LETTER

Filtering and Smoothing for Motion Trajectory of Feature Point Using Non-Gaussian State Space Model

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SUMMARY Filtering and smoothing using a non-Gaussian state space model are proposed for motion trajectory of feature point in image sequence. A heavy-tailed non-Gaussian distribution is used for measurement noise to reduce the effect of outliers in motion trajectory. Experimental results are presented to show the usefulness of the proposed method.

key words: *feature point tracking, image sequence, non-Gaussian state space model, sequential Monte Carlo method*

1. Introduction

Feature point tracking in an image sequence is essential to shape from motion, motion segmentation, and motion based control of active camera. False matching due to error in feature extraction and change in illumination between frames may occur in feature point tracking. The false matching leads to outliers in motion trajectory of feature point. A method to reduce the effect of outliers is important to give good data for the subsequent processing.

A method to reduce the effect of outliers is stochastic filtering using state space model for motion trajectory. The Kalman filter[1] is well known as a typical method. It has been used in many applications, but the lack of robustness for outliers has been pointed out; if measurement noise distribution deviates from Gaussian, the Kalman filter cannot estimate state properly, because linear Gaussian state space model is assumed in it[2]. Such deviation from Gaussianness can be frequently observed in real data.

Filtering and smoothing using a non-Gaussian state space model are proposed to reduce the effect of outliers. In the proposed method, heavy-tailed non-Gaussian distribution is used for measurement noise. We can represent both measurement noise with high probability and outliers with low probability using it. Thus state estimation which takes account of the existence of outliers can be realized.

A problem on the use of non-Gaussian distribution is state estimation; the Kalman filter cannot be applied to non-Gaussian model. We use the sequential Monte Carlo method(SMC)[3]-[6] for state estimation of non-Gaussian model.

Experimental results including the comparison with Gaussian model are presented to confirm the usefulness of the proposed method.

2. Non-Gaussian State Space Model for Motion Trajectory

2.1 State Vector, System Matrices and Measurement Matrix

A state space model is represented as follows:

$$\boldsymbol{x}_t = \boldsymbol{F}\boldsymbol{x}_{t-1} + \boldsymbol{G}\boldsymbol{v}_t \quad (1)$$

$$\boldsymbol{y}_t = \boldsymbol{H}\boldsymbol{x}_t + \boldsymbol{w}_t \quad (2)$$

where Eq.(1) is state transition equation, $\boldsymbol{x}_t$ is state vector and $\boldsymbol{v}_t$ is system noise vector. The matrix $\boldsymbol{F}$ and $\boldsymbol{G}$ are system matrices. Equation(2) is measurement equation, $\boldsymbol{y}_t$ is measurement vector and $\boldsymbol{w}_t$ is measurement noise vector. The matrix $\boldsymbol{H}$ is measurement matrix.

The measurement vector $\boldsymbol{y}_t = [x(t), y(t)]^T$ is coordinates of feature point in image. It is assumed as a prior knowledge that the second order difference of coordinates of feature point takes zero; if there is no noise, the coordinates of feature point satisfy the following equations:

$$x(t+1) = 2x(t) - x(t-1) \quad (3)$$

$$y(t+1) = 2y(t) - y(t-1) \quad (4)$$

From this assumption, state vector, system matrices, measurement matrix, measurement noise vector and system noise vector of the state space model are defined as follows:

$$\boldsymbol{x}_t = [x_s(t), y_s(t), x_s(t-1), y_s(t-1)]^T \quad (5)$$

$$\mathbf{F} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (6)$$

$$\boldsymbol{v}_t = [v_x(t), v_y(t)]^T \quad (7)$$

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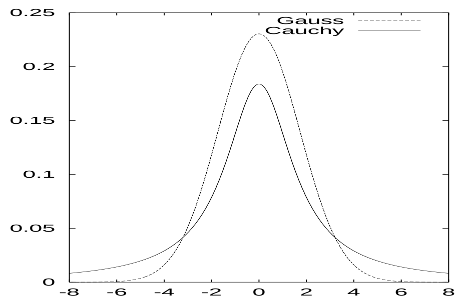


Fig. 1 Gaussian distribution (dotted line) and Cauchy distribution (solid line); location:0,scale:3.

$$\boldsymbol{w}_t = [w_x(t), w_y(t)]^T \tag{8}$$

where x_s and y_s are the coordinates of feature point to be estimated, v_x and v_y are the variables for system noise, w_x and w_y are the variables of measurement noise.

