The STAR-2 Model for Mapping Hierarchically Structured Analogs

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Although the importance of analogy in human cognition has long been recognized (e.g., Piaget, 1950; Spearman, 1923) and understanding of human analogical reasoning accelerated in the 1980s (Gentner, 1983, 1989; Gick & Holyoak, 1983) the explanatory potential of analogy has not been fully recognised. Computational modelling has advanced rapidly, as is clear from the chapters in *this volume*, and this has been accompanied by increasingly wide applications. A role has been been proposed for analogy in a variety of cognitive phenomena including scientific understanding (Dunbar, *this volume*; Gentner & Gentner, 1983), political reasoning (Holyoak & Thagard, 1995) and children's mathematics (English & Halford, 1995). Analogy is recognised as a natural mechanism for human reasoning, since it is possible from an early age (Goswami, 1992; Halford, 1993) and can even be performed by some nonhuman primates such as chimpanzees (Oden, Thompson & Premack, 1998; Premack, 1983). However, despite recognition of the power and widespread availability of analogy, it is not yet as widely utilised in modelling human cognition as we might have expected.

Part of the reason why the explanatory potential of analogy in human cognition has been under-utilised might be that it has sometimes been difficult to demonstrate its effects on human problem solving in the laboratory. Analogy is a structure-preserving map from a base or source to a target (Gentner, 1983) but unless participants are given extensive training on the base analog, they tend to focus on superficial attributes rather than recognising relations that form the deeper basis for the analogy (Gick & Holyoak, 1983). This is by contrast with real-life analogies where structural correspondences between situations tends to be more readily recognised. The chapter by Dunbar (*this volume*) is very timely in indicating possible reasons for the paradox that effective analogical reasoning occurs so readily in naturalistic settings yet is so difficult to demonstrate in laboratories. Analogies in real life tend to be idiosyncratic and ephemeral, often lasting only as long as necessary to solve a problem, and then being forgotten. Most importantly, people are more likely to focus on structural aspects when they generate

analogies for themselves. These factors makes analogical reasoning difficult to manipulate experimentally. If research such as that by Dunbar and his collaborators leads to more effective techniques for promoting analogical reasoning in laboratory studies it might increase experimentation on the explanatory power of analogy.

We have argued elsewhere (e.g. Halford, 1993) that there is potential for analogy theory, in combination with capacity theory, to explain phenomena in cognitive development. We will briefly review some examples to indicate the kind of hypotheses that can be generated.

Analogy as a Mechanism in Children's Reasoning

Developmental research on analogy has tended to focus on age of attainment (e.g. Goswami, 1992, *this volume*) rather than on the role of analogy as a model of children's reasoning. However we will illustrate its explanatory potential in two domains, transitive inference and class inclusion. It is well established that transitive inference can be performed by ordering premise elements into an array (Riley & Trabasso, 1974; Sternberg, 1980a, b) and Halford (1993) has pointed out that this can be interpreted as mapping premises into an ordering schema, as shown in Figure 1A. Given premises such as Tom is taller than James, Mike is taller than Tom, we can construct the order Mike, Tom, James. At first sight this might not seem to be a case of analogy, but on reflection we see that it amounts to assigning the three names to slots in an ordering schema. We all learn ordering schemas such as top-bottom or left-right at an early age, and they can function effectively as templates for ordering. The template is really the base for an analogy, and the ordered set of names is the target. This illustrates that analogy may play a subtle role in a lot of situations where it has not been recognised. Consequently, analogy theory, as it has been developed over the last two decades, has a lot of unrecognised potential for generating hypotheses about such tasks.

We have been interested in generating hypotheses based on the processing demands of analogical mapping. The load occurs in this case because in order to map Mike, Tom, James into top, middle, bottom respectively, both premises must be considered. The premise Tom is taller than James, taken alone, only tells us that Tom should go in top or middle position, and that James should go in middle or bottom position. Similarly for the premise Mike is taller than Tom. To assign Mike, Tom, James uniquely to ordinal positions both premises must be considered jointly. It is the integration of the premises, each of which represents a binary relation, into a ternary relation, that imposes the processing load for both children (Halford, Maybery, & Bain, 1986) and adults (Maybery, Bain, & Halford, 1986). This provides a possible explanation for the difficulty which young children experience with transitive inference (Andrews & Halford, 1998).

Another task which young children have found difficult for reasons that have not been easy to explain is class inclusion (Halford, 1993) as shown in Figure 1B. A class inclusion problem for children might entail presenting a small set of apples and oranges and asking "are there more fruit or more apples". Unlike transitive inference, the basis of solution is not known, and analogy theory can help fill this gap. In principle, the problem can be solved by mapping into a familiar schema that is isomorphic to the inclusion hierarchy. A suitable schema would be the family, because it includes parents and children. Fruit can be mapped into family, apples into (say) parents, and non-apples into children. Family is more numerous than parents, and this becomes a candidate inference for the target. By reverse mapping, it can be concluded that fruit are more numerous than apples.

The difficulties children have with these tasks can be explained by the complexity of information that must be processed to perform the correct mapping (Halford, 1993). Notice that, in order to determine the correct mapping, it needs to be recognised that fruit and family

are superordinates, whereas apples-nonapples, and parents-children, are subordinates. The difficulty of recognizing this is that the status of a category as superordinate or subordinate is not inherent in the category, but is defined by its relation to other categories. For example, neither fruit nor family is inherently a superordinate. If the hierarchy had been fruit, meat and food, fruit would have been a subordinate and food a superordinate. Fruit is a superordinate because it includes a subordinate and its complement, that is apples and nonapples. Similarly, family is a superordinate because it includes parents and children. To determine the correct mapping, relations between the three classes must be taken into account. This means mapping a ternary relation, between fruit, apples, nonapples to another ternary relation, between family, parents, children, as shown in Figure 1A. Mapping ternary relations imposes a high processing load (Halford, Wilson, & Phillips, 1998a) for both adults and children, which explains one source of difficulty.

Analogy in mathematics education

Another little recognised application for analogy occurs in mathematics education, where concrete aids representing mathematical concepts have been analysed as analogs. Differences in effectiveness can be explained by the complexity of the information that is required to determine the mapping from the analog to the concept represented (Halford, 1993; English & Halford, 1995).

Analogy theory in reasoning

There is also scope for analogy theory to have a greater explanatory role in logical inference. Some models are based on formal inference rules (Braine, 1978; Rips, 1989) but most theorists have chosen to model reasoning on the basis of alternative psychological mechanisms such as memory retrieval (Kahneman & Tversky, 1973) mental models (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991) or pragmatic reasoning schemas (Cheng & Holyoak, 1985). There are

also models based on more specialised mechanisms such as cheater detection (Cosmides, 1989; Cosmides & Tooby, 1992).

The major issues here can be exemplified in the Wason Selection Task (Wason, 1968) shown in Figure 2. In this task participants are given four cards containing p, ~p, q, ~q (where ~p means not-p) and asked which cards must be turned over to test the rule $p \rightarrow q$ (p implies q). The correct choices, p and ~q, are rarely chosen in abstract versions of the task (see reviews by Evans, 1982, 1989). The literature on techniques for improving performance on this task includes evidence for an effect of analogy (e.g. Cox & Griggs, 1982) but the scope for improvement has probably increased because of greater understanding of analogical reasoning processes. An example of this is that the improvement induced by pragmatic reasoning schemas, such as permission (Cheng & Holyoak, 1985) appears to be interpretable as analogical reasoning.

The general form of the permission schema is that in order to perform action p, it is necessary to have permission q. The disconfirming case is p and ~q, where the action is performed without permission. Pragmatic reasoning schemas are sometimes interpreted as being specialised for deontic reasoning (Oaksford & Chater, 1994; Rips, 1994; Almor & Sloman, 1996), but it appears to have been overlooked that they may be utilized by analogical mapping. We will use the definition of pragmatic reasoning schemas as structures of general validity that are induced from ordinary life experience. This includes not only deontic rules such as permission and obligation, but other rules such as prediction and cause, and also extends to social contract schemas such as cheater-detection. As Figure 2B shows, the elements and relations presented in the WST task can be mapped into a permission or prediction schema. This can be done by application of the principles that are incorporated in contemporary computational models of analogy (Falkenhainer, Forbus, & Gentner, 1989; Gray, Halford,

Wilson, & Phillips, 1997; Hummel & Holyoak, 1997; Mitchell & Hofstadter, 1990) and no special mechanism is required. Most of the major findings attributed to different formats might be more parsimoniously interpreted in terms of analogical mapping.

