Application of Formal Verification to Software Security

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Verification of Security Properties of Software

Generally speaking,

- Software security is difficult to define
  - Many unclear notions (e.g., "privacy")
  - Often many details (e.g., technical details)
- Pencil-and-paper verifications/proofs are difficult to check
  - Many abbreviations (e.g., "We see that...")
  - Often many cases (e.g., lengthy enumerations)

There is a need for:

1. Mathematical definitions of what to verify
2. Computer means to do (or at least check) verifications

Formal Verification

- Appropriate in the case of critical systems
- Formal verification consists of:
  1. A mathematical model $\mathcal{M}$ of the system
  2. A property φ expressed in a formal logic
  3. Techniques to prove and check that $\mathcal{M}$ satisfies φ
- There are mainly two approaches:
  - Proof-assistants
    - \+ Very expressive (infinite models handled by induction)
    - − Requires human interaction
  - Model-checking
    - \+ Automatic proof
    - − Finite models only (unless safe abstractions are made)

Proof-assistants

- A proof-assistant consists of:
  - A language for writing mathematical models $\mathcal{M}$, statements φ, and proofs that $\mathcal{M}$ satisfies φ
  - An automatic way to <u>check</u> proofs
  - An interactive way to <u>build</u> proofs

    Automatic discovery of proofs for simple statements only

- Worthwhile if the cost of mistakes is extremely high

  E.g., critical parts of microprocessor design

The Coq Proof-assistant [INRIA, 1984–2006]

- A programming language with powerful types...
  - Inductive/coinductive types for finite/infinite data structures
    Lists, trees, streams, etc.
  - Dependent types
    The output-type of a function can vary according to its argument
- ...for writing models, properties, and proofs:
  - Properties are types
  - Proofs are programs (Heyting semantics)
    In particular, proof-checking = type-checking
- Remarkable achievements:
  - Verification of virtual machines for smartcards
    [Trusted Logic, 2003]
  - The four color theorem [Gonthier and Werner, 2004]

The Four Color Theorem

Four colors are enough to color any geographical map in such a way that no neighboring two countries are of the same color.

- The proof requires the verification of many cases
- Long history:

  1853 first statement
  1976 first proof, using a computer
  2004 certified proof in Coq

- Practical application:

  reduce the number of used broadcasting frequencies for mobile phones

Verification of Functional Programs in Coq

General approach:

- Mathematical model $\mathcal{M}$: a function in the Coq language
- Property φ: a statement in the Coq language
- Verification that $\mathcal{M}$ satisfies φ: by interactive proof

Demo

Verification of Imperative Programs in Coq

- Problem: the Coq language is not imperative

  Imperative programs cannot be represented directly

- Solution: use the Coq language to model imperative programs

  This amounts to formalization of their semantics

- General approach:
  - Mathematical model $\mathcal{M}$:
    the formal model of an imperative program
  - Property φ: a statement in the Coq language
  - Verification that $\mathcal{M}$ satisfies φ: by interactive proof

Verification of Imperative Programs

Hoare Logic (1/2)

Empty statement axiom

$$\frac{}{\{P\}\ \mathsf{skip}\ \{P\}}$$

Assignment axiom schema

$$\frac{}{\{P[E/x]\}\ x{:=}E\ \{P\}}$$

Example: $\{x + 5 < 20\}\ x{:=}x + 5\ \{x < 20\}$

Sequence rule

$$\frac{\{P\}\ C\ \{Q\} \quad \{Q\}\ D\ \{R\}}{\{P\}\ C;D\ \{R\}}$$

Verification of Imperative Programs

Hoare Logic (2/2)

Conditional rule

$$\frac{\{E \wedge P\}\ C\ \{Q\} \quad \{\neg E \wedge P\}\ D\ \{Q\}}{\{P\}\ \text{if } E \text{ then } C \text{ else } D \text{ endif } \{Q\}}$$

While rule

$$\frac{\{E \wedge \boxed{Inv}\}\ C\ \{\boxed{Inv}\}}{\{\boxed{Inv}\}\ \text{while } E \text{ do } C \text{ done } \{\neg E \wedge \boxed{Inv}\}}$$