2.2 Non-Gaussian Measurement Noise Distribution

The variables in system noise vector $\boldsymbol{v}_t$ are independent, and their distributions are represented as $q\left(v; m_q,\ \tau^2\right)$, where m_q and τ are the parameters of location and scale. The Gaussian distribution $N\left(0, \tau^2\right)$ is used for system noise. The multivariate form of the system noise distribution is denoted as $q_v\left(\boldsymbol{v}_t; \boldsymbol{m}_{vq},\ \boldsymbol{T}\right)$ where $\boldsymbol{m}_{vq} = [0,0]^T$ and $\boldsymbol{T} = \mathrm{diag}\left(\tau^2, \tau^2\right)$.

The variables in measurement noise vector $\boldsymbol{w}_t$ are independent, and their distributions are represented as $r\left(w; m_r, \sigma^2\right)$, where m_r and σ are the parameters of location and scale. A heavy-tailed distribution is used as $r\left(w; m_r, \sigma^2\right)$ to represent the existence of the outliers due to false matching in tracking. The following Cauchy distribution $C\left(0, \sigma^2\right)$ is selected for measurement noise.

$$r\left(w; 0, \sigma^2\right) = \frac{\sigma}{\pi\left\{w^2 + \sigma^2\right\}} \tag{9}$$

It is heavy-tailed(Fig.1) and thus can represent both measurement noise with high probability and outliers with low probability. State estimation which takes account of the existence of outliers, therefore, can be realized. The multivariate form of the measurement noise distribution is denoted as $r_v\left(\boldsymbol{w}_t; \boldsymbol{m}_{vr},\ \boldsymbol{\Sigma}\right)$ where $\boldsymbol{m}_{vr} = [0,0]^T$ and $\boldsymbol{\Sigma} = \mathrm{diag}\left(\sigma^2, \sigma^2\right)$.

The parameters τ^2 and σ^2 are called the hyper-parameters, since they have the role to control state estimation. The hyper-parameters can be determined using log-likelihood as shown in Section 4.

3. State Estimation by Sequential Monte Carlo Method

Since Kalman filter commonly used for state estimation assumes Gaussian model, it cannot be used for the non-Gaussian model shown in Section 2. The sequential Monte Carlo method(SMC) which can be used for non-Gaussian model has been proposed recently[3]-[6]. The Monte Calro filter(MCF)[4] which is a kind of the SMC is applied to state estimation of the proposed model. The procedure of the MCF is briefly described in below.

In the MCF, the probability distributions used in state estimation are approximated by m particles, i.e. m realizations from the distributions, as follows:

$$\begin{aligned} \text{prediction} : p\left(\boldsymbol{x}_t \mid Y_{t-1}\right), &\quad \left\{\boldsymbol{p}_t^{(1)}, \ldots, \boldsymbol{p}_t^{(m)}\right\} \\ \text{filter} : p\left(\boldsymbol{x}_t \mid Y_t\right), &\quad \left\{\boldsymbol{f}_t^{(1)}, \ldots, \boldsymbol{f}_t^{(m)}\right\} \\ \text{smoothing} : p\left(\boldsymbol{x}_{t-L} \mid Y_t\right), &\quad \left\{\boldsymbol{s}_{t-L|t}^{(1)}, \ldots, \boldsymbol{s}_{t-L|t}^{(m)}\right\} \end{aligned}$$

where $Y_t = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t\}$ is a sequence of measurement vector and $L\,(>0)$ is the lag in fixed-lag smoothing. The system noise distribution is also approximated as follows:

$$\text{system noise} : q_v\left(\boldsymbol{v}_t; \boldsymbol{m}_{vq},\ \boldsymbol{T}\right), \quad \left\{\boldsymbol{v}_t^{(1)}, \ldots, \boldsymbol{v}_t^{(m)}\right\}$$

State estimation is carried out by repeating one step ahead prediction and filter process using these particles as summarized below.

[MCF algorithm]

[Step1:generation of particles of initial distribution] Generate m particles, i.e. m random vectors, from an initial distribution of state $p_0\left(\boldsymbol{x}\right)$. These particles $\boldsymbol{f}_0^{(i)}$ $(i = 1, \ldots, m)$ can be regarded as initial distribution of filter.