In this theory, a possible reason why induction of a permission schema improves performance is that, as Table 1 shows, permission is isomorphic to the conditional. Extending this argument, a possible reason for the tendency to respond in terms of the biconditional $p \leftrightarrow q$, is that in the standard version of the task participants may interpret the rule as a prediction. As Table 1 shows, prediction is isomorphic to the biconditional (Halford, 1993). It implies that the importance of permission is not that it is deontic, but that it is isomorphic to the conditional. As the canonical interpretation of the task is based on the truth-functional definition of the conditional, this mapping produces more responses that are deemed to be correct. By contrast, a lot of the errors produced by the standard version of the task are attributable to the rule being interpreted as a biconditional. This is what would be expected if the task were mapped into the prediction schema, because prediction is isomorphic to the biconditional.

While we would not suggest that this argument accounts for all the effects associated with either the Wason Selection Task or pragmatic reasoning schemas, it does serve to illustrate that analogy can serve as the basic mechanism even in tasks such as WST that might normally be considered to entail logical reasoning. Analogical mapping explanations of WST performance are not confined to schemas such as permission and prediction, but will apply in principle to any schema with a structure sufficiently close to the WST problem to permit a mapping to be made. It therefore has potential to subsume other explanations, including those based on social contract theory.

Cosmides and Tooby (1992) have argued that performance in the WST does not reflect general purpose reasoning mechanisms, but is based on a cheater detection schema that has

evolved because of its adaptive value in a social environment. Thus the rule p → q can be represented as:

"If you take the benefit, then you pay the cost" (benefit → cost);

The correct choice, p and ~q, corresponds to cases where the benefit is taken without paying the cost. One specific prediction made by social contract theory is that the choices should be the same if the rule is switched;

"If you pay the cost, then you take the benefit" (cost → benefit).

Social contract theory predicts that favoured choices with the switched rule will continue to be those where the benefit is taken without paying the cost. However because the rule is now "cost → benefit", the logically correct choices, corresponding to p and ~q, are those where cost is paid and the benefit is not accepted. The data favour the social contract theory prediction, that is, cases where the benefit is taken without paying the cost, are chosen.

It is possible however that the cheater detection schema could be used by analogical mapping as shown in Figure 2C. The rule benefit → cost is isomorphic to the conditional. It is really another interpretation of the permission rule. Therefore, while Cosmides and Tooby may well be correct in their claim that cheater detection is a powerful schema for reasoning about the WST, the phenomena they observe are quite consistent with the principle that performance on the task reflects a general purpose, analogical reasoning process.

The tendency to make the same choices with the original and switched rules might simply reflect a tendency by participants to transform the rule to a more familiar form. Thus the rule "If you pay the cost, then you take the benefit" might be transformed to "If you take the benefit, then you pay the cost". They might switch rules based on the principle of the complementarity of rights (Holyoak & Cheng, 1995). There is plenty of evidence throughout cognition and cognitive development that the problem participants solve may be very different from the one

that experimenter intends (e.g., Cohen, 1981). The undoubtedly powerful tendency to apply the cheater-detection schema, irrespective of the surface form of the problem, might simply reflect a universal tendency to map the problem into a familiar schema. Viewed this way, cheater-detection is another reasoning schema that is isomorphic to the conditional, and the reasoning mechanism might be another case of analogy.

Analogy and relations in higher cognition

We have been exploring the potential of relational knowledge to provide a basis for the theory of higher cognitive processes. We have argued that the implicit-explicit distinction may be captured by the contrast between associations and relations (Phillips, Halford & Wilson, 1995) and that the theory of relational knowledge captures the properties of higher cognition and accounts for processing capacity limitations (Halford et al., 1998a,b). This theory has been implemented as a neural net model (Halford et al., 1998a). Analogy plays a central role in this theory, so it is essential to demonstrate that analogy can be implemented in the architecture of the neural net model. That is the primary purpose of the STAR model of analogy, to be presented next.

The STAR model of analogy

The Structured Tensor Analogical Reasoning (STAR) model of analogical reasoning (Halford et al., 1994) is a neural net model, and the representations used are designed to be consistent with human processing capacity limitations.

An analogy is a structure-preserving map from a base or source to a target (Gentner, 1983). The structure of base and target are coded in the form of one or more propositions. An analogical mapping between base and target consists of a mapping of the propositions in the base to the propositions in the target. Each proposition consists of a binding between a relation-symbol (e.g. bigger-than or CAUSE) and a number of arguments (e.g. bigger_than(dog, cat) or CAUSE(pressure-difference, water-flow)). A proposition is like a relational instance in that a

proposition with *n* arguments and an n-ary relation both comprise a subset of the cartesian product of *n* sets. However a proposition, unlike a relational instance, need not be true. True and false propositions correspond to different subsets of the cartesian product (Halford et al., 1998a, section 2.2.2).

The STAR-1 model

The problem for any neural net model of analogy is how to represent the propositions that comprise the base and target of the analogy. This is essentially the same problem as how to represent relational instances. There are two major classes of approaches to this problem. One is based on synchronous oscillation model (Hummel & Holyoak, 1997; Shastri & Ajjanagadde, 1993) while the other is based on product operations such as circular convolution (Plate, 1998) or tensor products (Smolensky, 1990). Our approach is based on a tensor product representation of relational knowledge. The first version, which we now designate STAR-1, (Halford et al., 1994), is briefly described below. A further development, STAR-2, is described in detail in this chapter.

Our approach to representing propositions and relational instances is to represent the relation symbol and each argument by a vector. The binding is represented by computing the tensor product of the vectors. Thus the n-ary relation R on $A_1 \times A_2 \times ... \times A_n$ is represented on a tensor product space $V_R \otimes V_1 \otimes V_2 \otimes ... \otimes V_n$. The vectors used to represent concepts in each space V_i are orthnormal (i.e. orthogonal to each other and of length 1). To illustrate, binary relational instances mother-of(mare, foal)and loves(woman, baby) are represented by $v_{mother\text{-}of} \otimes v_{mare} \otimes v_{foal}$ and $v_{loves} \otimes v_{woman} \otimes v_{baby}$. The representing vectors $v_{mother\text{-}of}$ and v_{loves} are orthogonal, and so are the pairs v_{mare} and v_{woman}, and v_{foal} and v_{baby}. The representation handles all of the properties of relational knowledge as well as providing a natural explanation for limits to human information processing capacity (Halford et al., 1998a).

Information can be retrieved from this representation using a generalized dot-product operation. For example the query “what is mare the mother of?” can be answered by defining the probe mother-of(mare, -) and computing the dot product of the tensor product representing the probe with the tensor product representing the proposition, thus;

$$v_{\text{mother-of}} \otimes v_{\text{mare}} \bullet v_{\text{mother-of}} \otimes v_{\text{mare}} \otimes v_{\text{foal}} = V_{\text{foal}}$$

Any one or more components of the proposition can be retrieved in this way, as we will illustrate below.

