

cheat sheet `ssrnat.v` (SSREFLECT v1.15)

`ssrfun.v` naming conventions

K cancel
 LR move an op from the lhs of a rel to the rhs
 RL move an op from the rhs to the lhs

`ssrfun.v` notations

$f \dashv\vdash y$ $\text{fun } x \Rightarrow f \ x \ y$
 $p \cdot 1$ $\text{fst } p$
 $p \cdot 2$ $\text{snd } p$
 $f = 1 \ g$ $f \ x = g \ x$
 $\{\text{morph } f : x / aF \ x \ \>\> \ rR \ x\}$ $f \ (aF \ x) = rF \ (f \ x)$
 $\{\text{morph } f : x \ y / aDp \ x \ y \ \>\> \ rDp \ x \ y\}$ $f \ (aDp \ x \ y) = rDp \ (f \ x) \ (f \ y)$

`ssrfun.v` definitions

injective f forall x1 x2, f x1 = f x2 -> x1 = x2
 cancel f g g (f x) = x
 involutive f cancel f f
 left_injective op injective (op[~] x)
 right_injective op injective (op y)
 left_id e op e \square x = x
 right_id e op x \square e = x
 left_zero z op z \square x = z
 right_zero z op x \square z = z
 self_inverse e op x \square x = e
 idempotent op x \square x = x
 commutative op x \square y = y \square x
 associative op x \square (y \square z) = (x \square y) \square z
 right_commutative op (x \square y) \square z = (x \square z) \square y
 left_commutative op \underline{x} \square (y \square z) = y \square (\underline{x} \square z)
 left_distributive op add (x + y) * z = (x * z) + (y * z)
 right_distributive op add x * (y + z) = (x * y) + (x * z)
 left_loop inv op cancel (op x) (op (inv x))

`ssrbool.v` naming conventions

A associativity
 AC right commutativity
 b a boolean argument
 C commutativity/complement
 D predicate difference
 E elimination
 F/f boolean false
 T/t boolean truth
 U predicate union

`ssrnat.v` naming conventions

A(infix) conjunction
 B subtraction
 D addition
 p(prefix) positive
 S successor
 V(infix) disjunction

(* nat_scope *)

Notation "n .+1" := (succn n). Notation "n .*2" := (double n). Notation "n '!":=(factorial n).

Notation "n .-1" := (predn n). Notation "n ./2" := (half n).

Notation "m <n" := (m.+1 <=n). Notation "m ^ n" := (expn m n).

addOn/addn0 left_id 0 addn/right_id 0 addn
 addIn/addn1 1 + n = n.+1/n + 1 = n.+1
 addn2 n + 2 = n.+2
 addSn m.+1 + n = (m + n).+1
 addnS m + n.+1 = (m + n).+1
 addSnnS m.+1 + n = m + n.+1
 addnC commutative addn
 addnA associative addn
 addnCA left_commutative addn
 eqn_add2l (p + m == p + n) = (m == n)
 eqn_add2r (m + p == n + p) = (m == n)
 subOn/subn0 left_zero 0 subn/right_id 0 subn
 subnn self_inverse 0 subn
 subSS m.+1 - n.+1 = m - n
 subn1 n - 1 = n.-1
 subnDl (p + m) - (p + n) = m - n
 subnDr (m + p) - (n + p) = m - n
 addKn cancel (addn n) (subn[~] n)
 addnK cancel (addn[~] n) (subn[~] n)
 subSnn n.+1 - n = 1
 subnDA n - (m + p) = (n - m) - p
 subnAC right_commutative subn
 ltnS (m <n.+1) = (m <= n)
 prednK 0 <n -> n.-1.+1 = n
 leqNgt (m <= n) = [~]~ (n < m)
 ltnNge (m <n) = [~]~ (n <= m)
 ltnn n <n = false
 subnDA n - (m + p) = (n - m) - p
 leq_eqVlt (m <= n) = (m == n) || (m < n)
 ltn_neqAle (m <n) = (m !=n) &&& (m <=n)
 ltn_add2l (p + m < p + n) = (m < n)
 leq_addr n <= n + m
 addn_gt0 (0 < m + n) = (0 < m) || (0 < n)
 subn_gt0 (0 < n - m) = (m < n)
 leq_sub2r m <= n -> m - p <= n - p
 leq_subLR (m - n <= p) = (m <= n + p)
 ltn_sub2r p <n -> m <n -> m - p <n - p

ltn_subRL (n < p - m) = (m + n < p)
 subnKC m <= n -> m + (n - m) = n
 subnK m <= n -> (n - m) + m = n
 addnBA p <= n -> m + (n - p) = m + n - p
 subnBA p <= n -> m - (n - p) = m + p - n
 subKn m <= n -> n - (n - m) = m
 mulOn/muln0 left_zero 0 muln/right_zero 0 muln
 mulIn/muln1 left_id 1 muln/right_id 1 muln
 mul2n/muln2 2 * m = m.*2/m * 2 = m.*2
 mulnC commutative muln
 mulnA associative muln
 mulSn m.+1 * n = n + m * n
 mulnS m * n.+1 = m + m * n
 mulnDl left_distributive muln addn
 mulnDr right_distributive muln addn
 mulnBl left_distributive muln subn
 mulnBr right_distributive muln subn
 mulnCA left_commutative muln
 muln_gt0 (0 < m * n) = (0 < m) &&& (0 < n)
 leq_pmulr n > 0 -> m <= m * n
 leq_mul2l (m * n1 <= m * n2) = (m == 0) || (n1 <= n2)
 leq_pmul2r 0 < m -> (n1 * m <= n2 * m) = (n1 <= n2)
 ltn_pmul2r 0 < m -> (n1 * m < n2 * m) = (n1 < n2)
 leqP leq_xor_gtn m n (m <= n) (n < m)
 ltngtP compare_nat m n (m < n) (n < m) (m == n)
 expn0 m ^ 0 = 1
 expn1 m ^ 1 = m
 expnS m ^ n.+1 = m * m ^ n
 exp0n 0 < n -> 0 ^ n = 0
 expIn 1 ^ n = 1
 expnD m ^ (n1 + n2) = m ^ n1 * m ^ n2
 expn_gt0 (0 < m ^ n) = (0 < m) || (n == 0)
 fact0 0! = 1
 factS (n.+1)! = n.+1 * n!
 odd_add odd (m + n) = odd m (+) odd n
 odd_double_half odd n + n./2.*2 = n

Variant leq_xor_gtn m n : nat -> nat -> nat -> nat -> bool -> bool -> Set :=
 | LeqNotGtn of m <= n : leq_xor_gtn m n m n n true false
 | GtnNotLeq of n < m : leq_xor_gtn m n n m m false true.
 Variant compare_nat m n : nat -> nat -> nat -> nat -> bool -> bool -> bool -> bool -> bool -> Set :=
 | CompareNatLt of m < n : compare_nat m n m n n false false true false true
 | CompareNatGt of m > n : compare_nat m n n m m false true false true false
 | CompareNatEq of m = n : compare_nat m n m m m true true true true false false.