A slip pulse model with fault heterogeneity for low-frequency earthquakes and tremor along plate interfaces

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Abstract

Deep low-frequency earthquakes (LFEs) and nonvolcanic tremor have distinctive characteristics unlike those of regular earthquakes, including strong anisotropy in their migration velocity and source spectra displaying 1/f decay. We show that a physical model can explain these features in a simple framework with slip pulses originating on fault heterogeneity and triggered by slow-slip events. LFE/tremor source areas in the model consist of unstable patches sparsely and heterogeneously distributed following a Gaussian distribution. The difference in their migration speeds along dip and along strike was reproduced, without anisotropic rheological properties, by introducing alignments of their sources similar to observed streaks of LFEs/tremor. The key to reproducing inverse linear spectral decay is that the slip pulse has a constant mean moment rate. This model provides new insights into the physical source process of LFEs and tremor and should find practical use in assessing properties of deep plate interfaces.

1. Introduction

Recent precise observations in southwest Japan and Cascadia show that clusters of deep low-frequency earthquakes (LFEs) and nonvolcanic tremor tend to occur in association with slow slip events (SSEs) propagating along the strike of subduction plate interfaces [Rogers and Dragert, 2003; Obara et al., 2004; Ghosh et al., 2009a; Wech et al., 2009]. The waveforms of tremor and LFEs show similar spectral contents, consistent with explaining tremor as a swarm of LFEs [Shelly et al., 2007b; Brown et al., 2009]. These events propagate along strike at speeds about 10 km/day. Along-dip migration of LFEs/tremor has been documented at speeds on the order of 100 km/hr, several orders of magnitude faster than along-strike events [Shelly et al., 2007a; Ghosh et al., 2009b],
in which many of the fast migration appear to occur in streaky distribution of LFEs/tremor subparallel to the subduction direction (Similar streaks are found on a transform fault [Shelly, 2010]). It is not yet clear whether this strong anisotropy reflects physical differences on the plate boundary, such as anisotropy of rheological and hydraulic properties.

Ide, et al. [2007] showed that the high-frequency components of LFE/tremor source spectra decay at a rate inversely proportional to the frequency $f$, i.e., $|\Omega(f)| \propto 1/f$, and they documented a linear scaling relation between seismic moment and duration of a wide range of slow earthquakes. Given that the Fourier spectrum of a constant displays $1/f$ decay, this finding suggests that the moment rate of those events has a temporally constant mean value. As the moment rate $\dot{M}_o$ is written as

$$\dot{M}_o(t) = \mu [\dot{S}(t)D + \dot{S}\dot{D}(t)]$$  \hspace{1cm} (1)

where $\mu$ is rigidity, $D$ is displacement and $S$ a rupture area, the condition of a constant moment rate is a kinematic constraint on the source process:

$$\dot{S}D + \dot{S}\dot{D} = \text{Const.} \hspace{1cm} (2)$$

One well-known rupture mode satisfying equation (2) is a slip pulse propagating unilaterally or bilaterally with a constant pulse shape of a slip rate (i.e., $\dot{S} = 0$ and $\dot{D} \neq 0$), that is,

$$\dot{S}\dot{D} = \text{Const.} \hspace{1cm} (3)$$

Several mechanisms have been proposed to generate a pulse-like rupture mode, including a class of friction laws dependent on slip velocity [Heaton, 1990; Shibazaki and Shimamoto, 2007]. Another mechanism invokes the effect of heterogeneity in fault characteristics, sometimes called the pinning effect [Day et al., 1998]. These heterogeneities divide a fault into discontinuous segments, and slip is sustained only while a rupture propagates through individual segments that behave as isolated faults. If LFEs are the rupture of unstable patches in an otherwise stable region of the fault plane [e.g. Ito et al., 2007], this means that strength heterogeneity exist on the fault that can generate pulse-like ruptures caused by the pinning effect.

This paper proposes a comprehensive framework to explain the coincidental occurrence of LFEs and tremor with SSEs, the difference in the migration velocity of LFEs and tremor, and the source spectral characteristics of these events. Our model is developed on the basis of dynamics, not kinematics or quasi-statics.