Other propositions can be superimposed on the same represention. Thus mother-of(mare,foal) and mother-of(cat,kitten) can be superimposed by adding the corresponding tensor products thus;

$$v_{\text{mother-of}} \otimes v_{\text{mare}} \otimes v_{\text{foal}} + v_{\text{mother-of}} \otimes v_{\text{cat}} \otimes v_{\text{kitten}}$$

Simple proportional analogies of the form A is to B as C is to what (A:B::C:?) were simulated by superimposing an appropriate set of propositions on the same neural net representation as shown in Figure 3. We define a tensor product representation, T, of a set of propositions as;

$$T = v_{\text{mother-of}} \otimes v_{\text{mare}} \otimes v_{\text{foal}} + v_{\text{mother-of}} \otimes v_{\text{cat}} \otimes v_{\text{kitten}} + \ldots + v_{\text{larger-than}} \otimes v_{\text{mare}} \otimes v_{\text{rabbit}} + v_{\text{larger-than}} \otimes v_{\text{cat}} \otimes v_{\text{kitten}}$$

It is important that not all the stored propositions contribute to the required mapping. A proportional analogy, such as cat:kitten::mare:? can be solved by using the retrieval operation defined above. First we use “cat” and “kitten” as input, and the output is all the relation-symbols of the propositions that have “cat” in the first argument position and “kitten” in the second argument position. Thus the analogy cat:kitten::mare:foal can be performed by the following operations, as illustrated in Figure 3B .

$$v_{\text{cat}} \otimes v_{\text{kitten}} \bullet T = B_p = v_{\text{mother-of}} + v_{\text{feeds}} + v_{\text{protects}} + \ldots + v_{\text{larger-than}}$$

This output is called a “relation-symbol bundle”, and comprises the sum of the relation-symbols that are bound to cat and kitten. The relation-symbol bundle is then used as input in the second step.

$$B_p \otimes v_{mare} \bullet T = w_1.v_{foal} + w_2.v_{rabbit}$$

With w_1 and w_2 being the weighting of each vector in the output bundle, $w_1 > w_2$ due to foal satisfying more propositions in T than rabbit satisfies.

The ambiguity in the output is realistic, because cat:kitten::mare:rabbit is a valid, though not very satisying, analogy. Just as a cat is larger than a kitten, a mare is larger than a rabbit. However the solution “foal” is more satisfying because the arguments mare-foal are bound to more relation-symbols than the pair mare-rabbit. Thus we have mother-of(mare,foal), feeds(mare,foal), protects(mare,foal), larger-than(mare,foal), whereas the only proposition with the argument pair mare-rabbit is larger-than(mare,rabbit). The fact that $w_1 > w_2$ reflects the larger number of relation-symbols bound to mare and foal.

STAR-1 was also able to perform other forms of analogy, including those in which the relation-symbol was missing, and those where a base had to be retrieved. It also incorporated realistic processing capacity constraints, and was in fact the first model to do so. A metric for quantifying the complexity of structures that can be processed in parallel was required, and it was also necessary to explain how problems that exceed this capacity are processed. We proposed that a metric in which complexity of relations is quantified by the number of arguments is the best for this purpose (Halford et al., 1994; 1998a).

Capacity and Complexity

It is important that a model of human analogical reasoning should conform to human processing capacity limitations. This in turn raises the question of how to quantify processing capacity. The STAR model is designed to conform to the theory that capacity limitations can be defined by the complexity of relations that can be processed in parallel (Halford, 1993; Halford

et al., 1994; Halford et al., 1998a). In this section we will outline the relational complexity approach to defining processing capacity limitations.

The number of arguments of a relation corresponds to the dimensionality of the space on which the relation is defined. An N-ary relation can be thought of as a set of points in N-dimensional space. Each argument provides a source of variation, or dimension, and thereby makes a contribution to the complexity of the relation. A unary relation (one argument) is the least complex, and corresponds to a set of points in unidimensional space. A binary relation (e.g. BIGGER-THAN) has two arguments, and is defined as a set of points in two-dimensional space. A ternary relation has three arguments (e.g. love-triangle is a ternary relation, and has arguments comprising three people, two of whom love a third) is defined on three-dimensional space, and so on.

Relations of higher dimensionality (more arguments) impose higher processing loads. The working memory literature, plus some specific experimentation, has led to the conclusion that adult humans can process a maximum of four dimensions in parallel, equivalent to one quaternary relation (Halford et al., 1998a).

The tensor product representation provides a natural explanation for observations that processing load increases as a function of the arity of relations, and also explains why processing capacity would be limited to relations with only a few arguments. As Figure 3A illustrates, the number of binding units equals the product of number of units in all vectors, and increases exponentially with the number of arguments. Representation of an n-ary relation requires n+1 vectors, one for the relation symbol and one for each argument. If the relation symbol and each argument are represented by a vector with m elements, then the number of binding units for an n-ary relation is m^{n+1}.

Structures more complex than quaternary relations must be processed by either conceptual chunking or segmentation. Conceptual chunking is recoding a concept into fewer dimensions.

Conceptual chunks save processing capacity, but the cost is that some relations become temporarily inaccessible. Segmentation is decomposing tasks into steps small enough not to exceed processing capacity, as in serial processing strategies.

Even a relatively simple analogy, such as that between heat-flow and water-flow shown in Figure 4 has a structure that is too complex to be processed entirely in parallel by humans. Consequently it is segmented into propositions that do not exceed processing capacity, and which are processed serially. The most efficient way to do this is to construct a hierarhical representation, as illustrated for heat-flow and water-flow in Figure 4 . However STAR-1 was not able to process hierarchically structured knowledge representations. This therefore was the primary motivation for STAR-2.

The STAR-2 Model

STAR-2 processes complex structures that are represented hierarchically, as illustrated in Figure 4. Each argument is either an element, representing a basic object (e.g. water) or a chunked proposition (e.g. water-flow is a chunked representation of the proposition flow(vesselA, vesselB, water, pipe)). The height of a proposition in a hierarchy gives an indication of how much chunked structure it contains. First-order propositions have elements as arguments, while higher-order propositions have chunked propositions as arguments. First-order propositions are of height 2, and the height of a higher-order proposition is the height of the highest unchunked argument plus one.

The STAR-2 model forms mappings between domains containing multiple propositions while conforming to the working memory limitation of only mapping a single pair of quaternary propositions at a time. In order to do this, the model sequentially selects corresponding pairs of propositions from the base and target. The relation symbols and arguments in each selected pair of propositions are mapped in parallel before a new base and target pair of propositions are selected. The sequential selection of pairs of propositions can be

seen as a form of segmentation, sequentially focusing on propositions of acceptable dimensionality in order to form a mapping between higher dimensional concepts (e.g. the heat flow and water flow domains). Both the parallel mapping of relation symbols and arguments as well as the sequential selection of proposition pairs are performed by constraint satisfaction networks, indicating a degree of computational similarity between the sequential focus selection and the parallel mapping processes.

The model consists of three main structures: the Focus Selection Network, the Argument Mapping Network, and the information storage structures, which include a map-storing network. To illustrate the process of the model and the interaction of these structures we will consider the heat-flow/water-flow analogy (Figure 4). The basic steps involved in the model are as follows:

1. Initially, the information storage structures are established to represent the domains being mapped. These structures store information such as similarity of items in the domains, salience of propositions, item-types (e.g. relation symbol, number, animal) and chunked-proposition/unchunked-proposition associations. This information is specified as input to the model.
2. The focus selection network is established and used to select a single base/target pair of propositions that will form the first pair of propositions to be mapped (ie. the first focus). This selection is influenced by a number of heuristics detailed later. In the heat-flow/water-flow example the causal propositions of each domain were selected, largely due to their height and the "Cause" relation symbol that was common to the propositions in base and target.
3. The argument mapping network is established and forms mappings between the relation symbols and arguments of the currently selected propositions. In the heat-flow/water-flow example the mappings formed are Cause ↔ Cause, and the propositions Greater

Pressure ↔ Greater Temperature and Flow ↔ Flow. These mappings are then stored in the map storing network which maintains a store of all the mappings currently formed.

4. Connectivity in the focus selection network is modified to incorporate the mappings formed in step 3 and to ensure that previously selected propositions are not re-selected. The network then selects another pair of propositions to be mapped. In the example, the Greater Pressure proposition and the Greater Temperature proposition are selected due to their height and the mapping formed between them in step 3. These propositions then form the next focus.
5. The argument mapping network is then re-established to map the relation symbols and arguments of the current propositions. The mappings Greater ↔ Greater, Pressure(Vessel A) ↔ Temperature(Coffee) and Pressure(Vessel B) ↔ Temperature(Ice) are formed and stored in the map storing network.
6. The model then repeatedly:
   A. Updates the connectivity to the focus selection network.
   B. Selects a new pair of propositions.
   C. Maps the components of the selected propositions in the argument mapping network.
   D. Stores the new mappings in the map storing network,

   until a termination criterion is met. The order of selection of propositions in the example is indicated by the superscript numbers in Figure 4 and the order of mappings formed is also indicated in the figure.

