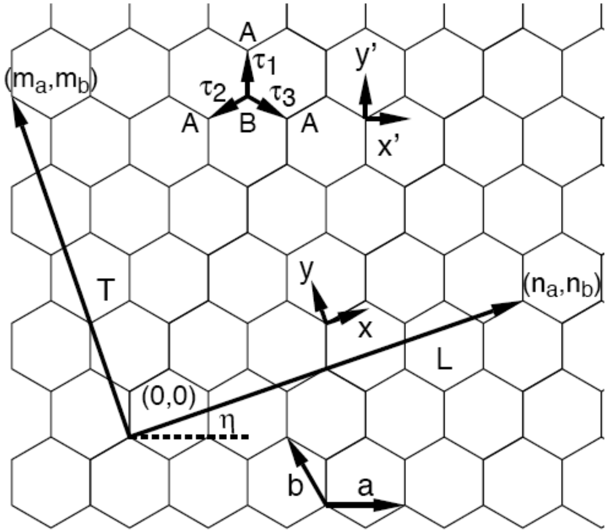
S. Maruyama, Univ. Tokyo (Animation)





First Brillouin Zone





sp^2 covalent bonding

single π band

tight–binding model

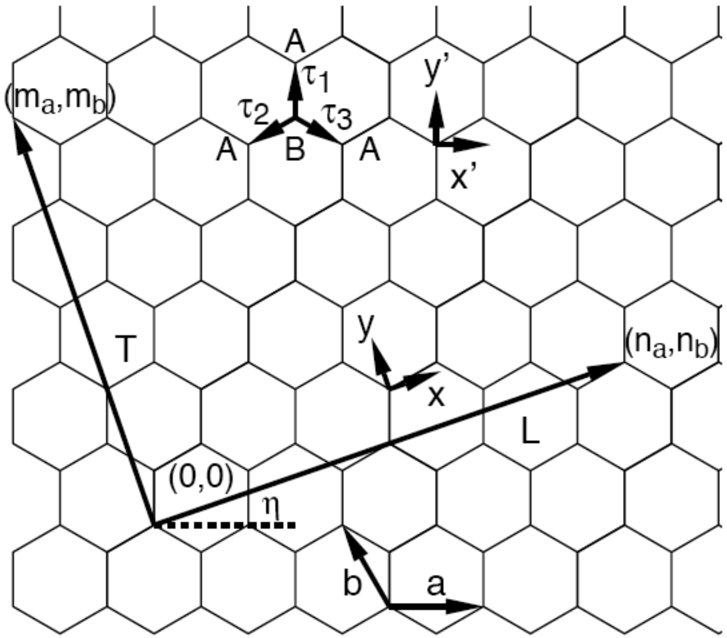
Nearest–neighbor

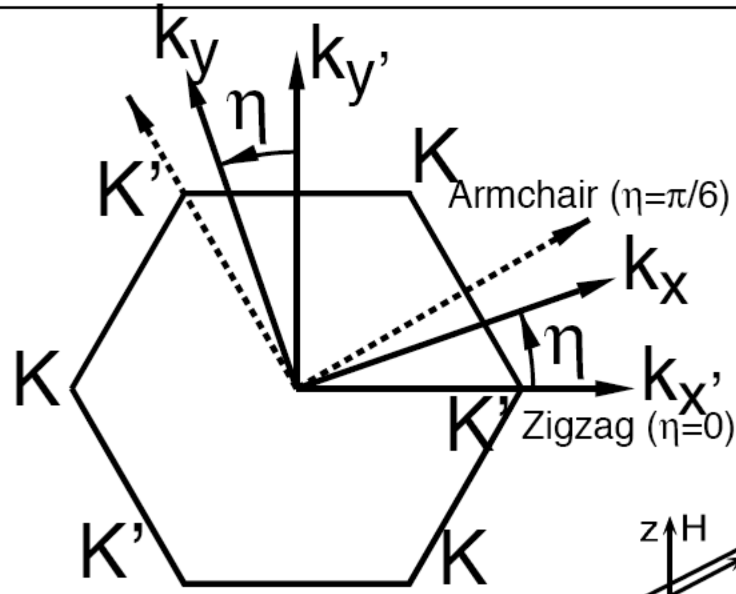
Transfer Integral: γ_0

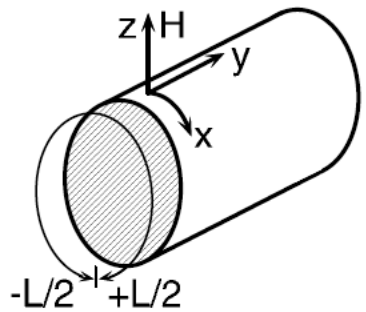
$$-\gamma_0 \sum_{l=1}^{3} \psi_B(\mathbf{R}_A - \vec{\tau}_l) \;=\; \varepsilon\psi_A(\mathbf{R}_A),$$

$$-\gamma_0 \sum_{l=1}^{3} \psi_A(\mathbf{R}_B + \vec{\tau}_l) \;=\; \varepsilon\psi_B(\mathbf{R}_B).$$

Graphite and Chiral Vector







$$
\begin{aligned}
\textbf{Chiral Vector:}\mathbf{L} &= n_a\mathbf{a} + n_b\mathbf{b} \equiv (n_a, n_b), \\
L = |\mathbf{L}| &= a\sqrt{{n_a}^2 + {n_b}^2 - n_a n_b}.
\end{aligned}
$$

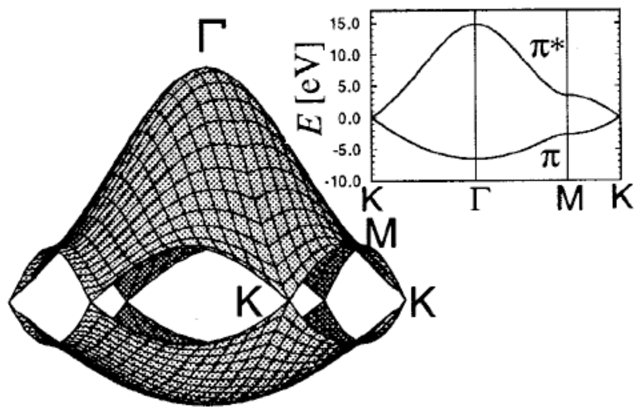
$(n_a, n_b) = (2,1)m :$ **armchair CN**

$(n_a, n_b) = (1,0)m :$ **zigzag CN**

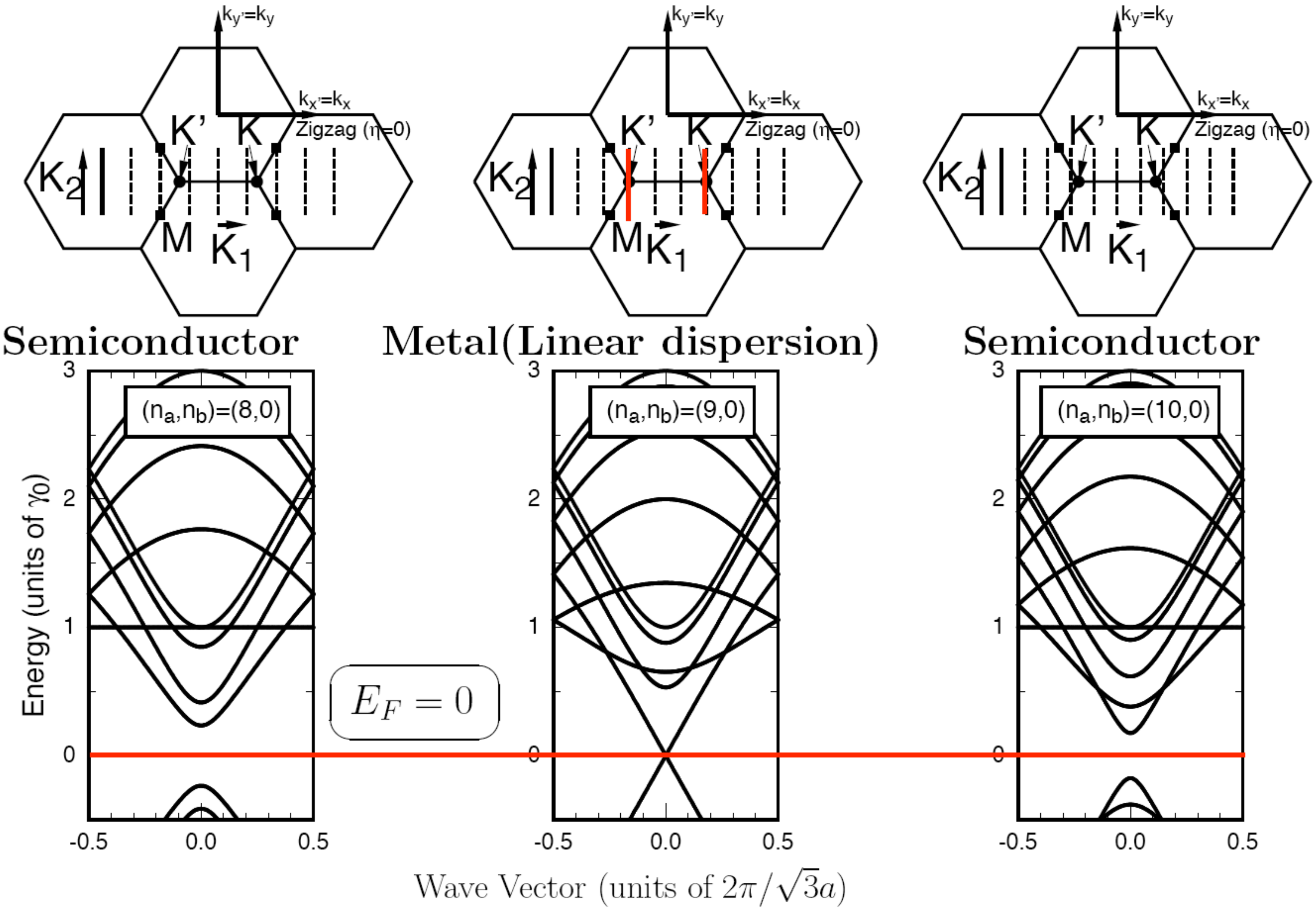
$$n_a + n_b = 3N + \nu$$

$\nu = 0$ **metallic CN**

$\nu = \pm 1$ **semiconducting CN**



Metallic and Semiconducting CN (Zigzag CN)