[Step2:filtering] Repeat next steps.

[Step2-1:generation of particles for system noise] Generate m particles $\boldsymbol{v}_t^{(i)} \sim q_v\left(\boldsymbol{v}; \boldsymbol{m}_{vq}, \boldsymbol{T}\right)$, $i = 1, \ldots, m$.

[Step2-2:one step ahead prediction] Compute the particles, $\boldsymbol{p}_t^{(i)}$, representing the distribution of prediction $p\left(\boldsymbol{x}_t \mid Y_{t-1}\right)$ using state transition equation:

$$\boldsymbol{p}_t^{(i)} = \boldsymbol{F}\boldsymbol{f}_{t-1}^{(i)} + \boldsymbol{G}\boldsymbol{v}_t^{(i)},\ i = 1, \ldots, m \tag{10}$$

$\left[\text{Step2-3:calculation of likelihood of } \boldsymbol{p}_t^{(i)}\right]$ Calculate the likelihood $\alpha_t^{(i)}$ of particle $\boldsymbol{p}_t^{(i)}$ using measurement vector $\boldsymbol{y}_t$ and measurement noise distribution as follows:

$$\alpha_t^{(i)} = r_v\left(\boldsymbol{y}_t - \boldsymbol{H}\boldsymbol{p}_t^{(i)}; \boldsymbol{m}_{vr}, \boldsymbol{\Sigma}\right),\ i = 1, \ldots, m \tag{11}$$

[Step2-4:calculation of filter distribution] Calculate particles $\boldsymbol{f}_t^{(i)}$ of filter distribution by resampling the particles $\boldsymbol{p}_t^{(i)}$ in accordance with the following probabilities:

$$Pr\left(\boldsymbol{f}_t^{(i)} = \boldsymbol{p}_t^{(i)}\right) = \frac{\alpha_t^{(i)}}{\alpha_t^{(1)} + \ldots + \alpha_t^{(m)}},\ i = 1, \ldots, m \tag{12}$$

The fix-lag smoothing algorithm can be obtained by resampling the past particles simultaneously with the current particles $\boldsymbol{p}_t^{(i)}$ [4]. That is, Step2-4 in the

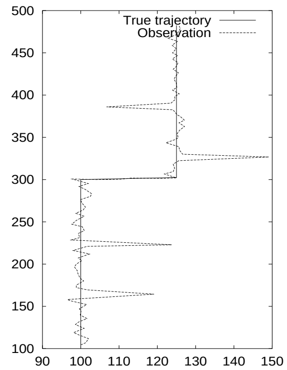


Fig. 2 Synthetic data used in the experiment. The solid line represents true trajectory and the dotted line represents observation.

Table 1 Determined hyper-parameters for the synthetic data. GM:Gaussian model, NGM:non-Gaussian model.

GM		NGM	
τ^2	σ^2	τ^2	σ^2
0.20	11	0.45	0.20

filtering algorithm should be modified in the fix-lag smoothing algorithm as follows:

[Step2-4:calculation of smoothing distribution] Calculate particles $s_{t-L|t}^{(i)}$ of fix-lag distribution $p\left(\boldsymbol{x}_{t-L} \mid Y_t\right)$ by resampling a set of particles $\left(\boldsymbol{s}_{t-L|t-1}^{(i)}, \ldots, \boldsymbol{s}_{t-1|t-1}^{(i)}, \boldsymbol{p}_t^{(i)}\right)$ in accordance with the probability of Eq.(12).

At the beginning of the fix-lag smoothing, particles $\left(\boldsymbol{f}_0^{(i)}, \boldsymbol{p}_1^{(i)}\right)$ are resampled, and the result is $\left(\boldsymbol{s}_{0|1}^{(i)}, \boldsymbol{s}_{1|1}^{(i)}\right)$ where $\boldsymbol{s}_{1|1}^{(i)} = \boldsymbol{f}_1^{(i)}$. This procedure is continued by adding particles $\boldsymbol{p}_t^{(i)}$ and resampling them with the stored particles $\boldsymbol{s}_{j|t-1}^{(i)}$ (if $t \leq L$ then $j = 0, \ldots, t-1$. if $t > L$ then $j = t-L, \ldots, t-1$.) at each time to obtain $\boldsymbol{s}_{t-L|t}^{(i)}$.