2. Model and method

We constrain our model of the brittle-ductile transition zone where SSEs, LFEs and tremor occur [Rogers and Dragert, 2003; Obara et al., 2004; Shelly et al., 2006], illustrated schematically in
Figure 1, based on three observational facts. First, given the coincidence of SSEs with tremor and with LFEs and their lateral migration, we hypothesize that LFEs and tremor are triggered by stress change due to SSEs. Second, given that seismic radiation is associated with LFEs/tremor but not with SSEs, and given that source sizes are considerably smaller for LFEs/tremor than for SSEs, we assumed a number of small frictionally unstable regions in otherwise stable background regions. Those unstable patches are assumed to be clustered to constitute LFE sources, a theoretical constraint to reproduce slip pulses through the pinning effect. LFE sources are assumed to be the constituent of tremor sources. Third, the LFE and tremor sources are assumed to be aligned, to account for the observed streaky distribution of LFEs and tremor subparallel to the subduction direction.

Following a previous simulation [Shibazaki and Shimamoto, 2007] as well as observations on migrating LFE/tremor cluster geometries [Ito et al., 2007], SSEs are assumed to span the brittle–ductile transition zone vertically and propagate horizontally with a pulse shape.

To model the source process, which radiates seismic waves, we employed the dynamic boundary integral equation method [Ando and Yamashita, 2007] in a 3D full space with a triangular mesh [Tada, 2006].

We simplified the model by introducing the stress change due to a slow slip front as an external load on a fault. We simplified the pulse shaped load as a cosine function with one peak parameterized by wavelength $L_w$, amplitude $A_w$, and propagation speed $V_{prop}$. Accordingly, shear traction is given by

$$\begin{align*}
\tau &= \tau_o + A_w \left[ 1 + \cos \left( \pi \frac{x - V_{prop}t}{L_w} \right) \right] / 2 \quad (\text{if } |X - V_{prop}t| < L_w / 2) \\
\tau &= \tau_o \quad \text{(elsewhere)}
\end{align*}$$

(4)

where $x$ is the position along strike and $\tau_o$ is the initial traction, whose values are assumed to be $\tau_o = A_o$ on the patches and $\tau_o = 0$ on the background. This simplified treatment allowed us to omit detailed numerical modeling of a large SSE and focus on the resultant smaller scale events, tremor and LFEs.

On the patches, fault strength $\tau_s$ is given by the following fault constitutive law,

$$\begin{align*}
\tau_s &= \tau_p \quad \text{(if } V = 0) \\
\tau_s &= \tau_r + \eta V \quad \text{(if } V > 0)
\end{align*}$$

(5)

where $\tau_p$ and $\tau_r$ (\(\tau_p > \tau_r\)) are the peak and residual strengths, $\eta$ (>0) is the viscosity factor, and $V$ is the slip velocity. The viscous damping term is necessary at the brittle–ductile transition depth, where damping effects decelerate ruptures, to reproduce low-frequency phenomena. Note that, in equation (5), slip weakening due to the relation $\tau_p > \tau_r$ leads to instability and velocity hardening due to the damping leads to stability. In the background region, fault strength is kept constant at $\tau_p$. We
assumed the values $\tau_p = A_w + A_o$ so that the rupture is triggered by the slow slip front. In the following, the case of $(A_w - \tau_r)/(A_o - \tau_r) = 1/5$ and $V_{prop} = 0.25 V_p (= 0.43 V_s)$ is shown as an example, where $V_p$ and $V_s$ are the P- and S-wave speeds.

Hereafter, we present physical quantities in nondimensionalized form; space is nondimensionalized by $\Delta s$, the side length of a regular triangular mesh to discretize the fault, time is by $\Delta s/V_p$, shear traction is by $\tau_p - \tau_r$, and slip is by $\Delta s(\tau_p - \tau_r)/\mu$.

Figure 2 (at $T = 0$) shows the assumed initial condition and fault region in our model. The locations of the unstable patches are shown by bright colors in the middle panel, denoting high traction. The patch distribution is multileveled and primarily characterized by an alignment of three large clusters of the patches as a streak with the inclination angle $\Phi$. Within these clusters, some fluctuations are added so that the locations and sizes of their constituent patches follow a Gaussian distribution (sufficient randomness is introduced by random variation between along-strike rows of patches). Hereafter we refer to the circular clusters of patches as simply LFE sources but we can regard them as tremor sources as in Figure 1.