Aharonov–Bohm (AB) effect

H. Ajiki and T. Ando, JPSJ 62 (1993) 1255.

$$\kappa_{\nu,\phi}(n) = \frac{2\pi}{L}(n - \nu/3 + \varphi)$$

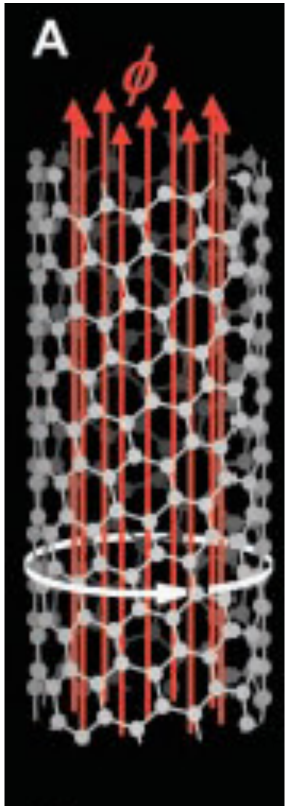
$$\varphi = \phi/\phi_0$$

$$\phi_0 = ch/e$$

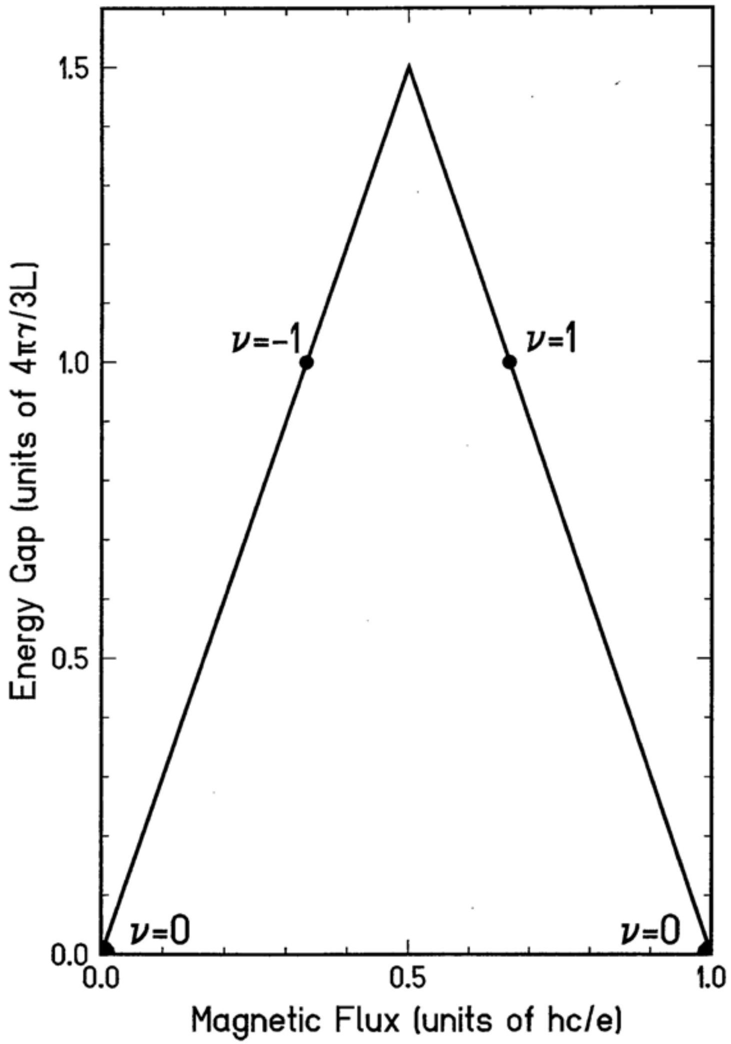
Energy Gap

$$E_g = \gamma\left|\kappa_{\nu,\phi}(n)\right|$$

5300T (d=1nm)



S.Zaric *et al*. Science 304, 1129 (2004)



Effective–Potential

T. Ando and T. Nakanishi, J. Phys. Soc. Jpn. **67**,1704 (1998)

Effective–Mass Equation

$$(\mathcal{H}_0 + V)\mathcal{F} = \varepsilon \mathcal{F}$$

$$\mathcal{H}_0 = \begin{pmatrix} 0 & \gamma(\hat{k}_x - i\hat{k}_y) & 0 & 0 \\ \gamma(\hat{k}_x + i\hat{k}_y) & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma(\hat{k}_x + i\hat{k}_y) \\ 0 & 0 & \gamma(\hat{k}_x - i\hat{k}_y) & 0 \end{pmatrix}, \mathcal{F} = \begin{pmatrix} F_A^K(\mathbf{r}) \\ F_B^K(\mathbf{r}) \\ F_A^{K'}(\mathbf{r}) \\ F_B^{K'}(\mathbf{r}) \end{pmatrix}$$

$$V = \begin{pmatrix} u_A(\mathbf{r}) & 0 & e^{i\eta} u'_A(\mathbf{r}) & 0 \\ 0 & u_B(\mathbf{r}) & 0 & -\omega^{-1} e^{-i\eta} u'_B(\mathbf{r}) \\ e^{-i\eta} u'_A(\mathbf{r})^* & 0 & u_A(\mathbf{r}) & 0 \\ 0 & -\omega e^{i\eta} u'_B(\mathbf{r})^* & 0 & u_B(\mathbf{r}) \end{pmatrix}$$

$$u_A = \frac{\sqrt{3}a^2}{2} \sum_{\mathbf{R}_A} \tilde{u}_A(\mathbf{R}_A),$$

$$u_B = \frac{\sqrt{3}a^2}{2} \sum_{\mathbf{R}_B} \tilde{u}_B(\mathbf{R}_B),$$

$$u'_A = \frac{\sqrt{3}a^2}{2} \sum_{\mathbf{R}_A} e^{i(\mathbf{K}'-\mathbf{K})\cdot\mathbf{R}_A} \tilde{u}_A(\mathbf{R}_A),$$

$$u'_B = \frac{\sqrt{3}a^2}{2} \sum_{\mathbf{R}_B} e^{i(\mathbf{K}'-\mathbf{K})\cdot\mathbf{R}_B} \tilde{u}_B(\mathbf{R}_B),$$

$\sqrt{3}a^2/2$: Area of a Unit Cell

Potential Range $d \ll$ Circumference $L = |\mathbf{L}|$

$$u_A(\mathbf{r}) = u_A \delta(\mathbf{r} - \mathbf{r}_0), \qquad u_B(\mathbf{r}) = u_B \delta(\mathbf{r} - \mathbf{r}_0),$$
$$u'_A(\mathbf{r}) = u'_A \delta(\mathbf{r} - \mathbf{r}_0), \qquad u'_B(\mathbf{r}) = u'_B \delta(\mathbf{r} - \mathbf{r}_0).$$

$\mathbf{r}_0$: Impurity Position

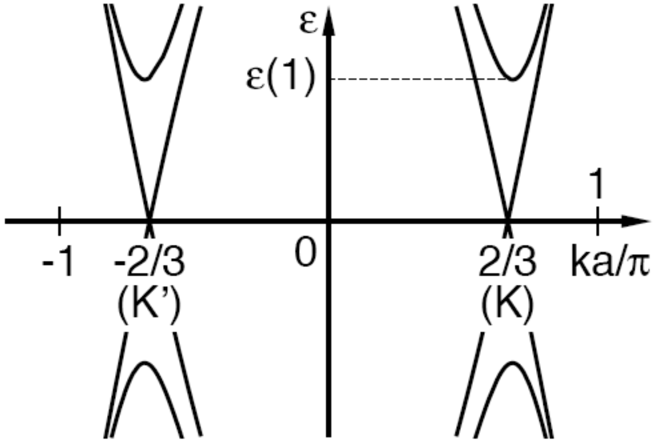
Slowly-varying Potential

Potential Range $d \gg a$

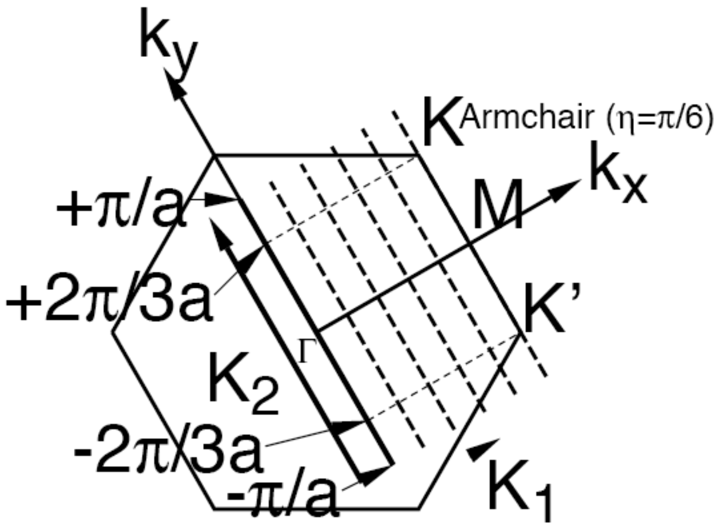
$u_A(\mathbf{r}) = u_B(\mathbf{r})$

$u'_A(\mathbf{r}) = u'_B(\mathbf{r}) = 0$

Right- and left-going channels



Metallic CN



$(n_a, n_b) = (2,1)m$

armchair CN

○**Solutions for** $V = 0, |\varepsilon| < \varepsilon(1) = \frac{2\pi\gamma}{L}$

$$\mathbf{F}^{K\pm} = \begin{pmatrix} F_A^K(\mathbf{r}) \\ F_B^K(\mathbf{r}) \end{pmatrix} = \frac{1}{\sqrt{2AL}} \begin{pmatrix} \mp i \\ 1 \end{pmatrix} \exp(iky),$$

$$\mathbf{F}^{K'\pm} = \begin{pmatrix} F_A^{K'}(\mathbf{r}) \\ F_B^{K'}(\mathbf{r}) \end{pmatrix} = \frac{1}{\sqrt{2AL}} \begin{pmatrix} \pm i \\ 1 \end{pmatrix} \exp(iky).$$

A: **Length of Nanotube**

Energy: $\varepsilon(k) = \pm\gamma k$

Group Velocity: $v = \pm\gamma/\hbar$

$$\pm \begin{cases} \text{Right–going } \mathbf{F}^{K+}, \mathbf{F}^{K'+} \\ \text{Left–going } \mathbf{F}^{K-}, \mathbf{F}^{K'-} \end{cases}$$