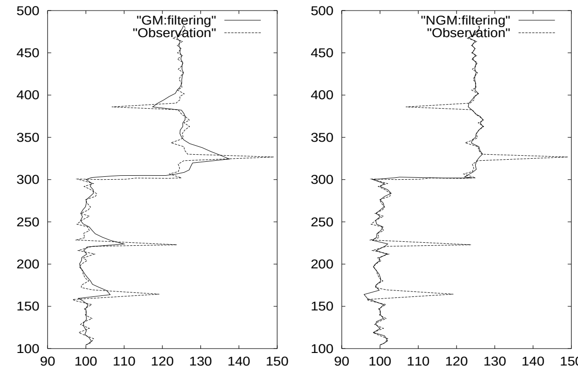
4. Experimental Results

The experimental results for synthetic and real data are shown. The comparison with Gaussian model with state estimation by the Kalman filter is also presented; the Gaussian distribution $N\left(0, \sigma^2\right)$ is used for measurement noise in the Gaussian model.

4.1 Synthetic Data

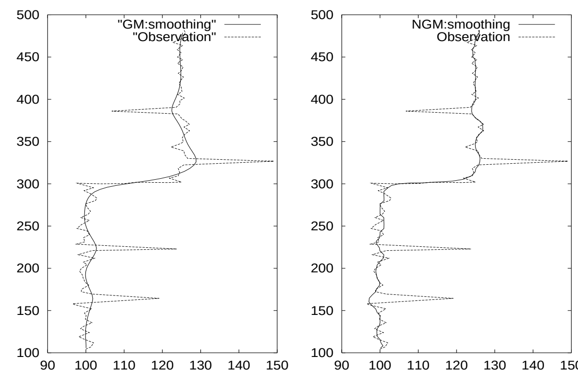
The synthetic data shown in Fig.2 was used: the solid line represents the true trajectory and the dotted line represents the observation with noise and outliers.

The log-likelihood can be used to determine the hyper-parameters τ^2 and σ^2[7]. The log-likelihood for the Gaussian model is represented as follows:



(a) (b)

Fig. 3 Filtering results for the synthetic data. (a) Gaussian model. (b) non-Gaussian model.



(a) (b)

Fig. 4 Smoothing results for the synthetic data. (a) Gaussian model. (b) non-Gaussian model.

$$l_{GM}\left(\tau^2, \sigma^2\right) = -\frac{1}{2}\left\{ nN \log 2\pi + \sum_{t=1}^{N} \log \left| \boldsymbol{V}_{t|t-1} \right| + \sum_{t=1}^{N} \left(\boldsymbol{y}_t - \boldsymbol{m}_{t|t-1}\right)^T \boldsymbol{V}_{t|t-1}^{-1} \left(\boldsymbol{y}_t - \boldsymbol{m}_{t|t-1}\right) \right\} \quad (13)$$

where n is the number of dimension of state vector, N is the length of the sequence of measurement Y_t, and $\boldsymbol{m}_{t|t-1}$ and $\boldsymbol{V}_{t|t-1}$ are the mean vector and the covariance matrix of prediction distribution calculated in the Kalman filter. The log-likelihood for the non-Gaussian model can be approximated as follows[4]:

$$l_{NGM}\left(\tau^2, \sigma^2\right) \cong \sum_{t=1}^{N} \log \left(\sum_{i=1}^{m} \alpha_t^{(i)} \right) - N \log m \quad (14)$$

where $\alpha_t^{(i)}$ is likelihood calculated in Step2-3 of the MCF algorithm. A coarse-to-fine method was used to determine the hyper-parameters; first, coarse grid $\left[\tau^2, \sigma^2\right]$: $\{1, 2, \ldots, 20\} \times \{1, 2, \ldots, 20\}$ was used as the candidates of the hyper-parameters and then more detailed grid was selected according to the maximum log-likelihood in coarse grid. The determined hyper-parameters for the synthetic data are shown in Table

Table 2 Mean squared error for the estimates of the synthetic data. GM: Gaussian model, NGM:non-Gaussian model, F:filtering, S:smoothing.

GM		NGM	
F	S	F	S
0.326	0.276	0.156	0.144

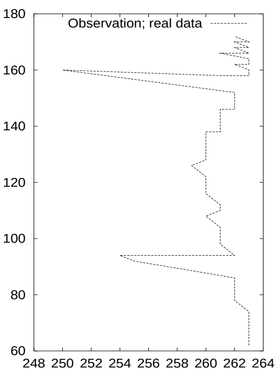


Fig. 5 Real data used in the experiment.