On termination of the algorithm, all mappings for the analogy are retrievable from the map storing network. We will now consider each of the components of the model in more detail.

Argument Mapping Network

This is a constraint satisfaction network[1] and consists of up to five rows and five columns of mapping nodes. Each row corresponds to either the relation symbol or an argument position of the currently selected base proposition, while each column corresponds to either the relation symbol or an argument position of the currently selected target proposition. Each mapping node, therefore, represents a potential mapping between a relation symbol/argument in the base proposition and a relation symbol/argument in the target proposition.

An example of the network is shown in Figure 5, mapping the top proposition of water-flow to heat-flow. That is, it is mapping the causal relation between pressure-difference and water-flow to the causal relation between temperature difference and heat flow. Each of the nodes has an associated activation value that is updated by the excitatory and inhibitory input from each of the other nodes. Inhibitory connections between all nodes in the same row or column tend to make mappings unique; that is each base element is mapped to at most one target element and vice verse. Excitatory connections exist between all nodes not in the same row or column to allow a stable growth of activation. When the activations stabilise, winning nodes are selected due to their greater activation, and indicate the mapping adopted. The shaded nodes in Figure 5 show the winning nodes, that represent mapping of the relation-symbol and arguments of the base to the relation-symbol and arguments of the target.

Mapping heuristics bias the mappings selected in the network towards conforming to various informational constraints. They are implemented by influencing nodes that provide constant excitatory or inhibitory input to the mapping nodes, biasing them towards or away from becoming winning nodes. These include:

Corresponding argument positions. This node provides a bias towards mapping relation symbol to relation symbol, and to mapping arguments in corresponding positions in base and target. For

[1] Constraint satisfaction networks have been used in a number of PDP analogy models since first introduced by ACME. A detailed explanation of constraint satisfaction operational mechanics as well as details of ACME

example, if the proposition R(a,b) was being matched to the proposition R'(c,d) then there would be a bias towards the mappings R ↔ R', a ↔ c, b ↔ d.

Similarity - there is a bias to map identical or similar entities. Semantic similarity of items is specified as input to the model, and identical items are identified through a common label.

Type - items are initially specified with a type such as relation symbol, object, number or animal and there is a bias to map items of identical or previously mapped types. As mappings of the elements of the domains are formed and stored in the map storing network (detailed later), the corresponding types of elements also form type-mappings, which are also stored to the map storing network. These mappings then influence later instances of the argument mapping network.

Salience. Items are initially specified with a salience and there is a bias towards mapping pairs of items with a higher salience and a bias away from mappings between items with a difference in salience.

Consistency. There is a bias towards mappings that are consistent with previous mappings and a bias against mapping unlike components, such as elements to propositions, or relation symbols to arguments. The strength of potential mappings is derived from the map storing network in the form of a mapping score (detailed later) as each argument mapping network is instantiated. This allows connectivity to be established that biases towards strong mappings, but is not a hard constraint so one-to-many and many-to-many mappings can still be formed if implied by the domains.

Focus Selection Network

The Focus Selection Network (see Figure 6) is a two-layer constraint satisfaction network designed to select the pair of propositions that will form the next focus, to be passed on to the

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can be found in Holyoak & Thagard (1989).

argument mapping network. Each layer has a structure similar to the argument mapping network. Within a layer each row represents a proposition from the base, and each column a proposition from the target. Thus each node represents a base/target pair of chunked propositions. For example, in the water-flow/heat-flow analogy in Figure 4, one node represents the pair of causal propositions from base and target. The constraint satisfaction process eventually results in a winning node that indicates the next pair of propositions to be mapped.

Layer One (the lower layer) is influenced by many of the same sources of activation as the mapping network, with additional heuristics, biasing towards selecting propositions with similar height in the hierarchy, similar number of arguments and with corresponding relation-symbols and arguments.

Excitatory connections are placed between focus selection nodes in which the propositions represented by one node are arguments to the propositions represented by the other node. For example, referring to Figure 4, in the heat-flow/water-flow domain an excitatory connection exists between the node representing the pair of CAUSE propositions (superscript 1) and the node representing the pair of FLOW propositions (supercript 3). This connectivity provides a bias towards matching similarly-shaped tree structures. To illustrate, if the node representing the pair of CAUSE propositions developed a significant activation, the excitatory connection to the node representing the pair of FLOW propositions would increase the activation of this node and also the node representing the pair of GREATER propositions (superscript 2). Over the entire analogy this will mean that nodes in the corresponding positions in the heat-flow and water-flow trees will support each other. The connectivity results in Layer 1 settling to a state in which a group of nodes develops a strong activation, representing a set of consistent base/target proposition pairs to potentially be mapped.

The first layer is influenced by a number of heuristics through influencing nodes, including heuristics based on the height and height difference of propositions, saliance, similarity, types associated with propositions and common or previously mapped propositions, arguments and relation symbols. The connectivity from the influencing nodes can be modified between runs of the network to accommodate newly formed mappings. For example, in heat-flow/water-flow analogy, if the argument mapping network had been loaded with the pair of CAUSE propositions, resulting in mappings being formed between the pair of GREATER and the pair of FLOW propositions, excitatory connectivity from the appropriate influencing node to the nodes representing the GREATER and FLOW proposition pairs will be established and increase the likelihood of these proposition pairs winning in the next selection.

Layer Two is designed to select a single winning pair of chunked propositions from the set of winning terms in layer one. Within layer two, inhibitory connections are formed from each mapping node to every other mapping node. This connectivity forms competition between all the layer two mapping nodes and therefore results in a final, stable state of only one node winning. This single winning node represents the pair of chunked propositions that becomes the next focus.

The connectivity between layer one and two is displayed in figure 6. It is unidirectional so layer two cannot affect layer one. An excitatory connection is formed from each node in layer one to the corresponding node in layer two. This connectivity results in strong units (winning nodes) of layer one providing strong input to the corresponding unit in layer two, and losing nodes of layer one providing no significant input to layer two. Additional connectivity is formed between unit *i* in layer one and unit *j* in layer two if the chunked propositions represented by unit *i* are arguments to the propositions represented by unit *j* or vice versa. This corresponds to the excitatory connections between mapping nodes in layer one, and provides a bias for layer one winning nodes which are part of a layer one ``winning tree". If a pair of

chunked propositions has already been a successful focus, then the weight of the connections between the units in layer one and the unit in layer two representing these chunked propositions, as well as all other units in the same row or column, is set to 0. This stops the input to these units, and ensures that a previous focus, or foci inconsistent with it, will not be selected in future focus selections.

In addition to the above, connectivity is formed from some of the influencing nodes in layer one to the focus selection nodes in layer two. While the influencing nodes in layer one biased which combination of nodes (tree) would win, in layer two the bias is to which of the nodes in the winning combination becomes the focus. The heuristics for influencing connectivity to layer two are based on the height of propositions and common or previously mapped relation symbols.

Information storage structures

Information storage structures are used to store information about the entities (propositions, elements and relation-symbols) in the base and target. A number of tensor networks store information about similarity between pairs of entities, salience of entities, entity - type associations and chunked proposition - unchunked proposition associations. In addition to these networks, a rank two tensor ``mapping tensor" (or “map storing network”) is used to store mappings between entities as they are formed by the argument mapping network.

Mapping Scores are designed to reflect the uniqueness of mappings from base to target, and also the salience of the mapped entities. Uniqueness is a soft constraint, and non-unique mappings can occur if an element in the target is mapped to different base elements, or *vice verse*, on different foci. When a base item *a* is mapped to a target item *b*, a weighted form of the tensor product $a \otimes b$ is superimposed on the mapping tensor. If a mapping formed by the argument mapping network is the same as one stored from a previous focus, the new mapping is still superimposed on the mapping tensor to increase the strength of the mapping.

