

Mathematical fixation: Search viewed through a cognitive lens*

Steven Phillips, Yuji Takeda

National Institute of Advanced Industrial Science and Technology (AIST),
AIST Tsukuba Central 6-11, 1-1-1 Higashi, Tsukuba, Ibaraki, 305-8566, Japan

Abstract

We provide a mathematical category theory account of the size and location of the authors' Functional View Field (FVF). Category theory explains systematic cognitive ability via universal construction, that is, a necessary and sufficient condition for composition of cognitive processes. Similarly, FVF size and location is derived from a (universal) construction called a fibre (pullback) bundle. Hulleman & Olivers (H&O) account for an impressively diverse array of visual search data with a single free parameter: the "size" of (number of items in) their putative Functional View Field (FVF). Nonetheless, we see two critical shortcomings: (1) FVF is purely descriptive and lacking independent motivation, which is indicative of an ad hoc assumption (Aizawa, 2003); and (2) FVF size is potentially infinite in continuous domains, making it unclear how such cases are supposed to be unified with search in finite settings. In support of the target article, we provide a mathematical (categorical/topological) basis for FVF (size and location), called a fibre (pullback) bundle (Husemoller, 1994), to help resolve these problems.

A category consists of a collection of objects, a collection of morphisms between objects, and an operation for composing morphisms (Mac Lane, 1998). For example, every topological space is a category whose objects are the subsets constituting the topology, and morphisms are inclusions. Categories can model cognition by interpreting objects as cognitive states or spaces, morphisms as cognitive processes between states/spaces, and the operation as composition of cognitive processes. A universal construction is an arrangement whereby every morphism is composed of a common morphism and a unique morphism. This arrangement generalizes and refines classical/symbolic (Fodor & Pylyshyn, 1988) and connectionist/functional (van Gelder, 1990) compositionality by providing a necessary and sufficient condition for the indivisibility of certain clusters of cognitive capacities (Phillips & Wilson, 2010, 2011, 2012), without the ad hoc assumptions that were problematic in other approaches (Aizawa, 2003). Moreover, every universal construction is optimal in a certain category-theoretical sense. Thus, the preference for aligning relations over features in analogy (Gentner, 1983), affording optimal transfer of source knowledge to target domain,

*Preprint: Phillips, S., & Takeda, Y. (2017). Mathematical fixation: Search viewed through a cognitive lens. *Behavioral and Brain Sciences*, 40,. Commentary on Hulleman, J., & Olivers, C. N. L. (same issue). "The impending demise of the item in visual search".

derives from a universal construction (Phillips 2014). Visual search also involves compositionality and systematicity, hence our pullback approach to some differences between feature versus conjunctive search (Phillips, 2014). Similar considerations motivate our fibre bundle approach to FVFs. We regard the FVF as a projection of visual information formalized as a fibre bundle (E, B, π, F) : a topological space E , called the total space, that is locally a product of base B and fibre F , together with a projection $\pi : E \rightarrow B$ that is a continuous surjective map. Projections can be filters, discussed in the target article, in the sense of maps from unfiltered display (total) spaces to filtered view (base) spaces. For example, the projection $\pi_1 : C \times O \rightarrow C$ filters out features in orientation space O so that attention is focused on features in colour space C —total space $C \times O$ is the product of base C and fibre O . Likewise, $\pi_2 : C \times O \rightarrow O$ filters out colour features to focus attention on orientation features, as the task demands. Search involves changes in fixation that are bundle maps. In particular, a pullback bundle is obtained by “pulling back” a fibre bundle along a continuous map $f : B' \rightarrow B$ between base spaces, obtaining total space f^*B of pairs (b', e) , in a way that preserves bundle structure. That is, Figure 1(a) is a commuting (pullback) square: $\pi(g(b', e)) = f(\pi'(b', e))$. Commute means that fixation change after filtered view is the same as filtered view after fixation change, so search over view space is effectively search over display space. This construction is likened to a database lens, developed for a conceptually similar view update problem (Johnson, Rosebrugh, & Wood, 2012), hence the expression “cognitive lens.”

FVF size and location are determined by the nature of the projection and an inverse. A fibre over a point $b \in B$, that is, the set of points in E that project to b , denoted $\pi^{-1}[b]$, corresponds to an FVF. Hence, the size of an FVF is the number of elements in $\pi^{-1}[b]$. A section of a fibre bundle is a continuous right inverse of its projection, that is, a function $\sigma : B \rightarrow E$ such that $\pi(\sigma(b)) = b$. The location of an FVF associated with point b in view space is the point $\sigma(b)$ in display space. The pullback condition (Mac Lane, 1998) restricts bundles to disjoint unions of fibres, an optimal partitioning that prohibits gaps between fibres or overlaps.