Rule of consequence

$$\frac{P \Rightarrow P' \quad \{P'\}\ C\ \{Q'\} \quad Q' \Rightarrow Q}{\{P\}\ C\ \{Q\}}$$

Verification of Imperative Programs

Example

$\{a > 0 \ \wedge \ b > 0\}$
$x{:=}a;\ y{:=}b;$
$\{\boxed{x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)}\}$
while $x \neq y$ do
  $\{x \neq y \ \wedge \ x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)\}$
  if $x < y$ then
    $\{x < y \ \wedge \ x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)\}$
    $y{:=}y - x$
    $\{x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)\}$
  else
    $\{x > y \ \wedge \ x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)\}$
    $x{:=}x - y$
    $\{x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)\}$
  endif
  $\{x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)\}$
done
$\{x = y \ \wedge \ \boxed{x > 0 \ \wedge \ y > 0 \ \wedge \ gcd(x, y) = gcd(a, b)}\}$

The conclusion implies that $x = gcd(a, b)$

Application to Software Security

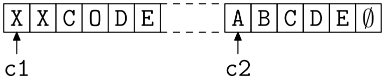
- Memory management in C
  - Buffer overflows
  - Security issues on multi-users systems
- Implementation of cryptographic devices (smartcards)
  - Efficient arithmetic on large integers

Buffer Overflow

- A dangerous program:

  ```
  for (c1=buf, c2=str; (*c1++ = *c2++)!=0; );
  ```

  - The buffer may be smaller than the string:

  - How can we prevent such bugs using formal verification?

Verification of Memory Management

Separation Logic (1/2)

- Hoare logic with a notion of mutable memory [Reynolds, 2002]
  - Singleton heap:

$$h \models (E \mapsto E') \text{ iff } \ dom(h) = E \ \wedge \ h(E) = E'$$

  - Memory accesses:

Mutation

$$\frac{}{\{E \mapsto ?\}\ [E] := E'\ \{E \mapsto E'\}}$$

Example:

$$\left\{\begin{array}{c}\boxed{?}\\ \uparrow\\ x\end{array}\right\} [x] := 4 \left\{\begin{array}{c}\boxed{4}\\ \uparrow\\ x\end{array}\right\}$$

is written $\{(x \mapsto ?)\}\ [x] := 4\ \{(x \mapsto 4)\}$

Lookup

$$\frac{}{\{E \mapsto E'\}\ x := [E]\ \{E \mapsto E' \ \wedge \ x = E'\}}$$

Verification of Memory Management

Separation Logic (2/2)

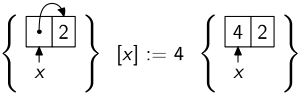
- Compositional reasoning using a logic extension
  - Compound heap:

$$h \models P \star Q \text{ iff}$$
$$\exists h_1, h_2 \text{ s.t. } h_1 \perp h_2 \;\wedge\; h_1 \uplus h_2 = h \;\wedge\; h_1 \models P \;\wedge\; h_2 \models Q$$

  - Frame Rule

$$\frac{\{P\}C\{Q\} \wedge \mathit{modified}(C) \cap \mathit{free}(R) = \emptyset}{\{P \star R\}C\{Q \star R\}}$$

Example:



is written

$\{(x \mapsto p) \star (p \mapsto 2)\}$ $[x] := 4$ $\{(x \mapsto 4) \star (p \mapsto 2)\}$

Verification of Memory Management

Example: Buffer Overflow

$\{buf \Mapsto B_0 \cdots B_{n-1} \star str \Mapsto S_0 \cdots S_{m-1}\}$

$c1 := buf; c2 := str; tmp := [c2];$

$$\left\{ \boxed{\begin{array}{l} buf \Mapsto S_0 \cdots S_{i-1} B_i \cdots B_{n-1} \star str \Mapsto S_0 \cdots S_{m-1} \wedge \\ c1 = buf + i \wedge c2 = str + i \wedge tmp = S_i \end{array}} \right\}$$

while $tmp \neq 0$ do

$\quad [c1] := tmp;$

$\quad c1 := c1 + 1;$

$\quad c2 := c2 + 1;$

$\quad tmp := [c2]$

done;

$$\left\{ tmp = 0 \wedge \boxed{\begin{array}{l} buf \Mapsto S_0 \cdots S_{i-1} B_i \cdots B_{n-1} \star str \Mapsto S_0 \cdots S_{m-1} \wedge \\ c1 = buf + i \wedge c2 = str + i \wedge tmp = S_i \end{array}} \right\}$$