Lowest Born Approximation

◯ Inter-valley Scattering

$$
\begin{aligned}
V_{K\pm K'+} &= \frac{1}{2AL}\int d\mathbf{r}\begin{pmatrix} \pm i & 1 \end{pmatrix}\begin{pmatrix} e^{i\eta}u'_A(\mathbf{r}) & 0 \\ 0 & -\omega^{-1}e^{-i\eta}u'_B(\mathbf{r}) \end{pmatrix}\begin{pmatrix} i \\ 1 \end{pmatrix} \\
&= \frac{1}{2AL}\int d\mathbf{r}\left\{\mp e^{i\eta}u'_A(\mathbf{r}) - \omega^{-1}e^{-i\eta}u'_B(\mathbf{r})\right\} \\
&= \frac{1}{2AL}(\mp u'_A e^{i\eta} - \omega^{-1}e^{-i\eta}u'_B) = V^*_{K'\pm K+}
\end{aligned}
$$

◯ Intra-valley Scattering

$$
\begin{aligned}
V_{K\pm K+} &= \frac{1}{2AL}\int d\mathbf{r}\begin{pmatrix} \pm i & 1 \end{pmatrix}\begin{pmatrix} u_A(\mathbf{r}) & 0 \\ 0 & u_B(\mathbf{r}) \end{pmatrix}\begin{pmatrix} -i \\ 1 \end{pmatrix} \\
&= \frac{1}{2AL}\int d\mathbf{r}\left\{\pm u_A(\mathbf{r}) + u_B(\mathbf{r})\right\} \\
&= \frac{1}{2AL}(\pm u_A + u_B) = V_{K'\pm K'+}
\end{aligned}
$$

Absence of back-scattering for slowly varying potential

$$V_{K-K'+} = V^*_{K'-K+} = 0, \qquad V_{K-K+} = V_{K'-K'+} \propto u_B - u_A = 0$$

Absence of Back Scattering$(d \gg a)$

$$T = V + V\frac{1}{\varepsilon - \mathcal{H}_0}V + V\frac{1}{\varepsilon - \mathcal{H}_0}V\frac{1}{\varepsilon - \mathcal{H}_0}V + \cdots$$

$$\mathcal{H}_0 = \gamma \begin{pmatrix} 0 & \hat{k}_x - i\hat{k}_y \\ \hat{k}_x + i\hat{k}_y & 0 \end{pmatrix}$$

Long range Potential

$$V = \begin{pmatrix} V(\mathbf{r}) & 0 \\ 0 & V(\mathbf{r}) \end{pmatrix}$$

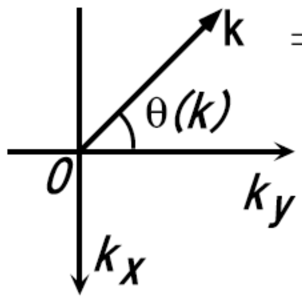
$$\begin{aligned} \mathbf{F}_{s\mathbf{k}}(\mathbf{r}) &= \frac{1}{\sqrt{LA}} \exp(i\mathbf{k}\cdot\mathbf{r})\,\mathbf{F}_{s\mathbf{k}}, \\ \varepsilon_s(\mathbf{k}) &= s\gamma|\mathbf{k}|, \\ s &= +1 \quad \textbf{conduction band} \\ s &= -1 \quad \textbf{valence band} \end{aligned}$$

$$\varepsilon_s(-\mathbf{k}) = \varepsilon_s(\mathbf{k})$$

$$\mathbf{F}_{s\mathbf{k}} = \exp[i\phi_s(\mathbf{k})]\,R^{-1}[\theta(\mathbf{k})]\,|s),$$

$$k_x + ik_y = i|\mathbf{k}|e^{i\theta(\mathbf{k})}$$

Spin-rotation operator



$$\begin{aligned} R(\theta) &= \exp\left(i\frac{\theta}{2}\sigma_z\right) \\ &= \begin{pmatrix} \exp(+i\theta/2) & 0 \\ 0 & \exp(-i\theta/2) \end{pmatrix} \end{aligned}$$

$$|s) = \frac{1}{\sqrt{2}} \begin{pmatrix} -is \\ 1 \end{pmatrix}$$

Absence of Back Scattering($d \gg a$)

T. Ando and T. Nakanishi, J. Phys. Soc. Jpn. **67**,1704 (1998)

$(p+1)$th order term

$$(s,-\mathbf{k}|T^{(p+1)}|s,+\mathbf{k}) = \frac{1}{LA}\sum_{s_1\mathbf{k}_1}\frac{1}{LA}\sum_{s_2\mathbf{k}_2}\cdots\frac{1}{LA}\sum_{s_p\mathbf{k}_p}$$

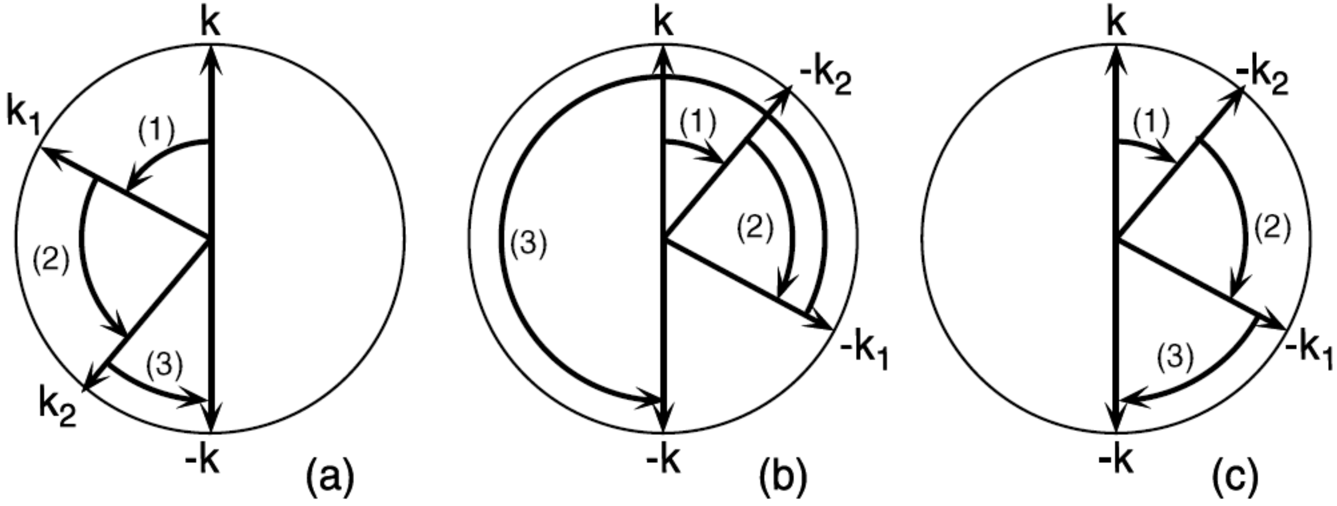
$$\times \frac{\boxed{V(-\mathbf{k}-\mathbf{k}_p)\cdots V(\mathbf{k}_2-\mathbf{k}_1)V(\mathbf{k}_1-\mathbf{k})}}{\boxed{[\varepsilon-\varepsilon_{s_p}(\mathbf{k}_p)]\cdots[\varepsilon-\varepsilon_{s_2}(\mathbf{k}_2)][\varepsilon-\varepsilon_{s_1}(\mathbf{k}_1)]}}$$

$$\boxed{\begin{aligned}&\times\ e^{-i\phi_s(-\mathbf{k})}(s|R[\theta(-\mathbf{k})]R^{-1}[\theta(\mathbf{k}_p)]|s_p)\\ &\times\ \cdots\times(s_2|R[\theta(\mathbf{k}_2)]R^{-1}[\theta(\mathbf{k}_1)]|s_1)\\ &\times\ (s_1|R[\theta(\mathbf{k}_1)]R^{-1}[\theta(\mathbf{k})]|s)e^{i\phi_s(\mathbf{k})}\end{aligned}}$$

time-reversal terms cancel out

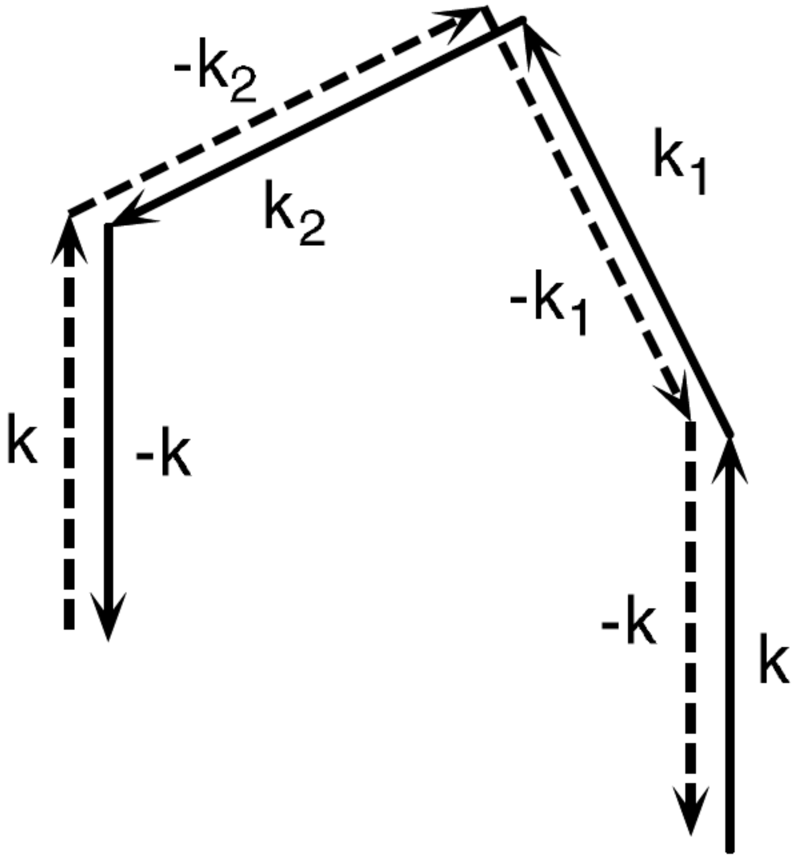
$$(s_1,k_1)\longrightarrow(s_p,-k_p),$$
$$(s_2,k_2)\longrightarrow(s_{p-1},-k_{p-1}),\ldots$$
$$R[\theta]=-R[\theta+2\pi]$$