Corresponding type mappings are stored as each atomic element and relation symbol mapping is stored.

To reflect salience, the tensor product $a \otimes b$ is weighted by multiplying the resulting binding units by either:

1. The average salience of the two propositions that formed the focus from which the mapping was made.

2. The single specified salience if only one of the two propositions has a specified salience.

3. The average of all specified saliences of propositions if neither of the two propositions has a specified salience.

This weighting results in mappings formed in salient foci being considered more important than mappings formed in less salient foci.

The first step in computing the mapping score is to retrieve the vector bundle comprising all target (base) elements mapped to base (target) item *a*, from the mapping tensor. The ``base to target mapping score" for the a-b mapping is calculated as follows:

base to target mapping score = b • vector_bundle/ |vector_bundle|

where vector_bundle is the vector retrieved from the calculation a • mapping tensor

This mapping score will be in the range 0 to 1 and indicates the degree of uniqueness of the mapping. A value of 0 means that b has never been mapped to a, and 1 means that b is the only item mapped to a. A value between 0 and 1 (exclusive) indicates the number of items, including b, that have been mapped to a. The higher the value the greater the strength of the a-b mapping relative to other mappings into which a has entered.

The target to base mapping score can be calculated in analogous manner, but it is not necessarily equal to the base to target mapping score (e.g. if item *a* in the base was mapped to several items in the target, including *b*, but item *b* was only mapped to item *a* in the base). To

provide an overall indication of the strength of the a-b mapping, an overall mapping score is calculated as follows:

overall mapping score = (base to target mapping score + target to base mapping score) / 2.

This overall mapping score is used to determine biases in the Argument Mapping Network and the Focus Selection Network.

Termination Criteria

In the current implementation a test for termination occurs when either the Focus Selection Network selects a previous focus, or a focus that is inconsistent with a previous focus, or the Argument Mapping Network converges to a state in which a set of mappings cannot be interpreted from the result. The percentage of chunked propositions in the smaller domain that have formed a focus is calculated and labeled the percentage focused. If this percentage focused is greater than 90% then the algorithm terminates and has successfully found an analogy. Successful termination also occurs when all chunked propositions in the smaller domain have formed a focus. None of the tested domains required a failure termination.

Analogies Solved

In addition to heat-flow/water-flow, a number of other analogies have been solved by the model. These include:

The Rutherford analogy between the structure of the solar system and the structure of the hydrogen atom has more propositions than heat-flow/water-flow and has a more complex structure, but is successfully handled by the STAR-2. The same representation adopted for SME (Falkenhainer et al., 1989) was used, as shown in Figure 7. This included a structured representation of the solar system involving the gravitational attraction of the sun and planets, the mass difference of the sun and planets and the revolution of the planets around the sun. The corresponding structure of the atom was partly represented, omitting some of the higher order

relations. Irrelevant details were added to both the solar system and the atom in the form of the temperature difference of the sun and planets and the mass difference of the electron and the nucleus.

STAR-2 successfully mapped all the corresponding structure of the two domains, initially focusing on the highest available corresponding causal relationships, and then moving down the corresponding structures. Once the corresponding structures were mapped, STAR then went on to map the irrelevant sun/planet temperature difference to the irrelevant electron/nucleus mass difference.

Jealous animals problem is an analogy between isomorphic children's stories where animals play the roles in the story (Gentner & Toupin, 1986) (See Figure 8). A number of versions were tested in which the corresponding animals' similarity was varied as well as the presence of higher order propositions (see Holyoak & Thagard, 1989 for details of these analogies). The model correctly solved most versions of the analogy, but also made incorrect mappings on versions that are difficult for humans (e.g. where animal similarity worked against the structurally correct mappings). Six versions, designed to vary systematicity (defined by existence of higher-order relations) and transparency (defined as the same animal filling corresponding roles in base and target stories) in the same manner as Gentner and Toupin, were tested. Animal similarity was specified as an element similarity in the input. The solutions produced by STAR-2 to this analogy corresponded closely to those found by Gentner and Toupin (1986). For the systematic versions, all representations resulted in correctly mapped corresponding chunked propositions and relation symbols. For the unsystematic versions, the case where animal similarity worked against the structurally correct solution resulted in incorrect mappings of both animals, relation symbols and propositions. The other cases were mapped correctly.

Addition/Union was solved by ACME to demonstrate the ability to find isomorphisms without semantic or pragmatic information (Holyoak & Thagard, 1989). It is an analogy between the properties of associativity, commutativity and the existence of an identity element on numeric addition and set union. The properties of addition and union are in fact isomorphic, involve higher order propositions but have no common relation-symbols or arguments between the two domains (see Figure 9). STAR-2 solves this analogy despite the lack of common items.

In the representation solved by ACME all propositions were considered to be single level (height = 1), with additional elements introduced to represent intermediate results of additions (unions). For example, commutativity of addition (a + b = b + a) would be represented as Sum (a, b, c), Sum (b, a, d), Number_equal (c, d). In this form STAR-2 was unable to solve the analogy, as there was no heuristics to distinguish between the many first level propositions, a lot of which adopted the same relation-symbol (Sum or Union). STAR was however able to solve the same problem with the domains re-represented to incorporate chunking and higher order relations. In the modified representation commutativity of addition would be represented as follows:

Sum (a, b) chunk as sum_ab, and considered to be a number.

Sum (b, a) chunk as sum_ba and consider to be a number.

Number_equal (sum_ab, sum_ba) chunk as commutativity

The representation containing higher order relations corresponds to our introspections when performing this analogy, and appears to be more cognitively realistic. In a complex physics equation components would be chunked and higher order relations adopted that use the chunked components. For example velocity is distance divided by time, but is normally chunked as a single variable (e.g. d/t is chunked into v). Then v is used to represent acceleration = $(v_2 - v_1) / (t_2 - t_1)$. Here, acceleration could be considered a higher order concept than

velocity, rather than both single level concepts with intermediate values used to hold the results (as ACME's representation would imply).

Adopting these hierarchical representations, STAR was able to successfully map the entire analogy representing commutativity, associativity and identity. It would first focus on a pair of higher order propositions (associativity of union and addition) and then focus on the arguments of the selected higher order propositions, moving down the hierarchical structure. It would then repeat this for commutativity and identity existence.

The boy-dog analogy (Holyoak & Thagard, 1989) is an analogy in which the base and target have isomorphic structure, but there are no higher order propositions and no common relation symbols or arguments (see figure 10). The basic version is difficult for humans to solve, and it is not clear how participants succeed on it. One possibility would be back tracking, that is partially undoing an incorrect solution. Back tracking was not used in STAR2 because there does not yet appear to be definitive evidence that humans partially undo a failed attempt at analogy. Alternatively participants might start the analogy again, avoiding previously incorrect solutions. Neither approach is implemented in STAR2 as it stands, with the result that it fails the basic version of the boy-dog analogy. However humans are more successful with a modified order of presentation or additional information about similarities of relation symbols (see Keane, Ledgeway & Duff, 1994, for details on the versions of this analogy). Order of presentation was handled in STAR2 by an additional higher order proposition indicating which proposition was presented first (see Figure 10). Two additional versions with different sets of similarity ratings were also used. In accordance with human results STAR2 failed the basic analogy but was able to form all of the modified analogies.

Conclusion

The STAR-2 model of analogical mapping maps complex analogies through a combination of serial and parallel processing. Base/target pairs of propositions are selected sequentially

while mappings between the components of the propositions are formed in parallel. This corresponds to a form of segmentation over capacity limited relational domains and thus conforms to observed psychological capacity limitations in the complexity of relations that can be processed in parallel. The model has been tested on five analogies and displays a correspondence with psychological results.