### 3. Simulations

#### 3.1. SSE triggered LFE and slip pulse

Figure 2 shows snapshots of the rupture process in our model. The arrival of the front is seen at $T = 10$ (enlarged view) as a dark red band along the left edge of the middle panel showing traction. The incoming stress wave triggers the rupture of nearby patches, and slip is accelerated on the top LFE source as seen in the velocity distribution (left panel). The rupture propagates laterally in the LFE source without apparent vertical expansion as seen at $T = 25$. As the slow slip front propagates rightward, the second and the third LFE sources begin passively rupturing at $T = 37.5$ and $50$, respectively. Once triggered, these ruptures expand to the whole LFE source areas spontaneously at a speed much slower than the terminal speed $V_s$ as the traction drops on the patches; it is shown that the ruptures propagate to the right ahead of the slow slip front as seen at $T = 80$ (enlarged view) for example as the irregularly expanded areas of relatively high values in velocity and traction ahead of the vertical bands of the low values associated with the slow slip front.

It is important to confirm that the simulated rupture becomes pulse-like. The distribution of slip rates shows that slip occurs within limited bands as pulses during the passage of the rupture across LFE sources. A pulse arises there because the sparsely distributed unstable patches interact only weakly mainly through stress change, which decreases rapidly away from the stress sources, patches. Thus these patches tend to slip only while the stress is dropping there.

#### 3.2. Anisotropic LFE migration velocity

It is also important to discriminate the different mechanisms of rupture nucleation and propagation within LFE sources and beyond them. Within a LFE source, the patches are close enough for their
interaction to lead to spontaneous rupture over the source although the abovementioned weak interaction causes slow rupture velocities. However, LFE sources are too far apart to generate spontaneous rupture between sources; thus, rupture propagation is passive, triggered by a slow slip front. Likewise, we can assume that lateral propagation across alignments or streaks of LFE sources is limited to the speed of the slow slip front.

We can explain the observations of faster migration speed along dip than along strike in terms of this passive mechanism. As demonstrated in the simulation, this happens when a slow slip front intersects an alignment of LFE sources at an acute angle $\Phi$. Under the simplest situation neglecting fluctuations in their geometry, the strike and dip components of the LFE migration velocity ($V_{\text{strike}}$, $V_{\text{dip}}$) can be written as

$$(V_{\text{strike}}, V_{\text{dip}}) = \{V_{\text{prop}} / \tan(\Phi)\}.$$  

(6)

The observation in section 1 that $V_{\text{dip}} / V_{\text{strike}} = 1 / \tan(\Phi) \sim 10^2$ means that $\Phi \sim 0.57^\circ$.

Note that we cannot eliminate a possibility that the fast along-dip migration is a spontaneous phenomenon similar to rupture in a LFE source without the above passive sequential initiation by slow slip. This can occur in our model if the neighboring LFE sources are close enough along dip to allow a spontaneous rupture propagation beyond them once initiated by slow slip or the other stress disturbance. Obviously, for both the cases, the difference of $V_{\text{dip}}$ and $V_{\text{strike}}$ depends on a fault geometrical feature rather than on anisotropic rheological properties.

### 3.3. Slip-pulse-like moment rate function

Figure 3a shows moment rate functions obtained for the three LFE sources in our model simulation. The three sources can be identified by the timing of the rupture onset. The moment rates show common characteristic behaviors such as a speedy initial rise coinciding with the rupture nucleation on the left side of the sources, a middle phase with fluctuations around certain constant values corresponding with the lateral rupture propagation phase in the LFE sources, and a rapid decay phase when the rupture reaches the other end.

The middle phase with roughly constant moment rate is the manifestation of the pulse-like rupture mode. Fluctuations in this underlying feature arise from the random variation in sizes and locations of the patches. As the randomness is assumed to follow a Gaussian distribution with a constant mean value, the underlying feature is generally robust.

The moment rate function including all three LFE sources is shown in Figure 3b, which is a linear superposition of them. This is interpreted to coincide with a tremor event. Given our model’s size limit, the modeled tremor duration is similar to those of the modeled LFEs, but if it included more LFE sources we could obtain a longer duration with constant average moment rate. Similarly, if the LFE sources were more sparsely distributed, isolated events corresponding to single LFE events
would be apparent in the moment rate function.