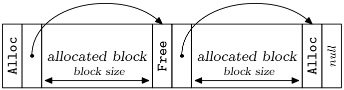
$[c1] := tmp$

$\{buf \Mapsto S_0 \cdots S_{m-1} \star \mathrm{T} \star str \Mapsto S_0 \cdots S_{m-1}\}$

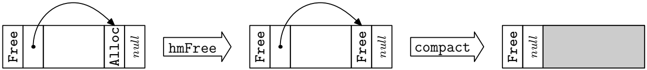
Possible only if $n \geq m$

Memory Management and Multi-users Systems

- Security issue: privacy of the data of users
- Example: memory management in O.S. [Marti et al., 2006]
  - Dynamically memory uses linked lists:



  - Separation property:
    "Newly allocated blocks do not override old ones"
  - Related problem found during verification of existing code:
    - Memory exhaustion:



Verification of the Implementation of Cryptosystems

- Algorithms <u>and</u> their implementation must be certified
- Cryptographic devices require low-level programming
- In low-level languages, properties depend on physical data:
  - Counter-intuitive arithmetic properties
    - Machine integers wrap around (integer overflow)
    - Confusing conversions:

      ```
      unsigned int u;
      ...
      if (u > -1) ... /* always false! */
      ```

    - The sign of the remainder of an integer depends on its size
  - Unsafe casts
    - Ariane 5 bug:
      Conversion from 64-bit floating-point to 16-bit signed integer

Formalization of Machine Integers in Coq (1/2)

- A machine integer is a list of bits
  - Examples:

    `i::i::i::i::nil` stands for (1111)

    `o::o::o::i::nil` stands for (0001)

- Hardware circuitry is a set of recursive functions
  - Example: “strictly less than”

```
Fixpoint listbit_lt (a b:list bit) {struct a} : bool :=
  match a with
    o::tla => match b with
                o::tlb => listbit_lt tla tlb
              | i::_ => true
              | _ => false
              end
  | i::tla => match b with
                o::_ => false
              | i::tlb => listbit_lt tla tlb
              | _ => false
      end
  | _ => false
  end.
```

Formalization of Machine Integers in Coq (2/2)

- Signed integers in two's complement notation:
  - Definitions:

$$
\begin{aligned}
(a_n \dots a_0)_u &= a_n 2^n + \cdots + a_0 \\
(a_n \dots a_0)_s &= -a_n 2^n + a_{n-1} 2^{n-1} + \cdots + a_0
\end{aligned}
$$

  - Examples:
    - $(0001)_u = (0001)_s$ but $(1111)_u \neq (1111)_s$
    - In Coq:

```
[[ o::o::o::i::nil ]]u = 1  | [[ i::i::i::i::nil ]]u = 15
[[ o::o::o::i::nil ]]s = 1  | [[ i::i::i::i::nil ]]s = -1
```

- We retrieve the "expected" properties:
  - $-1 \not< 1$
  - In Coq:

```
listbist_lt (i::i::i::i::nil) (o::o::o::i::nil) = false
```

Verification of Efficient Arithmetic on Large Integers

Formalization of machine integers is necessary because of:

- Target functions in assembly
  - Resource constraints
  - Application-specific extensions (e.g., SmartMIPS)
- Specifications at the bit-level
  - Carries and flags

Formal Verification of the Modular Multiplication in Coq

- Specification of the Montgomery algorithm:

$$\left\{\begin{array}{lll} X, Y, M & \text{such that} & |X|, |Y|, |M| = k \text{ and } X, Y < M \\ Z & \text{such that} & |Z| = k+1 \text{ and } Z = 0 \\ \alpha & \text{such that} & \alpha * M_0 \equiv -1[\beta] \end{array}\right\}$$

$$\texttt{montgomery}\ X\ Y\ M\ Z\ \alpha$$

$$\{\ \beta^k * Z \equiv X * Y[M] \text{ and } Z < 2 * M\ \}$$

- Example: $10^5 * 39796 \equiv 5792 * 1229\ [72639]$
- Basic idea: zero the least significant word of partial products