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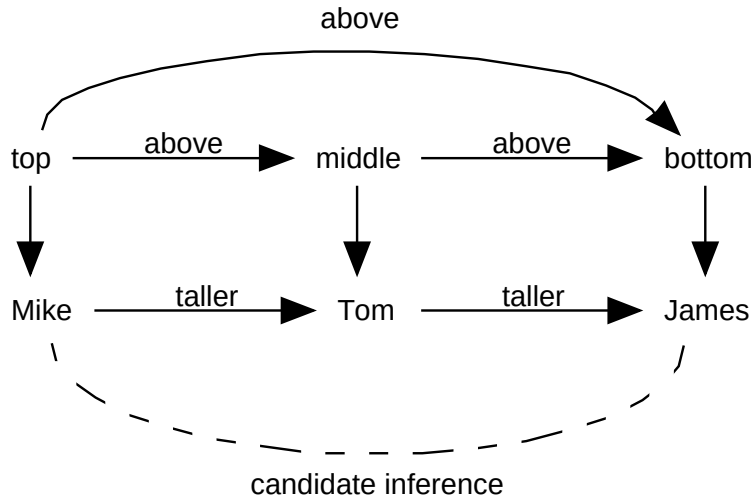
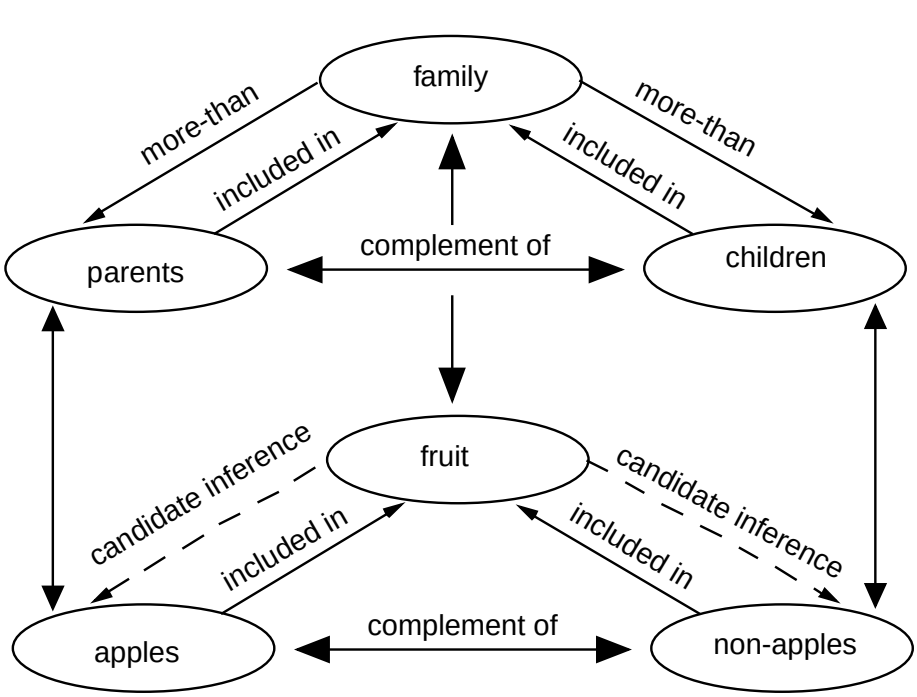
A


above
top
above
middle
above
bottom
Mike
taller
Tom
taller
James
candidate inference

B


family
more-than
included in
more-than
included in
parents
complement of
children
fruit
candidate inference
included in
candidate inference
included in
apples
complement of
non-apples

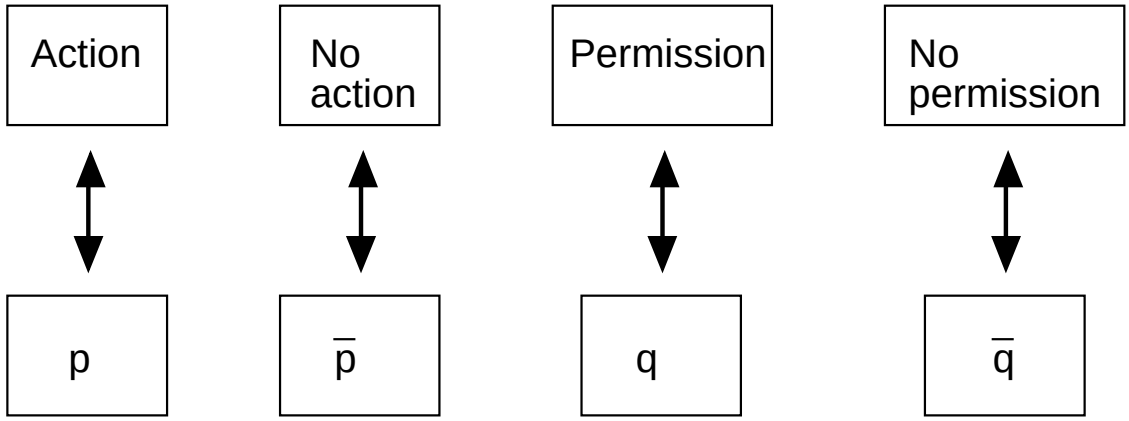
A

Each card contains a letter on one side and a number on the other side. Which cards must be turned over to determine if the rule “if there is an A on one side there is a 4 on the other side” is valid?

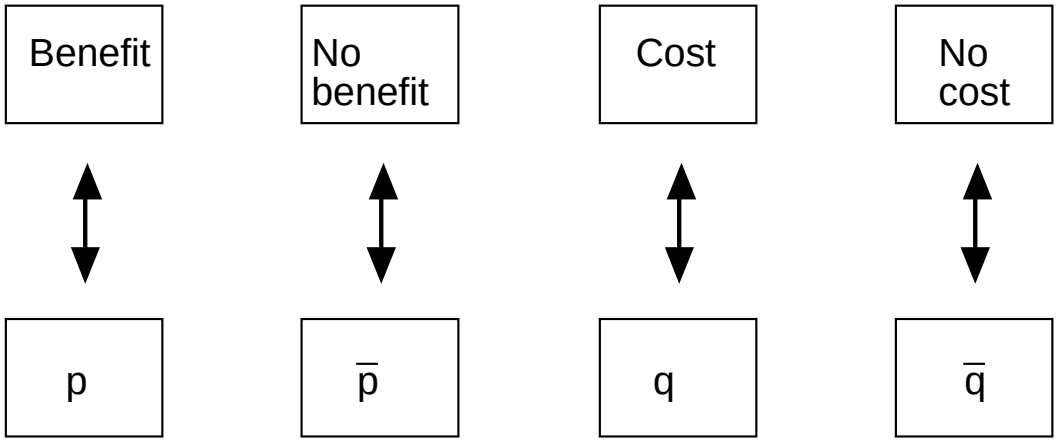
The rule, A $\rightarrow$ 4 is equivalent to ~~p~~$\rightarrow$ q.