(a) (b)

Fig. 6 Image sequence used in the experiment. (a) first frame. (b) frame 60. The black rectangles in image are feature points.

1.

Filtering and fixed-lag smoothing with $L = 25$ were carried out. The number of particles in the MCF was 10,000. The initial distribution of state $p_0(\boldsymbol{x})$ was Gaussian with the mean $[x(1), \ y(1), x(1), y(1)]^T$ and the identity matrix as covariance one. In the MCF, the estimated coordinates of feature point were obtained from the mode of 2D histogram of $x_s(t)$ and $y_s(t)$ computed from particles $\boldsymbol{f}_t^{(i)}$ or $\boldsymbol{s}_{t-L|t}^{(i)}$.

The results of filtering and smoothing show that the effect of outliers was appeared in the estimates of the Gaussian model (Fig.3 (a) and Fig.4 (a)) while it was clearly reduced by the non-Gaussian model(Fig.3 (b) and Fig.4 (b)). The usefulness of the non-Gaussian model was also verified quantitatively by the mean squared errors between estimates and true trajectory(Table 2).

4.2 Real Data

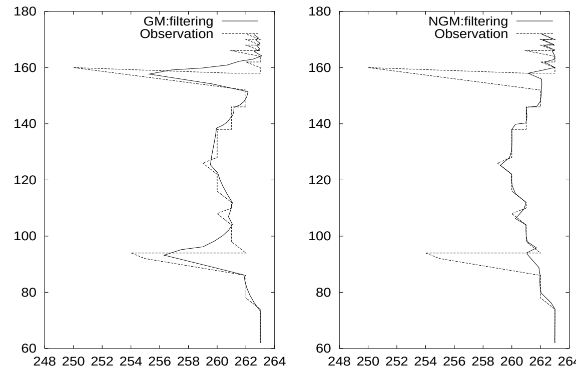
The motion trajectory of feature point shown in Fig.5 was obtained from the real image sequence shown in Fig.6; the motion trajectory of the feature point on the bottom of the book grabbed by the right hand was selected, since the false matchings were contained. The feature points in the image sequence, the black rectangles in Fig.6, were extracted by the corner detector shown in [8], and they were tracked using block matching based on normalized correlation. The hyper-parameters for the real data were also determined using the log-likelihood(Table 3), and the conditions, e.g. the number of particles, for the synthetic data were used.

Since the book in the image sequence was moved smoothly, smooth estimates should be obtained. The estimates by the Gaussian model, however, were clearly affected by the outliers due to the false matchings (Fig.7(a) and Fig.8(a)). On the other hand, the non-Gaussian model reduced the effect of the outliers, thus its estimates were desirable(Fig.7(b) and Fig.8(b)).

These experimental results show the usefulness of the proposed method.

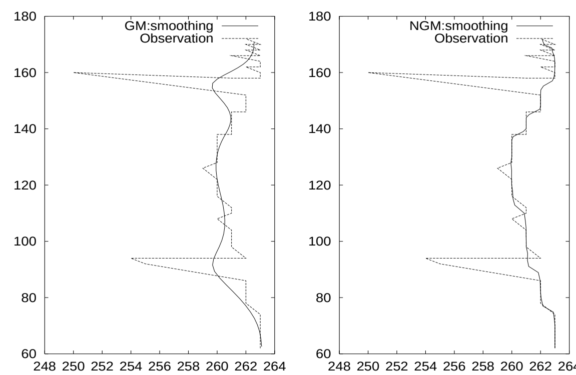
Table 3 Determined hyper-parameters for the real data.

GM		NGM	
τ^2	σ^2	τ^2	σ^2
0.055	2.5	0.03	0.07



(a) (b)

Fig. 7 Filtering results for the real data. (a) Gaussian model. (b) non-Gaussian model.



(a) (b)

Fig. 8 Smoothing results for the real data. (a) Gaussian model. (b) non-Gaussian model.

5. Conclusions

Filtering and smoothing using the non-Gaussian state space model have been proposed. The proposed method can reduce the effect of outliers in motion trajectory of feature point; its usefulness was confirmed by the experiments using the synthetic and real data.

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