0	0	0	0	0	0	5	8	3	5	7	0
0	0	0	0	0	6	5	0	5	3	0	0
0	0	0	0	5	0	9	4	9	0	0	0
0	0	0	3	4	7	6	5	0	0	0	0
0	0	3	9	7	9	6	0	0	0	0	0

- Verification of a SmartMIPS implementation in Coq using machine integers and Hoare logic [Affeldt and Marti, 2006]

Other Applications of Proof-assistants to Software Security

- Proof-carrying code [Hamid et al., 2002]
  - Mobile code sent <u>with</u> its safety proof
- Security protocols [Paulson, 1998]
  - Inductive proofs in the Isabelle proof assistant
- Internet applications
  - Mail server using a Coq implementation of the π-calculus and temporal logic [Affeldt et al., 2005]

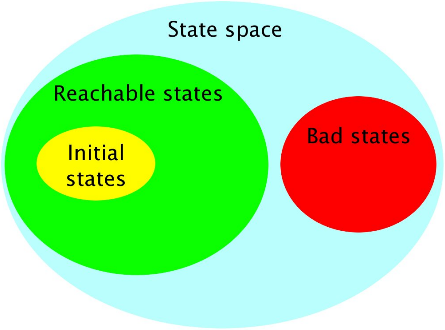
Model-checking

- The system is represented by a transition system, i.e., a directed graph where:
  - Nodes represent states
  - Edges represent changes of states
- Verification is done by exploring the transition system
  - The transition system should be finite
    (not necessarily the model)
  - Execution paths can be infinite (cycles)
- Mainly two families of specifications:
  1. State properties: reachability of a particular state
  2. Path properties: feasible of particular executions

Verification of State Properties

Example of state properties:

- Deadlocks (absence of successors)
- Satisfaction/violation of assertions

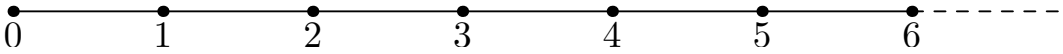


$$\mathit{Reachable}(\mathit{Init}) \cap \mathit{Bad} \; = \; \emptyset$$

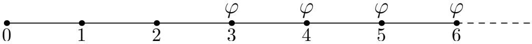
Specification of Path Properties

Path properties are better expressed with temporal logics

- A path is a sequence of states:



- Sample path properties
  - Stability: "There will be a state from which φ is always true."



    Linear Temporal Logic (LTL) notation: $\Diamond\Box\varphi$

  - Response:
    "Always, whenever there is a request, there will be eventually a reply."

    LTL notation: $\Box(\mathit{Request} \rightarrow \Diamond \mathit{Reply})$

Application to Software Security

Example

A simple client-server application:

- The server serves up-to-date files
- The client wants the latest version

We want to verify that:

- After a session, the client has an up-to-date file
- LTL notation: $\Box\Diamond(\mathit{client\_version} = \mathit{server\_version})$

For concreteness, we will use the Spin model-checker

Overview of the Basic Model

In Spin, transition systems are written
using concurrent processes, communicating via channels

```
/********************
  global definitions
 ********************/
typedef Message {
 int file_version;
 mtype signature
}

mtype = { client_key, server_key }

chan server_chan =
   [0] of { Message, chan };

int client_version = 100;
int server_version = 102;
```

```
/*********************
  processes skeletons
 *********************/
proctype client () {
 /* next slides */
}

proctype server (int version_number) {
 /* next slides */
}

init {
 run client ();
 run server (server_version)
}
```

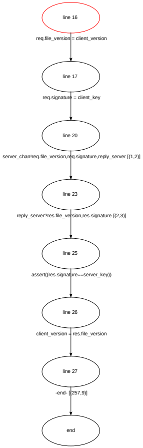
```
/********************
  property to verify
 ********************/
[] (<> (client_version == server_version))
```