The correct choices, A and 7, are equivalent to $\bar{p}$ and q

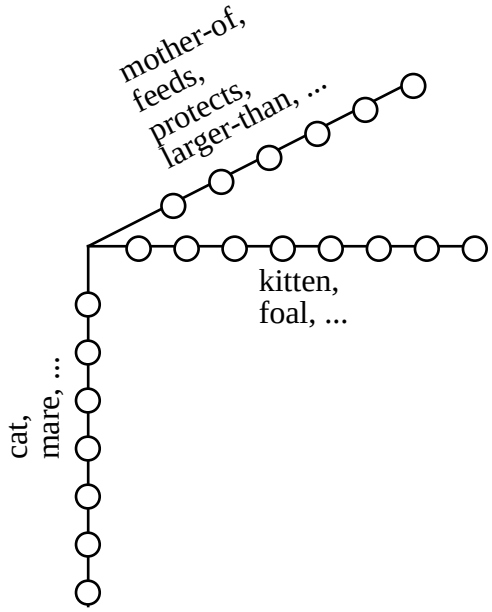
B



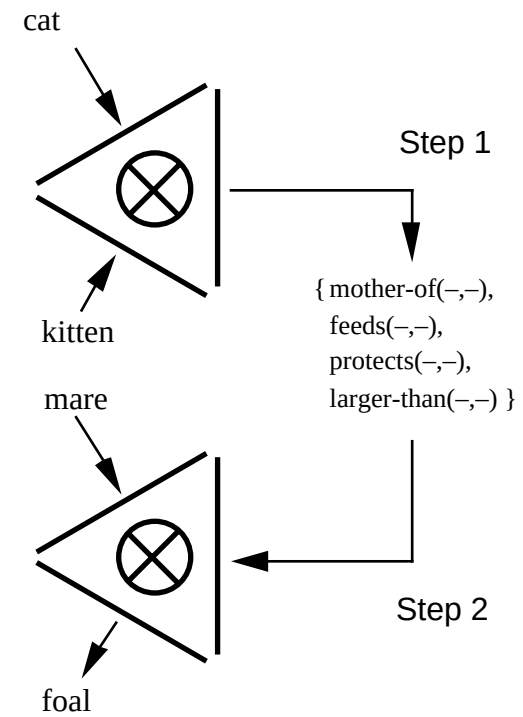
C



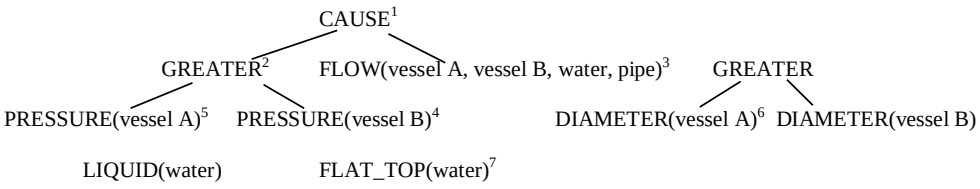
A.


mother-of,
feeds,
protects,
larger-than, ...
kitten,
foal, ...
cat,
mare, ...

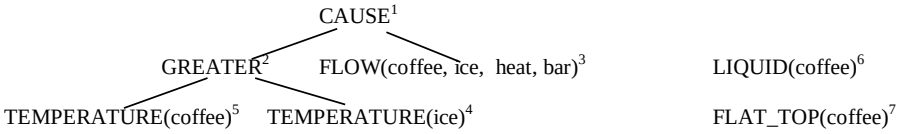
B.


cat
Step 1
{ mother-of(–,–),
feeds(–,–),
protects(–,–),
larger-than(–,–) }
kitten
mare
Step 2
foal

Water-flow

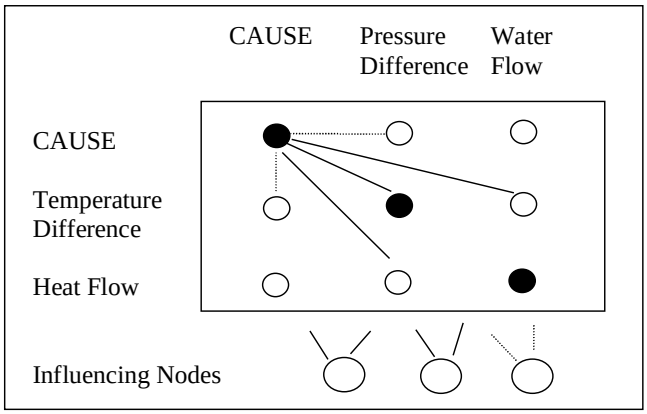


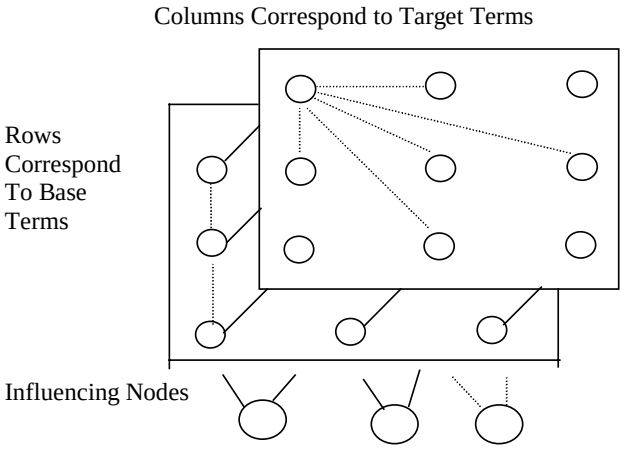
Heat-flow



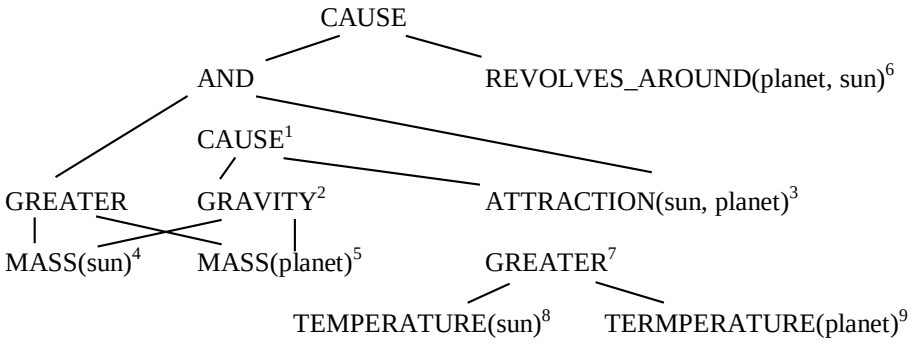
Mappings

Focus No.	Water-flow	Heat-flow	Final Overall Mapping Score
1	Cause Term	Cause Term	
1	CAUSE	CAUSE	1
1	Greater Pressure Term	Greater Temperature Term	
1	Flow Term	Flow Term	
2	GREATER	GREATER	1
2	Pressure Vessel A Term	Temperature Coffee Term	
2	Pressure Vessel B Term	Temperature Ice Term	
3	FLOW	FLOW	1
3	vessel A	coffee	0.9743
3	vessel B	ice	1
3	water	heat	0.8535
3	pipe	bar	1
4	PRESSURE	TEMPERATURE	1
6	Diameter Vessel A Term	Liquid Term	
6	DIAMETER	LIQUID	1
7	Flat Top Term	Flat Top Term	
7	FLAT_TOP	FLAT_TOP	1
7	water	coffee	0.5117

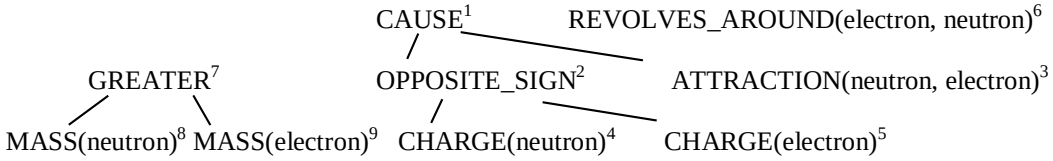

CAUSE
Pressure Difference
Water Flow
CAUSE
Temperature Difference
Heat Flow
Influencing Nodes


Columns Correspond to Target Terms
Rows Correspond To Base Terms
Influencing Nodes