Projections based on convex hulls of (topologically) neighbouring elements are one way to realize FVF for search in both natural and laboratory settings. For example, Figure 1(b) depicts a “natural” scene (E). Each convex hull (smallest set) enclosing one of the three scene components, that is, two people, a dog, and a tree, has a centre of mass, h_i . The projection (π) sends each point $e \in E$ to the closest centre. The fibre F_i is the region containing h_i , with boundaries indicated by dashed lines (cf. Voroni diagram). The base (B) is the (discrete) topological space on the three-centre set H , that is, the set of all subsets of H (cf. Delauney diagram). FVF location is the corresponding centre. An “X” indicates fixation before and after attentional shift, and dashed circles indicate corresponding items in the base. Pullback squares compose (Mac Lane, 1998). So response time corresponds to the number of composed squares to termination. By commutativity, search need only involve the three items in the base, rather than a very large (potentially infinite) number of locations in the display; greater resolution implies more locations (cf. texture-based search). An analogous situation applies to feature versus conjunctive search, shown in Figure 1(c, d). Lines connecting bars indicate neighbours in topological space; connected graphs correspond to fibres, which are larger in feature than conjunctive search, hence

feature search is generally more efficient. Off-item fixation corresponds to a virtual bar at the “centre of mass” of a multinode graph, so fixation need not coincide with a displayed bar. This situation is akin to perceptual grouping (Duncan & Humphreys, 1989). Similar considerations apply to conjunctive search (Wolfe, Cave, & Franzel, 1989): Nearby items are less likely to be of the same kind, hence FVFs are smaller and the number of fibres greater, implying the observed steeper search slope. A base can contain non-visual items, for example, categories affording category-guided search (Zelinsky, Adeli, Peng, & Samaras, 2013). The challenge is to develop an FVF model incorporating the mathematical theory.

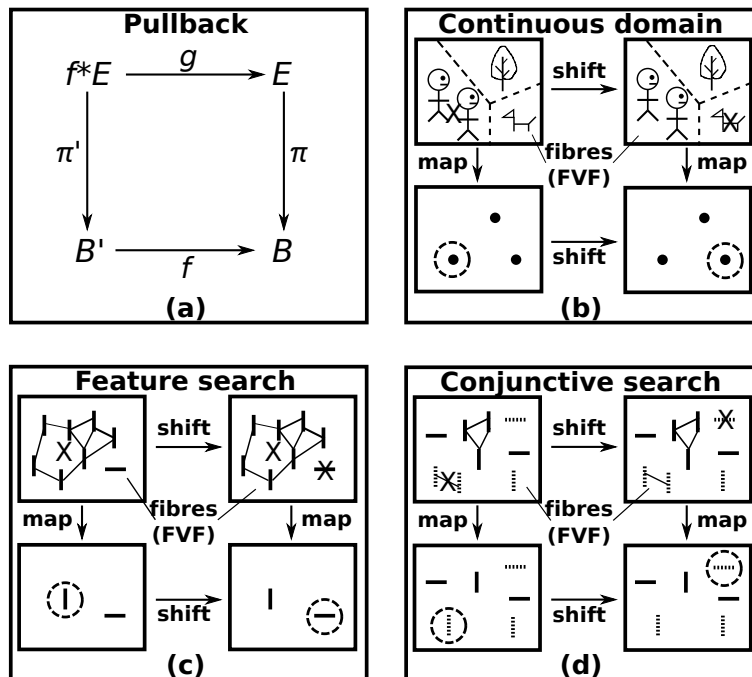


Figure 1: Fibre bundle (a) commuting/pullback square, and corresponding examples for (b) a natural scene, (c) feature search, and (d) conjunctive search.

References

- Aizawa, K. (2003). *The systematicity arguments*. New York: Kluwer Academic.
- Duncan, J., & Humphreys, G. W. (1989). Visual search and stimulus similarity. *Psychological Review*, *96*(3), 433–458.
- Fodor, J. A., & Pylyshyn, Z. W. (1988). Connectionism and cognitive architecture: A critical analysis. *Cognition*, *28*(1–2), 3–71.
- Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, *7*(2), 47–59.
- Husemoller, D. (1994). *Fibre bundles* (3rd ed.). New York, NY: Springer-Verlag.
- Johnson, M., Rosebrugh, R., & Wood, R. (2012). Lenses, fibrations and universal translations. *Mathematical Structures in Computer Science*, *22*, 25–42.

- Mac Lane, S. (1998). *Categories for the working mathematician* (2nd ed.). New York, NY: Springer.
- Phillips, S. (2014). Analogy, cognitive architecture and universal construction: a tale of two systematicities. *PLoS ONE*, *9*(2), e89152.
- Phillips, S., & Wilson, W. H. (2010). Categorical compositionality: A category theory explanation for the systematicity of human cognition. *PLoS Computational Biology*, *6*(7), e1000858.
- Phillips, S., & Wilson, W. H. (2011). Categorical compositionality II: Universal constructions and a general theory of (quasi-)systematicity in human cognition. *PLoS Computational Biology*, *7*(8), e1002102.
- Phillips, S., & Wilson, W. H. (2012). Categorical compositionality III: F-(co)algebras and the systematicity of recursive capacities in human cognition. *PLoS ONE*, *7*(4), e35028.
- van Gelder, T. (1990). Compositionality: A connectionist variation on a classical theme. *Cognitive Science*, *14*, 355–384.
- Wolfe, J. M., Cave, K. R., & Franzel, S. L. (1989). Guided search: An alternative to the feature integration model for visual search. *Journal of Experimental Psychology: Human Perception and Performance*, *15*(3), 419–433.
- Zelinsky, G. J., Adeli, H., Peng, Y., & Samaras, D. (2013). Modelling eye movements in a categorical search task. *Philosophical Transactions of the Royal Society of London B: Biological Sciences*, *368*(1628), 20130058.