

$$\text{big_morph} \quad (\forall xy, f(x+y) = f(x) + f(y)) \rightarrow f(0) = \hat{0} \rightarrow f\left(\sum_{P(i)}^{i \leftarrow r} F(i)\right) = \sum_{P(i)}^{i \leftarrow r} f(F(i))$$

Section Extensionality

$$\begin{aligned} \text{eq_big1} & \quad P_1 =_1 P_2 \rightarrow \sum_{P_1(i)}^{i \leftarrow r} F(i) = \sum_{P_2(i)}^{i \leftarrow r} F(i) \\ \text{eq_bigr} & \quad (\forall i, P(i) \rightarrow F_1(i) = F_2(i)) \rightarrow \sum_{P(i)}^{i \leftarrow r} F_1(i) = \sum_{P(i)}^{i \leftarrow r} F_2(i) \\ \text{big_nil} & \quad \sum_{P(i)}^{i \leftarrow \emptyset} F(i) = 0 \\ \text{big_pred0} & \quad P =_1 \text{xpred0} \rightarrow \sum_{P(i)}^{i \leftarrow r} F(i) = 0 \\ \text{big_pred1} & \quad P =_1 \text{pred1}(i) \rightarrow \prod_{P(j)}^j F(j) = F(i) \\ \text{big_ord0} & \quad \sum_{P(i)}^{i < 0} F(i) = 0 \\ \text{big_tnth} & \quad \sum_{P(i)}^{i \leftarrow r} F(i) = \sum_{P(r_i)}^{i < \text{size}(r)} F(r_i) \\ \text{big_nat_recl} & \quad m \leq n \rightarrow \sum_{m \leq i < n+1} F(i) = F(m) + \sum_{m \leq i < n} F(i+1) \\ \text{big_ord_recl} & \quad \sum_{i < n+1} F(i) = F(\text{ord0}) + \sum_{i < n} F(\text{lift}((n+1), \text{ord0}, i)) \\ \text{big_const_ord} & \quad \sum_{i < n} x = \text{iter}(n, \lambda y. x + y, 0) \end{aligned}$$

Section MonoidProperties

$$\begin{aligned} \text{big1} & \quad (\forall i, P(i) \rightarrow F(i) = 1) \rightarrow \prod_{P(i)}^{i \leftarrow r} F(i) = 1 \\ \text{big_nat_recl} & \quad m \leq n \rightarrow \prod_{m \leq i < n+1} F(i) = \left(\prod_{i < n} F(i)\right) \times F(n) \end{aligned}$$

Section Abelian

$$\begin{aligned} \text{big_split} & \quad \prod_{P(i)}^{i \leftarrow r} (F_1(i) \times F_2(i)) = \prod_{P(i)}^{i \leftarrow r} F_1(i) \times \prod_{P(i)}^{i \leftarrow r} F_2(i) \\ \text{bigU} & \quad A \cap B = \emptyset \rightarrow \prod_{i \in A \cup B} F(i) = \left(\prod_{i \in A} F(i)\right) \times \left(\prod_{i \in B} F(i)\right) \\ \text{partition_big} & \quad (\forall i, P(i) \rightarrow Q(p(i))) \rightarrow \prod_{P(i)}^{i \leftarrow s} F(i) = \prod_{j:J} \prod_{P(i)}^i F(i) \\ \text{reindex_onto} & \quad (\forall i, P(i) \rightarrow h(h'(i)) = i) \rightarrow \prod_{P(i)}^i F(i) = \prod_{P(h(j))}^j F(h(j)) \\ \text{pair_big} & \quad \prod_{P(i)}^i \prod_{Q(j)}^j F(i, j) = \prod_{P(p) \wedge Q(q)}^{(p, q)} F(p, q) \\ \text{exchange_big} & \quad \prod_{P(i)}^{i \leftarrow rI} \prod_{Q(j)}^{j \leftarrow rJ} F(i, j) = \prod_{Q(j)}^{j \leftarrow rJ} \prod_{P(i)}^{i \leftarrow rI} F(i, j) \end{aligned}$$

Section Distributivity

$$\begin{aligned} \text{big_distr1} & \quad \left(\sum_{P(i)}^{i \leftarrow r} F(i)\right) \times a = \sum_{P(i)}^{i \leftarrow r} (F(i) \times a) \quad (\text{also } \text{big_distr}) \\ \text{big_distr_big_dep} & \quad \prod_{P(i)}^i \sum_{Q(j)}^j F(i, j) = \sum_{f \in \text{pfamily}(j_0, P, Q)} \prod_{P(i)}^i F(i, f(i)) \\ \text{big_distr_big} & \quad \prod_{P(i)}^i \sum_{Q(j)}^j F(i, j) = \sum_{f \in \text{pfun_on}(j_0, P, Q)} \prod_{P(i)}^i F(i, f(i)) \\ \text{bigA_distr_big} & \quad \prod_i \sum_{Q(j)}^i F(i, f(i)) = \sum_{f \in \text{ffun_on}(Q)} \prod_i F(i, f(i)) \\ \text{bigA_distr_bigA} & \quad \prod_{i \in I} \sum_{j \in J} F(i, j) = \sum_{f \in J^I} \prod_{i \in I} F(i, f(i)) \end{aligned}$$

$\text{pfamily}(j_0, P, Q) \simeq \text{functions } Q^P$

$$\begin{aligned} \text{partition_big_imset} & \quad \sum_{i \in A} F(i) = \sum_{i \in h \circ e: A} \sum_{h(i)=j}^{i \in A} F(i) \\ \text{big_trivIset} & \quad \text{trivIset}(P) \rightarrow \sum_{x \in \text{cover}(P)} E(x) = \sum_{A \in P} \sum_{x \in A} E(x) \\ \text{partition_disjoint_bigcup} & \quad (\forall i, j, i \neq j \rightarrow F(i) \cap F(j) = \emptyset) \rightarrow \sum_{x \in \bigcup_i F(i)} E(x) = \sum_i \sum_{x \in F(i)} E(x) \end{aligned}$$