$$\boxed{\varepsilon_s(-\mathbf{k})=\varepsilon_s(\mathbf{k})}$$



Berry's Phase and Absence of Back Scattering

T. Ando, T. Nakanishi, and R. Saito, J. Phys. Soc. Jpn. **67**,2857 (1998)



$$\psi_s(\mathbf{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} -is \\ \exp[i\theta(\mathbf{k})] \end{pmatrix}$$

$$\psi_s(\mathbf{k}) \longrightarrow \psi_s(\mathbf{k}) \exp(-i\varphi)$$

Berry's Phase

$$\varphi = -i \int_0^T dt \left\langle \psi_s[\mathbf{k}(t)] \middle| \frac{d}{dt} \psi_s[\mathbf{k}(t)] \right\rangle = \pi$$

$$R[\theta - 2\pi] = -R[\theta]$$

$$R[-\pi] = -R[\pi]$$

Impurity Scattering

- **Absence of back scattering**
  T. Ando and TN, JPSJ 67 (1998) 1704
  - Long-range potential
    Diagonal effective potential
  - Linear band in Metallic CN
- Short-range potential
  Off-diagonal effective potential
- δ: short-range/long-range

$$V_L(\mathbf{r}) \begin{array}{cccc} \mathrm{KA} & \mathrm{KB} & \mathrm{K'A} & \mathrm{K'B} \\ \end{array}$$

$$V_L(\mathbf{r})\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$V_S^A(\mathbf{r})\begin{pmatrix} 1 & 0 & e^{i\phi} & 0 \\ 0 & 0 & 0 & 0 \\ e^{-i\phi} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\phi = (\mathbf{K}' - \mathbf{K})\mathbf{R}_i^A - \eta$$

Boltzumann Conductivity

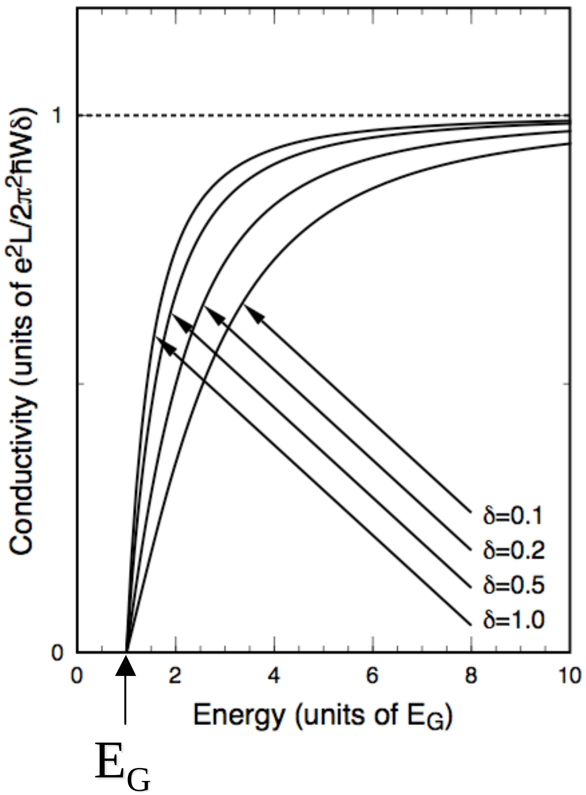
Metallic Nanotube

$$\sigma(\varepsilon) = \frac{e^2 L}{\pi^2 \hbar W} \frac{1}{2\delta}$$

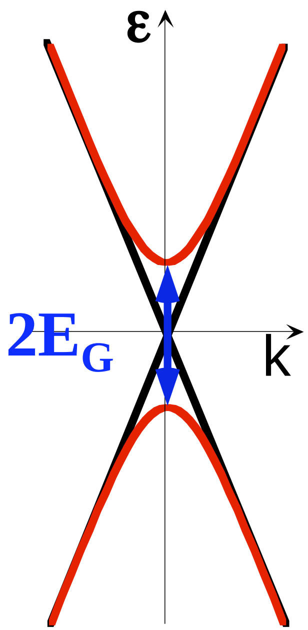
Perfect Conductor w/o short-range potential

(δ: ratio of short-range potential)

Non-Metallic Nanotube



Nanotubes with
Energy Gap E_G
(2 Bands)



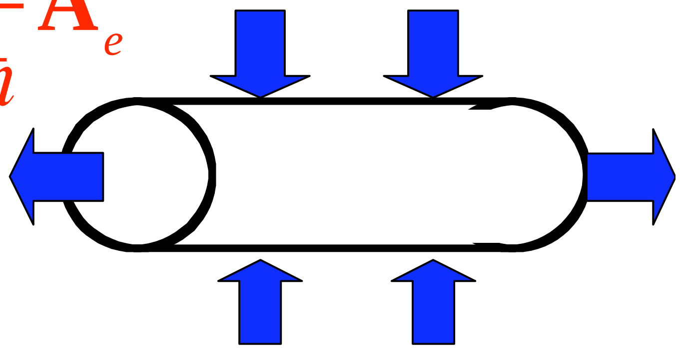
Strain (stretch, stress, *etc*) effects

$$H_K = \gamma \begin{pmatrix} 0 & \hat{k}_x - i\hat{k}_y \\ \hat{k}_x + i\hat{k}_y & 0 \end{pmatrix} + \begin{pmatrix} V_1 & V_2 \\ V_2^* & V_1 \end{pmatrix}$$

$$H_{K'} = \gamma \begin{pmatrix} 0 & \hat{k}_x + i\hat{k}_y \\ \hat{k}_x - i\hat{k}_y & 0 \end{pmatrix} + \begin{pmatrix} V_1 & -V_2^* \\ -V_2 & V_1 \end{pmatrix}$$

$$V_2 = g_2 e^{3i\eta}(u_{xx} - u_{yy} + 2iu_{xy})$$

$$\hat{\mathbf{k}} = -i\vec{\nabla} + \frac{e}{c\hbar}\mathbf{A} \rightarrow -i\vec{\nabla} + \frac{e}{c\hbar}\mathbf{A} + \frac{e}{c\hbar}\mathbf{A}_e$$



$$\mathbf{A}_e = \frac{c\hbar}{e}\Re V_2/\gamma$$

Suzuura and Ando,
PRB 65 (2002) 235412

Effective flux of strain effect

$$\frac{\phi}{\phi_0} = \frac{1}{2\pi\gamma}\oint_C \Re V_2 dx$$

$$= \frac{Lg_2}{2\pi\gamma}\left[(u_{xx} - u_{yy})\cos 3\eta - 2u_{xy}\sin 3\eta\right]$$

stretch,
breating(stress)

twisting

$$(u_x \neq 0, u_y = u_z = 0)$$

- Zigzag(η=0)
  Stretch, Breathing(stress)
- Armchair(η=π/6)
  Twist

$$\mathbf{u}(\mathbf{r}) = \mathbf{u}_{nq}\exp[i\chi(n)x + iqy]$$

$$\chi(n) = 2\pi n / L$$

n=0

T.Ando: JPSJ **73** (2004)3351

Effective flux of curvature effect

T. Ando: JPSJ **69** (2000)1757; **74** (2005) 777

π, σ mixing

max. in Zigzag(η=0), not in Armchair (η=π/6)

$$\boxed{\frac{\phi}{\phi_0} = -\frac{2\pi}{4\sqrt{3}}\frac{a}{L}p\cos 3\eta}$$

(K point) -ϕ (K' point)

$$p = 1 - \frac{3}{8}\frac{\gamma'}{\gamma} \qquad \gamma' = -\frac{\sqrt{3}}{2}V_{pp}^{\pi}a \qquad V_{pp}^{\pi} = -\gamma_0$$

|p|<1

$$\gamma = -\frac{\sqrt{3}}{2}(V_{pp}^{\sigma} - V_{pp}^{\pi})a$$

$$\boxed{\phi/\phi_0 \sim 0.05\,p\cos 3\eta \quad (L \sim 5\mathrm{nm})}$$

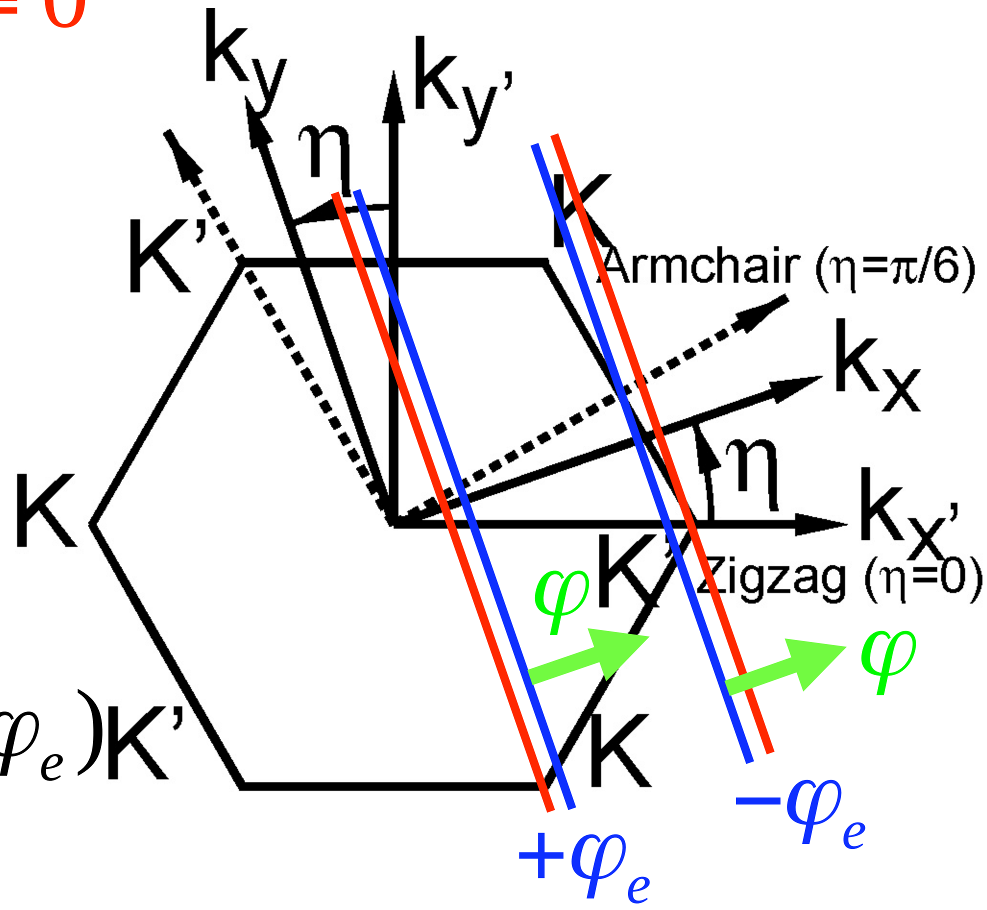
$$\Delta k_y = -\frac{\pi^2}{4\sqrt{3}}\frac{a}{L^2}\left(\frac{5}{8}\frac{\gamma'}{\gamma} - 1\right)\sin 3\eta$$