Model of the Client

Promela code:

```
proctype client () {

 /* request construction */
  Message req;
  req.file_version = client_version;
  req.signature = client_key;

 /* request to the server */
  chan reply_server = [0] of { Message };
  server_chan ! req, reply_server;

 /* response from the server */
  Message res;
  reply_server ? res;

 /* signature and version checks */
  assert (res.signature == server_key);
  assert (res.file_version >= client_version);
  client_version = res.file_version
}
```

Transition system:

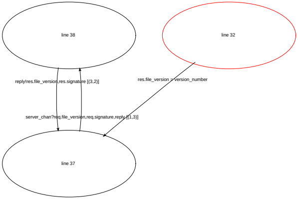


Model of the Server

Promela code:

```
proctype server (int version_number) {

 /* response construction */
  Message res;
  res.file_version = version_number;
  res.signature = server_key;

 /* repeatedly answers response */
  Message req;
  chan reply;
  do
  :: server_chan ? req, reply; reply ! res
  od
}
```

Transition system:



Verification of the Property for the Basic Model

- The property also can be represented as a transition system:

```
/*******************
  property to verify
 *******************/
[] (<> (client_version == server_version))
```

$\longrightarrow$

```
never {    /* !([] <> p) */
T0_init:
 if
 :: (! ((p))) -> goto accept_S4
 :: (1) -> goto T0_init
 fi;
accept_S4:
 if
 :: (! ((p))) -> goto accept_S4
 fi;
}
```

  - The resulting transition system loops as long as p is false
- Transition systems can be composed into a global one (product of *automata*)
- Verification amounts to look for a cycle in the global system

Model of the DNS

Usually, internet connections rely on a DNS:

```
/*****************
  model of the DNS
 *****************/
chan server_chan = [0] of { Message, chan };
chan dns_chan = [0] of { mtype, chan }

mtype = { server_ip }

proctype dns () {
 mtype ip;
 chan reply;
 do
 :: dns_chan ? ip, reply; reply ! server_chan
 od
}
```

Corresponding change in the client model:

```
/* request to the server */
 chan reply_server = [0] of { Message };
 server_chan ! req, reply_server;
```

$\longrightarrow$

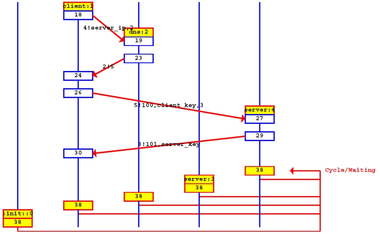
```
/* internet connection */
 chan socket = [0] of { Message, chan };
 chan reply_dns = [0] of { chan };
 dns_chan ! server_ip, reply_dns;
 reply_dns ? socket;

/* request to the server */
 chan reply_server = [0] of { Message };
 socket ! req, reply_server;
```

An Attack found by Model-checking

A spoofed DNS can invalidate $\Box\Diamond(\mathit{client\_version} = \mathit{server\_version})$:

```
/***********************
  model of a spoofed DNS
 ***********************/
chan server_chan = [0] of { Message, chan };
chan bad_server_chan = [0] of { Message, chan };
chan dns_chan = [0] of { mtype, chan }

mtype = { server_ip, bad_server_ip }

proctype dns () {
 mtype ip;
 chan reply;
 do
 :: dns_chan ? ip, reply;
  if
  :: true -> reply ! server_chan
  :: true -> reply ! bad_server_chan
  fi
 od
}
```

Counter-example:



$\Rightarrow$ The application is vulnerable to *replay attacks*

It is possible to enforce a downgrade despite encryption

Applications to Software Security

We have applied model-checking to verification of:

- An existing web-application
- An embedded operating system [Marti et al., 2006]

BTW, verification of crytographic protocols are carried out similarly

Conclusion

In this talk, we had:

- An introduction to formal verification
  - Proof-assistants
  - Model-checking
- Application to software security
  - Memory management in C
  - Implementation of cryptographic devices
  - Verification of internet applications

The slides and the examples are available at `http://staff.aist.go.jp/reynald.affeldt/isss2006/`.

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