Solar System



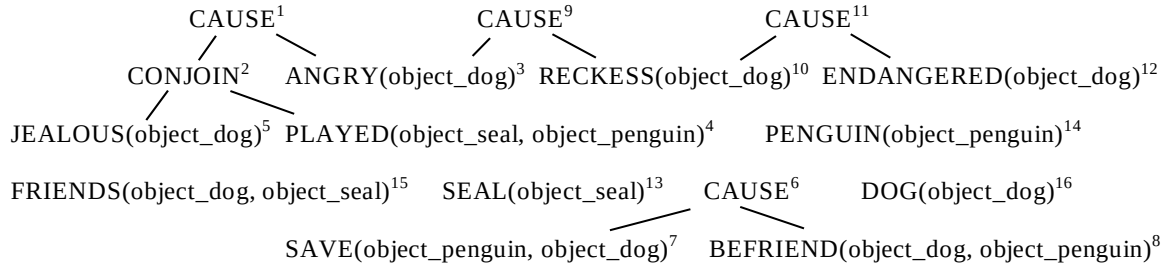
Atom



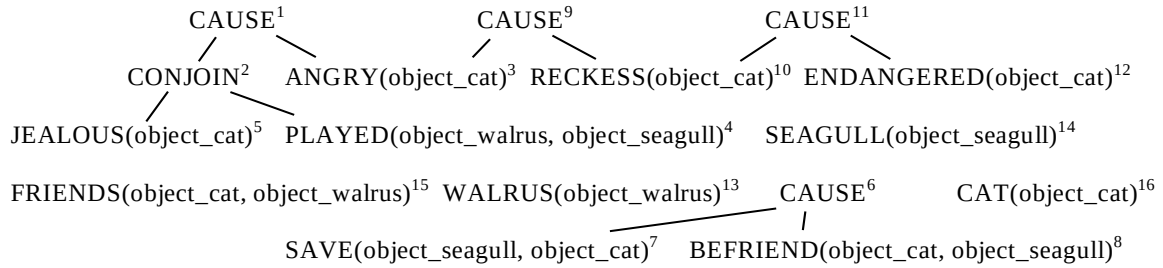
Mappings

Focus	Solar System	Atom	Final Overall Mapping Score
1	Attraction Cause Term	Attraction Cause Term	
1	CAUSE	CAUSE	1
1	Gravity Term	Opposite Sign Term	
1	Attraction term	Attraction term	
2	GRAVITY	OPPOSITE_SIGN	1
2	Sun Mass Term	Neutron Charge Term	
2	Planet Mass Term	Electron Charge Term	
3	ATTRACTION	ATTRACTION	1
3	sun	neutron	1
3	planet	electron	1
4	MASS	CHARGE	1
6	Revolves Term	Revolves Term	
6	REVOLVES_AROUND	REVOLVES_AROUND	1
7	Greater Temerature Term	Greater Mass Term	
7	GREATER	GREATER	1
7	Sun Temperature Term	Neutron Mass Term	
7	Planet Temperature Term	Electron Mass Term	
8	TEMPERATURE	MASS	1

Jealous Animals (dog, seal, penguin)



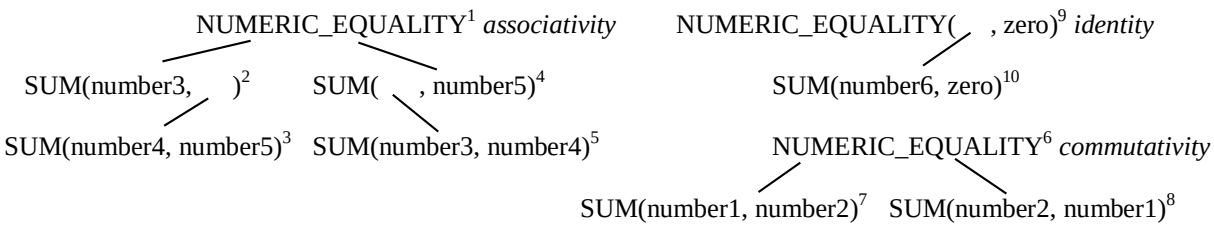
Jealous Animals (cat, walrus, seagull)



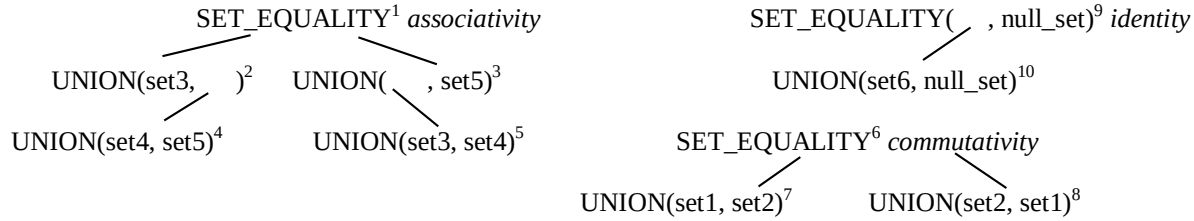
Mappings

Focus No.	**(dog, seal, penguin)**	**(cat, walrus, seagull)**	**Overall mapping score**
1	Conjoin Cause Angry Term	Conjoin Cause Angry Term	
1	CAUSE	CAUSE	1
1	Conjoin Term	Conjoin Term	
1	Angry Term	Angry Term	
2	CONJOIN	CONJOIN	1
2	Jelous Term	Jelous Term	
2	Played Term	Played Term	
3	ANGRY	ANGRY	1
3	object_dog	object_cat	1
4	PLAYED	PLAYED	1
4	object_seal	object_walrus	1
4	object_penguin	object_seagull	1
5	JEALOUS	JEALOUS	1
6	Save Cause Befriend Term	Save Cause Befriend Term	
6	Save Term	Save Term	
6	Befriend Term	Befriend Term	
7	SAVE	SAVE	1
8	BEFRIEND	BEFRIEND	1
9	Angry Cause Reckless Term	Angry Cause Reckless Term	
9	Reckless Term	Reckless Term	
10	RECKLESS	RECKLESS	1
11	Reckless Cause Endangered Term	Reckless Cause Endangered Term	
11	Endangered Term	Endangered Term	
12	ENDANGERED	ENDANGERED	1
13	Seal Term	Walrus Term	
13	SEAL	WALRUS	1
14	Penguin Term	Seagull Term	
14	PENGUIN	SEAGULL	1
15	Friends Term	Friends Term	
15	FRIENDS	FRIENDS	1
16	Dog Term	Cat Term	
16	DOG	CAT	1

ADDITION



UNION

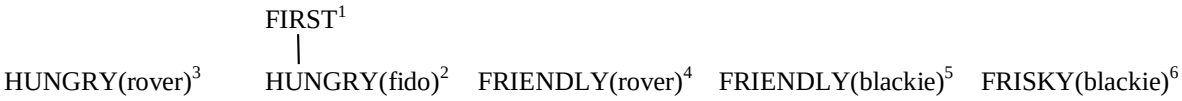


Mappings

Focus	Addition	Set	Final overall mapping score
1	Associativity Term	Associativity Term	
1	NUMERIC_EQUALITY	SET_EQUALITY	1
1	Sum Number 3_45 Term	Union Set 3_45 Term	
1	Sum Number 34_5 Term	Union Set 34_5 Term	
2	SUM	UNION	1
2	number3	set3	1
2	Sum Number 4_5 Term	Union Set 4_5 Term	
3	Sum Number 3_4 Term	Union Set 3_4 Term	
3	number5	set5	1
4	number4	set4	1
6	Commutativity Term	Commutativity Term	
6	Sum Number 1_2 Term	Union Set 1_2 Term	
6	Sum Number 2_1 Term	Union Set 2_1 Term	
7	number1	set1	1
7	number2	set2	1
9	Identity Term	Identity Term	
9	Sum Number 6_0 Term	Union Set 6_null Term	
9	zero	null_set	1
10	number6	set6	1

Boy

Dog



Mappings

Focus	Boy	Dog	Final overall mapping score
1	First Term	First Term	
1	FIRST	FIRST	1
1	Smart Steve Term	Hungry Fido Term	
2	SMART	HUNGRY	1
2	steve	fido	1
3	Smart Bill Term	Hungry Rover Term	
3	bill	rover	1
4	Tall Bill Term	Friendly Rover Term	
4	TALL	FRIENDLY	1
5	Tall Tom Term	Friendly Blackie Term	
5	tom	blackie	1
6	Timid Tom Term	Frisky Blackie Term	
6	TOM	FRISKY	1

Table 1

The Structure of the Permission Schema

			Permission (Symbolic)		*Schema*	*Conditional*			*Biconditional (Prediction)*
Permission schema		*Action → Permission*			$A \rightarrow P$	A	P	$A \rightarrow P$	$A \leftrightarrow P$
Action	permission	allowed	A	P	+	1	1	1	1
Action	no permission	not allowed	A	$\overline{\text{P}}$	-	1	0	0	0
No action	permission	allowed	$\overline{\text{A}}$	P	+	0	1	1	0
No action	no permission	allowed	$\overline{\text{A}}$	$\overline{\text{P}}$	+	0	0	1	1