Effective flux

metal: $n = \varphi = \varphi_e = \kappa = 0$

Effective flux for strain, curvature and stress

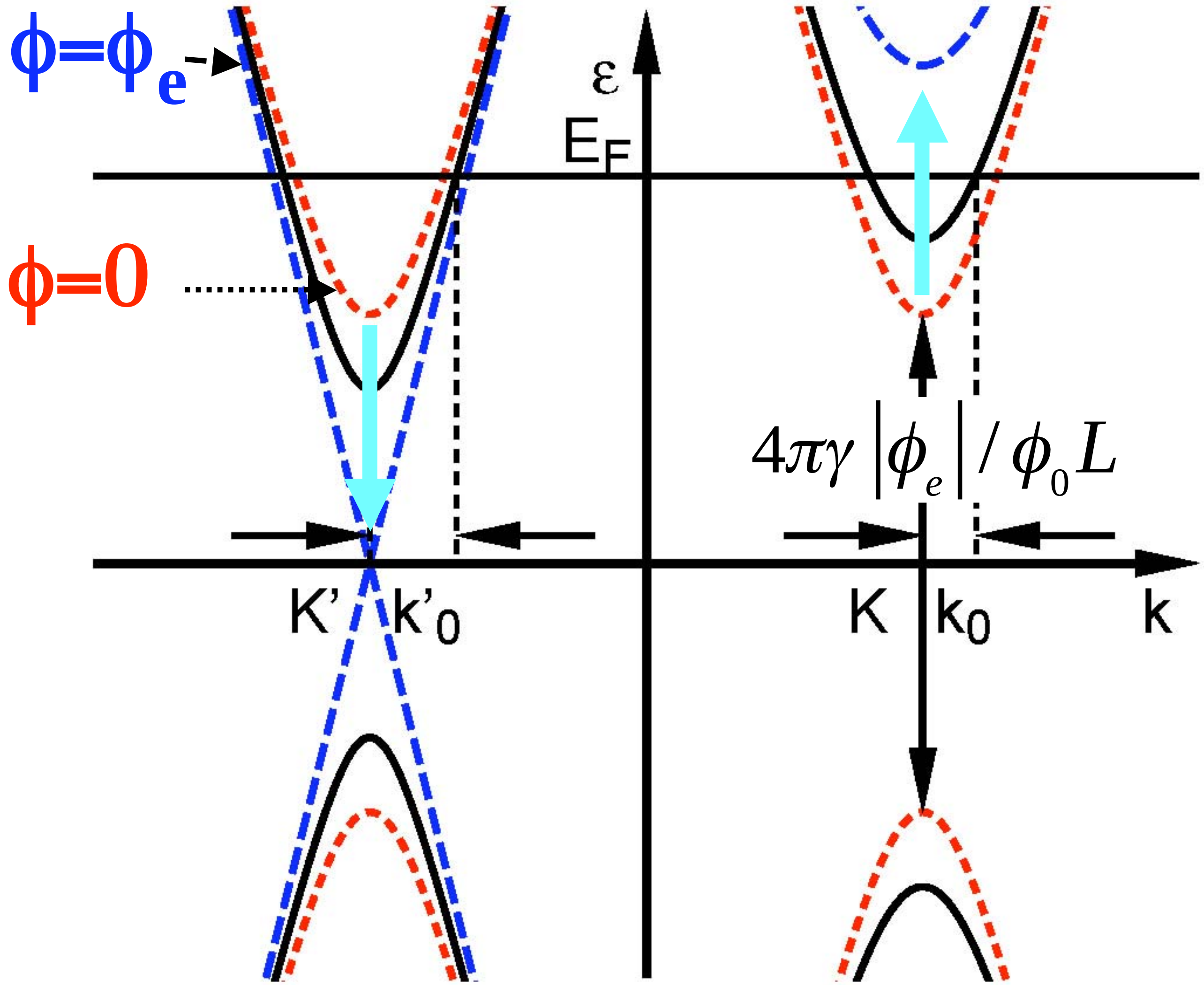
$$\varphi_e = \phi_e / \phi_0$$



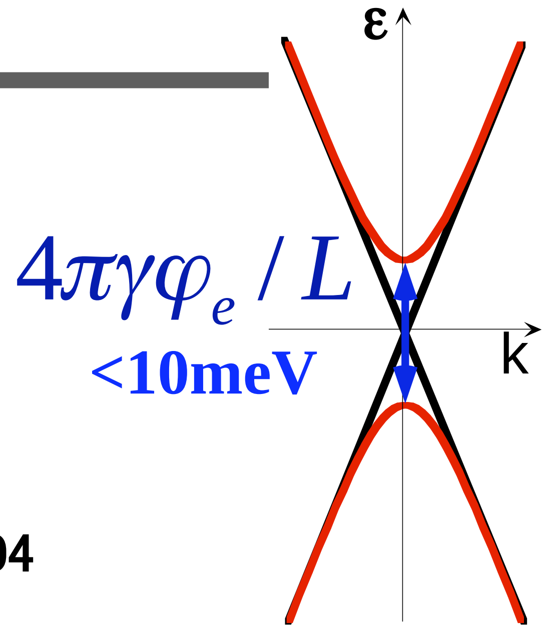
$$\kappa^{\mathrm{K}}_{\varphi_e,\varphi}(n) = \frac{2\pi}{L}(n+\varphi+\varphi_e)$$

$$\kappa^{\mathrm{K'}}_{\varphi_e,\varphi}(n) = \frac{2\pi}{L}(n+\varphi-\varphi_e)$$

Schematic View of Bands



Method



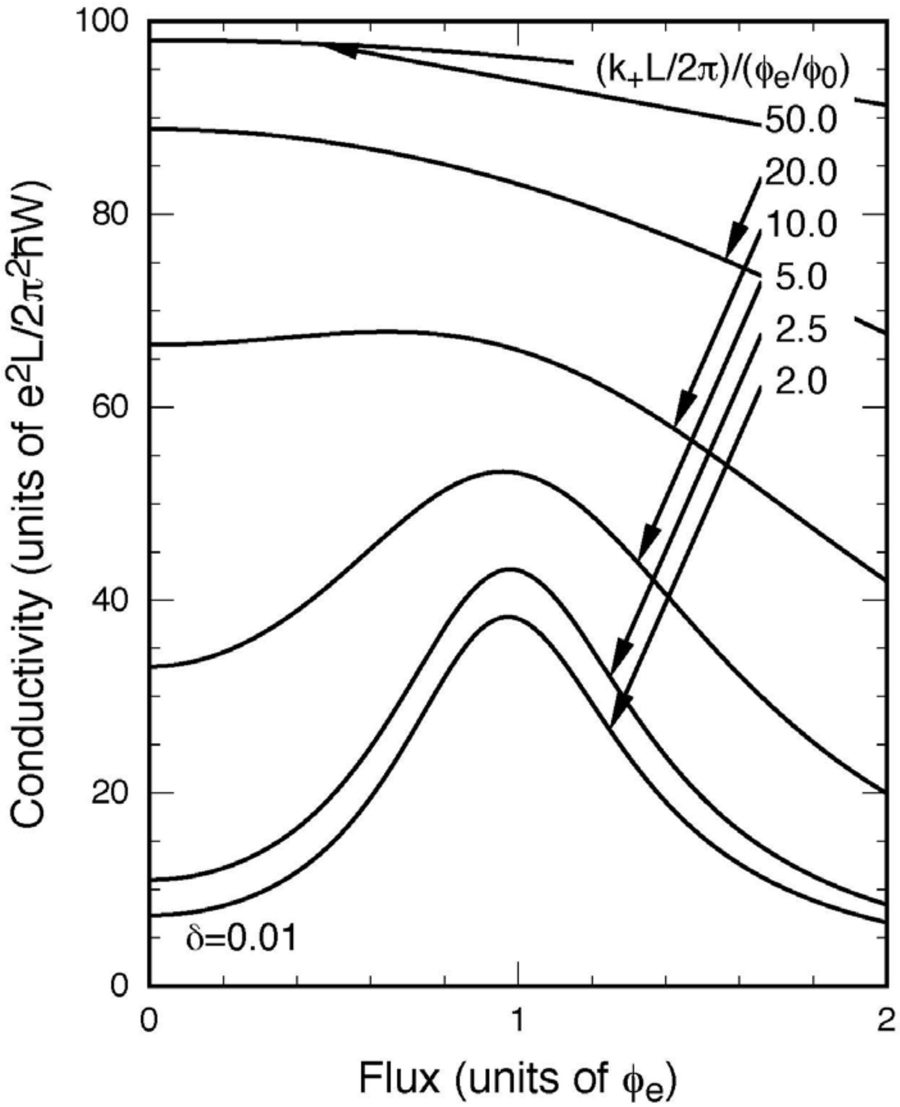
- Effective-mass scheme
- Boltzmann Conductivity
- Effective Potential
  - **Impurity:** T. Ando and TN, JPSJ 67 (1998) 1704
  - **Phonon:** T. Ando and H. Suzuura, JPSJ 71 (2002) 2753
- Strain, curvature and stress effects

$$\varphi_e = \phi_e / \phi_0$$

$$\kappa^{\mathrm{K}}_{\varphi_e,\varphi}(n) = \frac{2\pi}{L}(n + \varphi + \varphi_e)$$

$$\kappa^{\mathrm{K'}}_{\varphi_e,\varphi}(n) = \frac{2\pi}{L}(n + \varphi - \varphi_e)$$