Figure 3a also shows the moment rate for a single patch in a LFE source pointed by the white arrow at $T = 10$ in Figure 2. The form of this function is unlike that of the modeled LFEs and very similar to that of a regular earthquake, usually modeled as a circular crack, showing the rapid initial increase associated with the expansion of the slip area until the rupture reaches a maximum, followed by a monotonic decline \cite{Madariaga, 1977}. The absence of the plateau of constant moment rate is the important difference from the slip pulse explaining LFEs.

Here we would like to clarify how constituent crack-like moment rate functions produce an ensemble behavior like a pulse. If the constituent moment rate function is simplified as a simple triangular shape, as Figure 3c illustrates, a steady series of events can be superposed to simulate a function with a persistent constant peak. In the case including randomness, this condition is stochastically satisfied when the patch sizes and configurations follow a Gaussian distribution. The assumption of a Gaussian distribution in stochastic processes might have some similarities with the kinematics-based Brownian walk model proposed by Ide \cite{2008}.

3.4. Source spectrum

Figure 4 shows Fourier spectral amplitudes of the calculated moment rate functions. The source spectra of LFEs (warm colors) and their combination (black) show common behavior consisting of a high-frequency asymptote decaying inversely proportional to frequency $f$ (as $1/f$) and the low-frequency asymptote becoming flat. The $1/f$ decay is what we expect from the constant moment rate due to a slip pulse. A single patch produces $1/f^2$ decay, known as the omega-square model, due to linear inclinations shown in moment rate functions \cite{Madariaga, 1977}.

By constructing a physical dynamic model, we could successfully reproduce the observed $1/f$ decay of source spectra in high-frequency components \cite{Ide et al., 2007}, showing a distinct difference from a crack model case. The similarity in the source spectra of modeled LFEs and modeled tremor is consistent with the explanation that tremor can be a swarm of LFEs \cite{Shelly et al., 2007b}. The detailed structure of the spectra will be investigated with finer numerical resolution.

4. Conclusions

We propose a new mechanical model to explore the observed characteristics of LFE migrations and source spectra by considering slip pulses triggered by slow slip and sustained by fault heterogeneity. The underlying assumption is that a LFE source consists of heterogeneously and sparsely distributed unstable patches in a stable background. This heterogeneity might reflect differences in the onset temperature of plasticity or of dehydration \cite{Hacker et al., 2003; Shelly et al., 2006; Peacock, 2009} among the minerals forming fault rocks. Heterogeneous permeability might be also responsible through pore fluids.

We reproduced two independent observations for LFEs and tremor in this model. First, by
introducing streaks of LFE/tremor sources subparallel to fault dip, this model explains the considerably different migration speeds along dip and strike of subduction interfaces as a geometric effect without the need to posit anisotropy in rheological properties. Second, we also reproduced the $1/f$ decay of source spectra as a result of slip pulses. The key aspect reproducing the $1/f$ decay is that the mean of the moment rate is temporally invariant, resulting as the slip pulse propagates on unstable patches randomly distributed in a Gaussian distribution. This constraint may help us to infer the faults heterogeneity in the brittle–ductile transition zone.

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References


Figure 1. Schematic diagram of slow slip propagation and strength heterogeneity on a fault in the brittle–ductile transition zone. The slow slip front propagates to the right at $V_{prop}$. The black circles illustrate unstable patches in the otherwise relatively stable background region colored gray. The rectangles enclosed by broken lines are the region in the simulation.
Figure 2. Snapshots of the rupture process from time $T = 0$ to 137.5. The three panels in each snapshot represent (left) slip velocity, (middle) shear traction, and (right) slip. Enlarged views are separately presented for $T = 10$ and 80. Bright spots in the middle panel are locations of unstable fault patches.
Figure 3. Moment rate functions of modeled LFEs and tremor. (a) Moment rate functions obtained for three LFE sources in the model simulation (yellow, orange, and red). Blue curve is the moment rate function of a single unstable fault patch. (b) Moment rate functions obtained by superposing all LFE sources. (c) Schematic diagram showing production of a plateau function by superposition of sequential single-peak functions.

Figure 4. Source spectra of modeled LFEs and tremor. The color coding is the same as in Figure 3. Dotted lines indicate slopes of inverse linear frequency (1/f) and square of frequency (1/f²).