Nanotubes with small gap

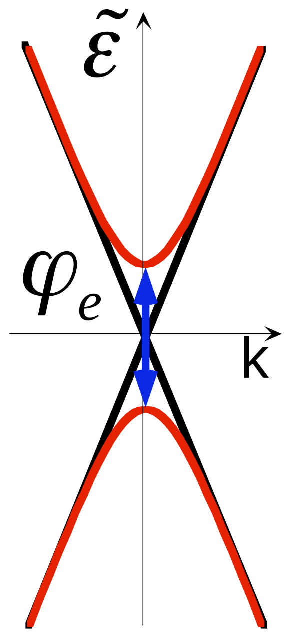


- **Peak in Conductivity**

$$\sqrt{\delta/(1-\delta)}(Lk_+/2\pi) < |\varphi_e|$$

- **k_+: Charge Density**
- **δ: ratio of short-range potential**
- **In available magnetic field**

~10T



Boltzmann Conductivity

$$\sigma(\varphi) = \frac{e^2}{\pi\hbar}(\Lambda^{K} + \Lambda^{K'})$$

$$\frac{1}{\Lambda^{K'}} = \frac{1}{\Lambda_L^{K'}} + \frac{1}{\Lambda_S^{K'}},$$

$\phi=\phi_e$

$$\kappa_{\varphi+\varphi_e} = \frac{2\pi}{L}(\varphi + \varphi_e)$$

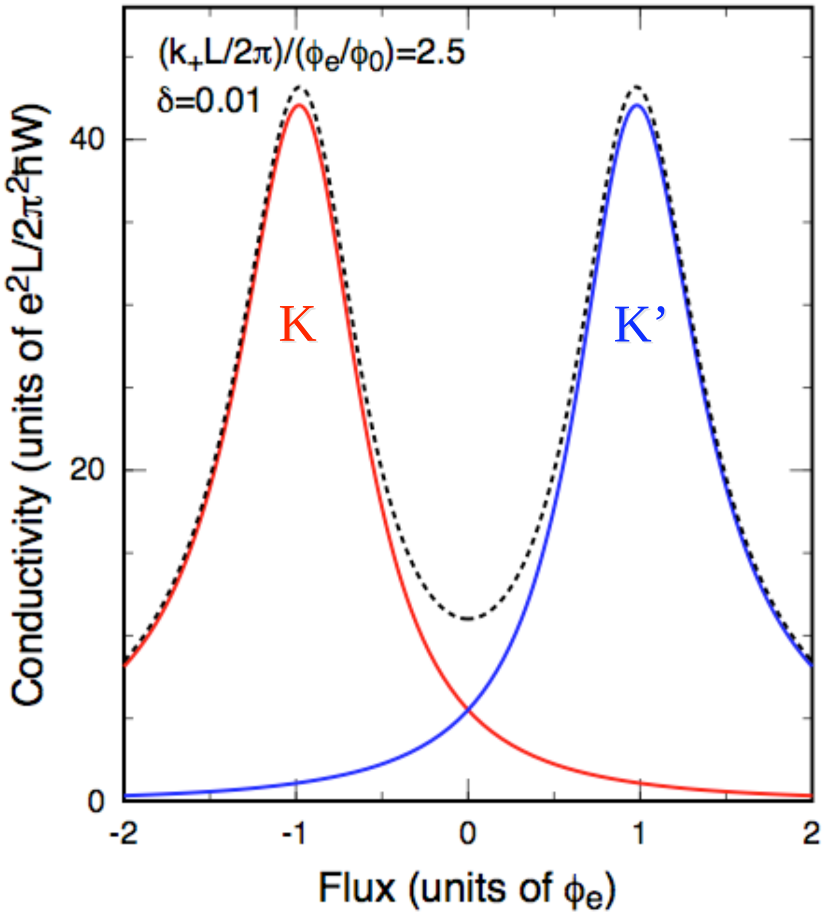
$$\frac{1}{\Lambda_L^{K'}} = \frac{2\pi W(1-\delta)}{L} \frac{\cancel{\kappa_{\varphi-\varphi_e}^2}}{k_0'^2},$$

$$\frac{1}{\Lambda_S^{K'}} = \frac{2\pi W\delta}{L} \sqrt{1 + \frac{\cancel{\kappa_{\varphi-\varphi_e}^2}}{k_0'^2}}$$

$$\times \left[\sqrt{1 + \frac{\kappa_{\varphi+\varphi_e}^2}{k_0^2}} + \sqrt{1 + \frac{\cancel{\kappa_{\varphi-\varphi_e}^2}}{k_0'^2}} \right],$$

- k_0: **Charge Density**
- **δ: ratio of short-range**
- **W:Total scattering strength**

Peak in Conductivity



$$\sigma(\varphi) = \frac{e^2}{\pi\hbar}\left(\boxed{\Lambda^K} + \boxed{\Lambda^{K'}}\right)$$

Lorentzian-like peak

$$\frac{1}{\Lambda^{K'}} = \frac{2\pi W(1-\delta)}{L}\left(\frac{2\pi}{Lk_0'}\right)^2\left[(\varphi-\varphi_e)^2 + \Delta\varphi^2\right],$$

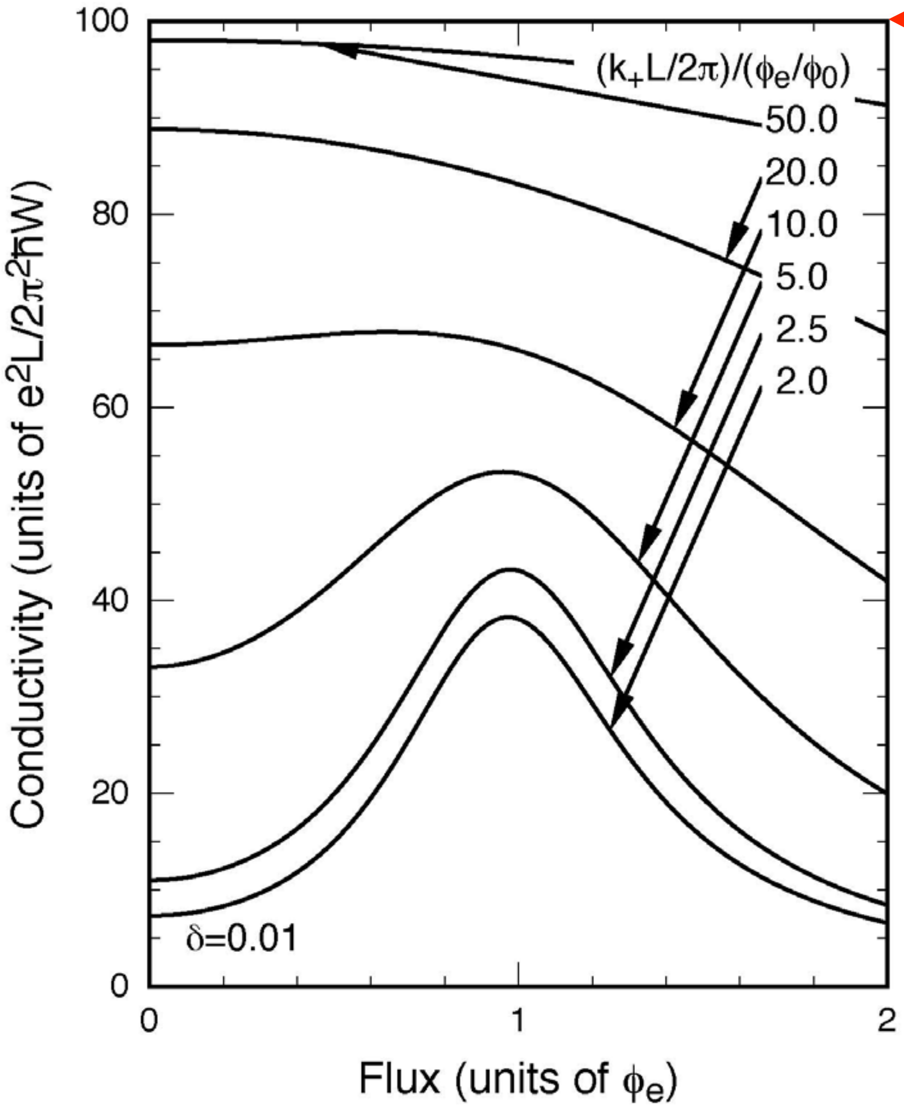
$$\Delta\varphi = \boxed{\sqrt{\frac{\delta}{1-\delta}\frac{Lk_0'}{2\pi}}}\sqrt{1+\sqrt{1+4\varphi_e^2\left(\frac{2\pi}{Lk_0}\right)^2}}$$

Condition to see the peak:

$$\sqrt{\delta/(1-\delta)}\,(Lk_+/2\pi) < |\varphi_e|$$

$k_+ = (k_0 + k_0')/2 \approx k_0'$ for large density

Large density and Scale



$$\sigma(\varepsilon)=\frac{e^2L}{\pi^2\hbar W}\frac{1}{2\delta}$$

Gap-less

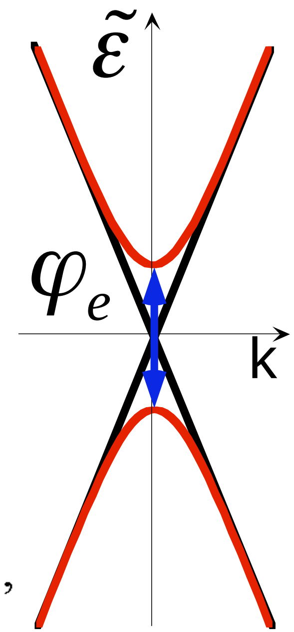
$$\boxed{\frac{\kappa_{\varphi-\varphi_e}}{k_0'}}=\left(\frac{\varphi_e}{\varphi}+1\right)\Bigg/\left(\frac{Lk_0'}{2\pi}\frac{1}{\varphi_e}\right)$$

$$\frac{1}{\Lambda^{K'}}=\frac{1}{\Lambda_L^{K'}}+\frac{1}{\Lambda_S^{K'}},$$

$$\frac{1}{\Lambda_L^{K'}}=\frac{2\pi W(1-\delta)}{L}\boxed{\frac{\kappa^2_{\varphi-\varphi_e}}{k_0'^2}},$$

$$\frac{1}{\Lambda_S^{K'}}=\frac{2\pi W\delta}{L}\sqrt{1+\boxed{\frac{\kappa^2_{\varphi-\varphi_e}}{k_0'^2}}}$$

$$\times\left[\sqrt{1+\boxed{\frac{\kappa^2_{\varphi+\varphi_e}}{k_0^2}}}+\sqrt{1+\boxed{\frac{\kappa^2_{\varphi-\varphi_e}}{k_0'^2}}}\right],$$

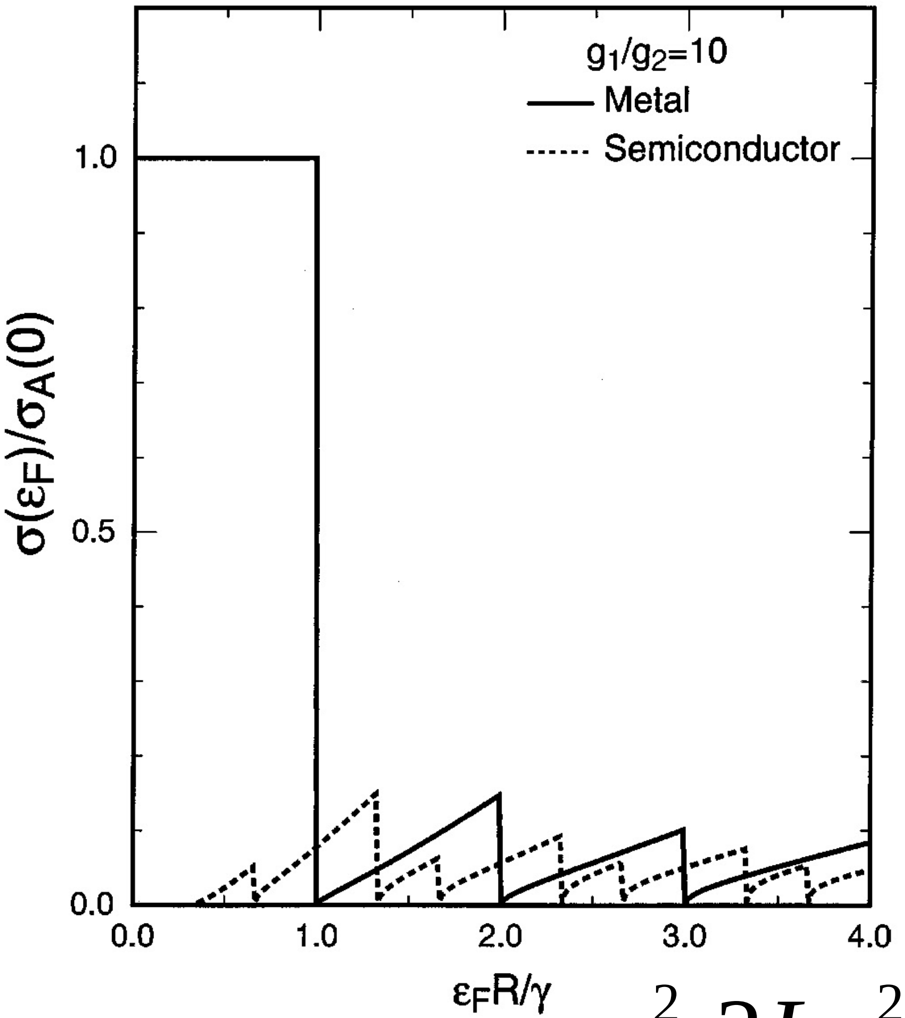


$$\sqrt{\delta/(1-\delta)}\,(Lk_+/2\pi)<|\varphi_e|$$

Electron-Phonon Interaction

- Suzuura and Ando, PRB 65 (2002) 235412
- Effective-Mass Theory
- Continuum model for acoustic phonons
- Elastic scattering approximation
- (High temperature approximation)
- Boltzmann Conductivity
  Relaxation time approximation
- C. L. Kane and E. J. Mele, PRL 78 (1997) 1932

Effective Potential



$$\begin{array}{cccc} KA & KB & K'A & K'B \end{array}$$

$$\begin{pmatrix} V_1 & V_2 & 0 & 0 \\ V_2^* & V_1 & 0 & 0 \\ 0 & 0 & V_1 & -V_2^* \\ 0 & 0 & -V_2 & V_1 \end{pmatrix}$$

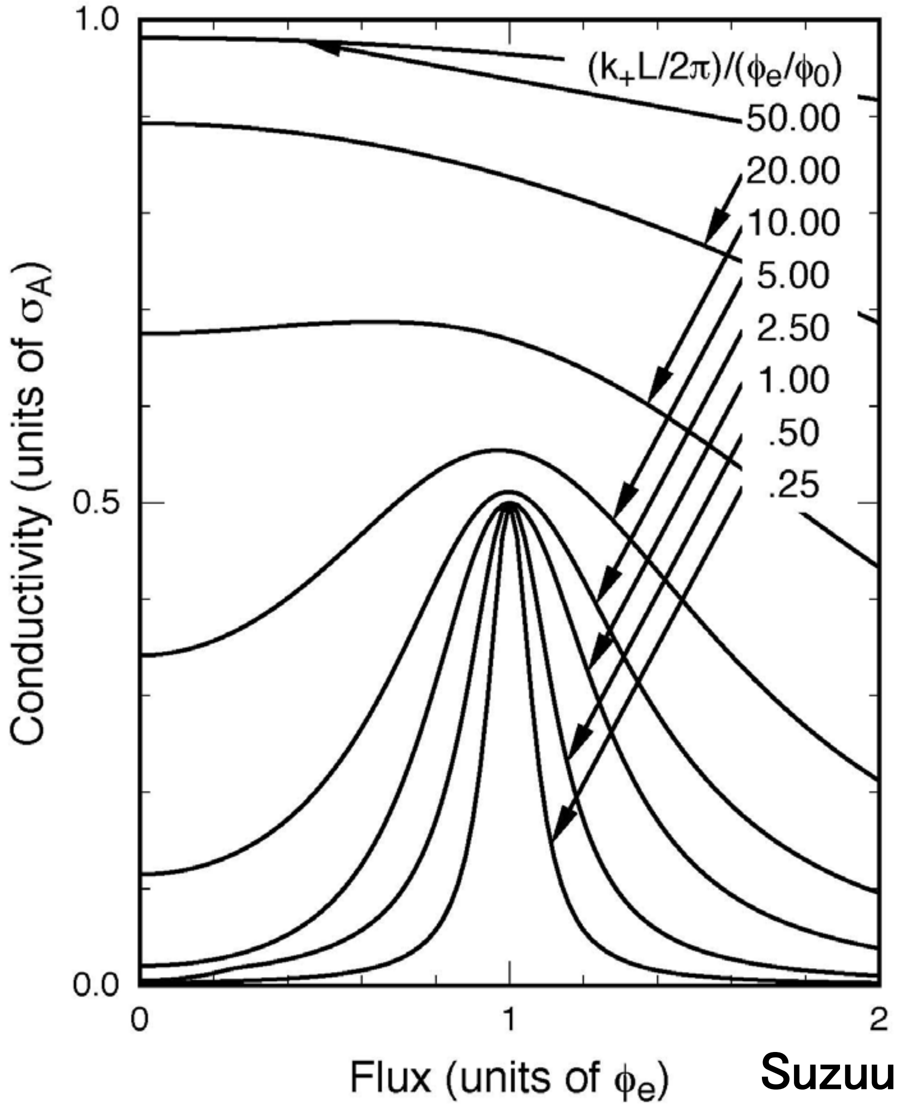
$$|V_1| >> |V_2|$$

$$\sigma_A = \frac{e^2}{h} \frac{2L\gamma^2\mu}{g_2^2 k_B T}$$

Λ~600L~1μm at room temp.
(10,10) armchair CN
Ballistic conductor

$$\sigma_A = \frac{e^2}{h}\frac{2L\gamma^2\mu}{g_2^2 k_B T}$$

Electron-Phonon



- **Similar peak**

$$|V_1| >> |V_2|$$

- **Effective potential**

$$\begin{array}{cccc} KA & KB & K'A & K'B \end{array}$$

$$\begin{pmatrix} V_1 & V_2 & 0 & 0 \\ V_2^* & V_1 & 0 & 0 \\ 0 & 0 & V_1 & -V_2^* \\ 0 & 0 & -V_2 & V_1 \end{pmatrix}$$

Suzuura and Ando, PRB 65 (2002) 235412

Semiconducting Nanotubes

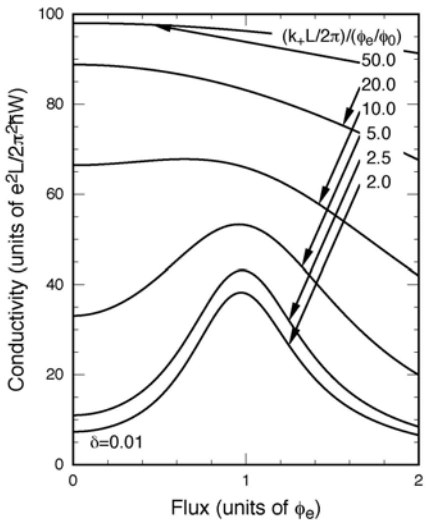
$\varphi_e \approx \pm 1/3$ too large to see the peak

$$\sigma(\varphi) = \sigma(0) + \frac{1}{2}\sigma''(0)\varphi^2 + \cdots$$

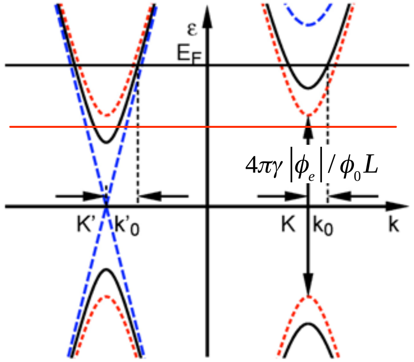
$$\sigma''(0) = \frac{4e^2}{\pi\hbar W}\frac{2\pi}{L}\frac{1-4\delta-\delta^2}{(1+\delta)^3}\frac{1}{k_+^2}$$

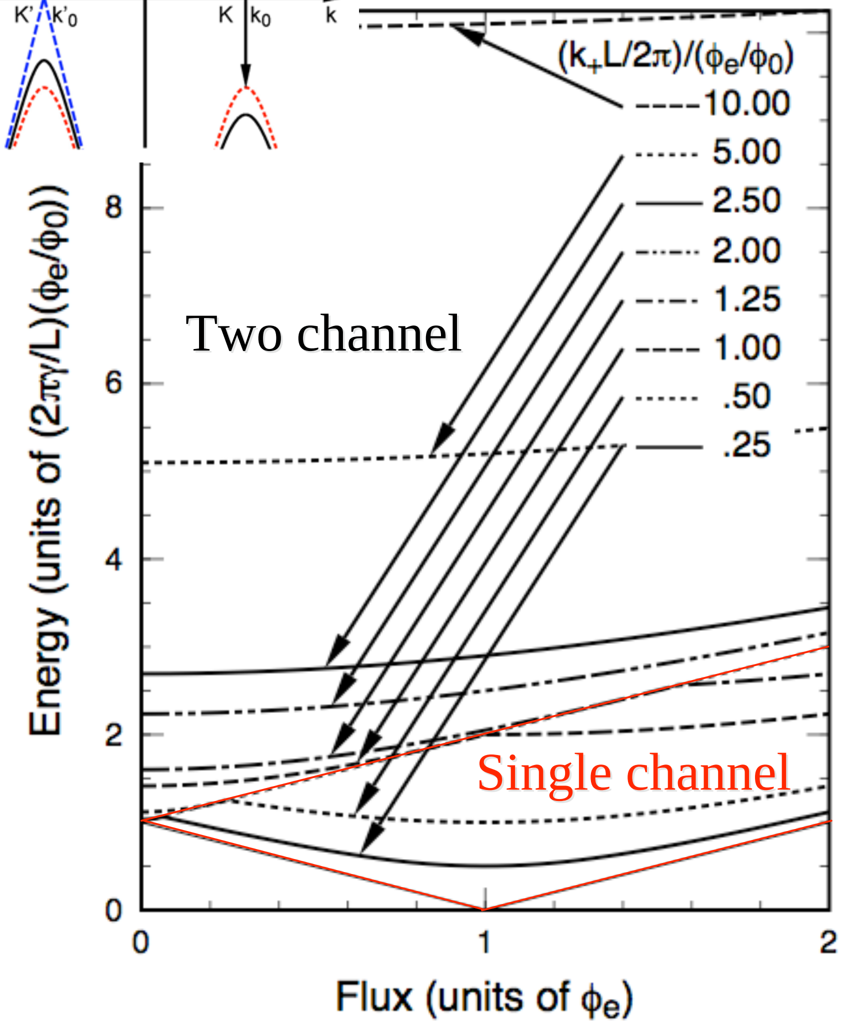
$\delta < \sqrt{5} - 2 = 0.236\cdots$ Positive magnetoconductivity (σ''>0)

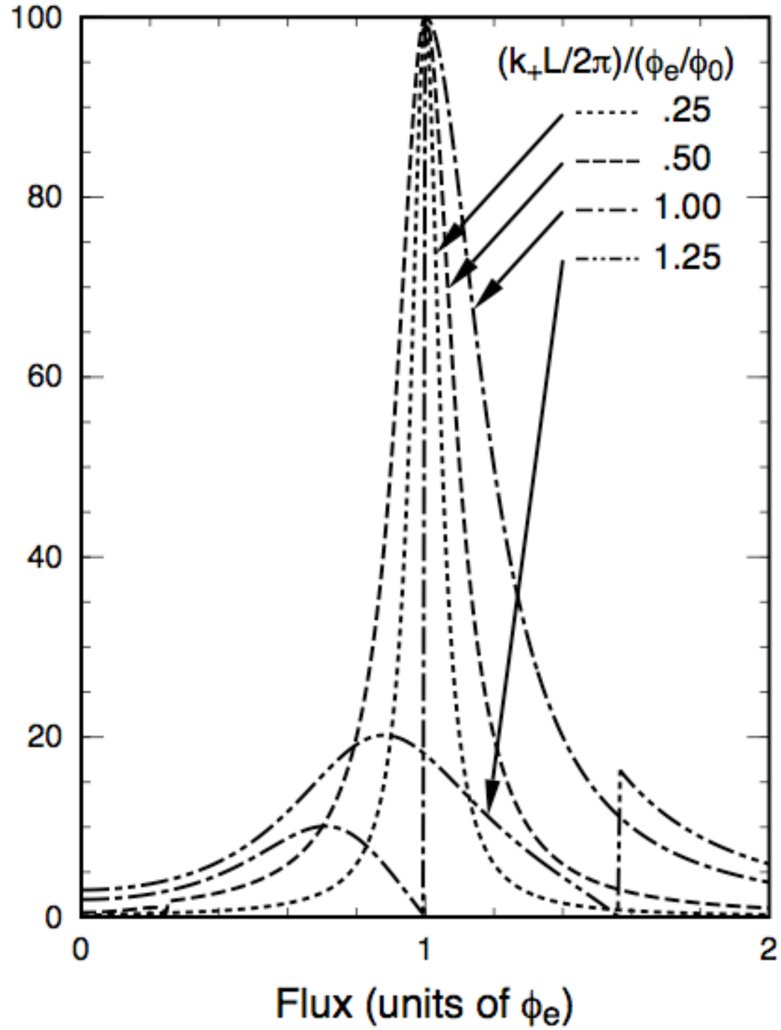
$\delta > \sqrt{5} - 2$ Negative magnetoconductivity (σ''<0)



Band depopulation

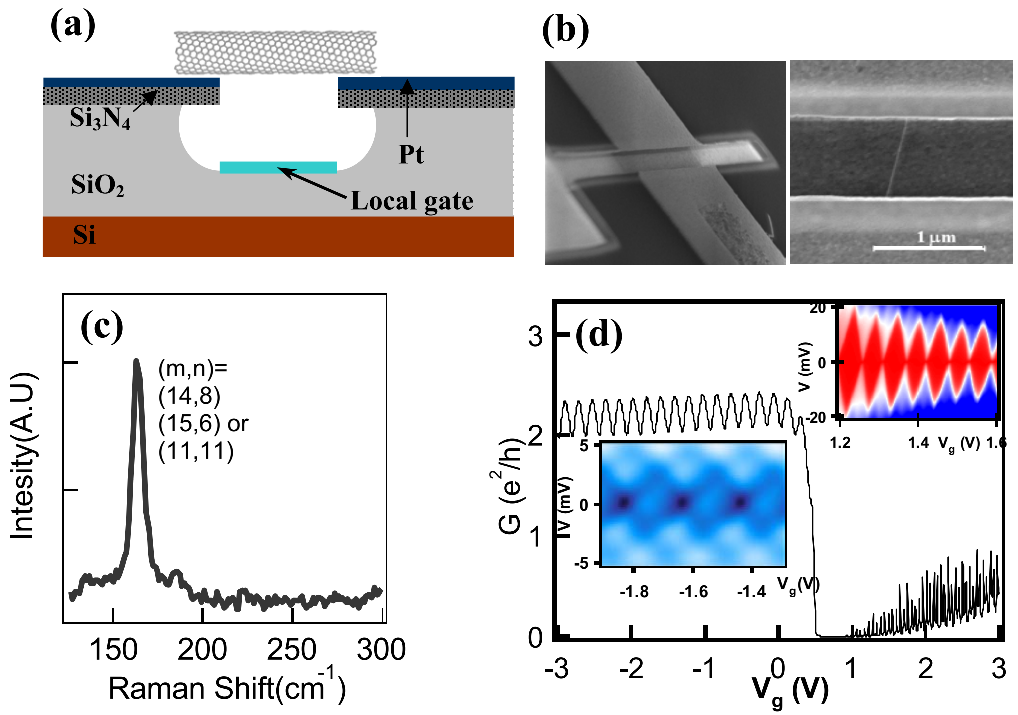


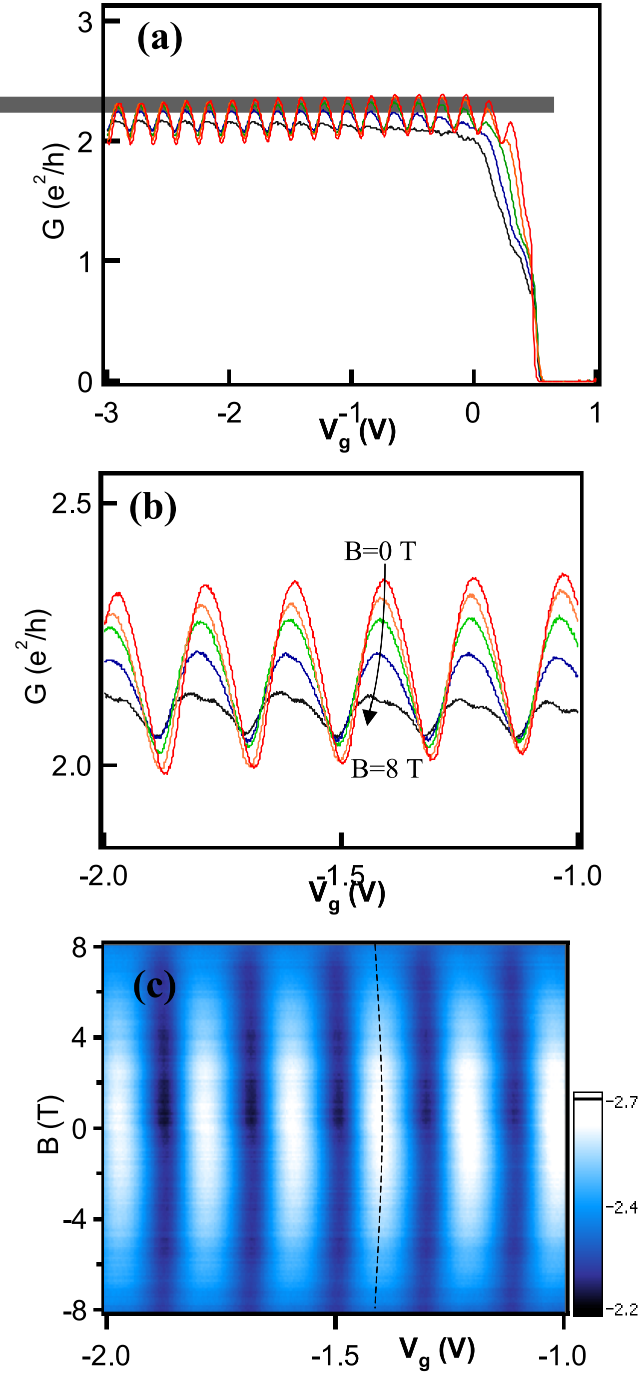




AB effect

J. Cao *et al.* (Stanford)
PRL 93, 216803 (2004)





Conclusion

- Boltzmann conductivity in carbon nanotubes with magnetic flux in axis direction
- Impurity and electron-phonon scatterings
- Prediction of an experiment to know
  - strain, curvature and stress effects
  - ratio of short- and long- range potential
  - ratio of deformation potential and bond-length change
- TN and T. Ando, JPSJ 74 (2005) 3027