VOLUME DATA CODING BASED ON REGION SEGMENTATION USING FINITE MIXTURE MODEL

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ABSTRACT

A volume data coding method based on the region segmentation is proposed in the present paper. The volume data is the three dimensional one, thus the visualization method is needed. To visualize the desired portions in the volume data, the region segmentation is indispensable. Hence, the region-oriented coding is suitable for the volume data.

The finite mixture model is used to estimate the probability density function of the feature vectors obtained from the volume data. The volume data is segmented using a posteriori probability calculated from this finite mixture model. The information of the segmented regions are represented as follows:(i)the contours of the regions are represented by the chain code in each slice,(ii)the colors of the regions are represented as the coefficients of the polynomial which is used to approximate those colors,(iii)since the posteriori probabilities can be regarded as the opacities of the voxels, the opacities which are needed to visualize the segmented regions are represented as the estimated parameters of the finite mixture model. An experimental result for the CT data is shown.

1. INTRODUCTION

The region-oriented coding is one of the methods utilizing the structural information for coding the image. In this method, the image is segmented into the homogeneous regions, and then the contours and the colors of the regions are coded separately. This method has been applied to the gray-scale image, the color image and the image sequence coding [1] [2] [3].

The volume data coding method based on the region segmentation is proposed in the present paper. A volume data $V(x, y, z)$ is the sampled function of three spatial dimensions. The volume data is useful to represent the internal structure of an object. However, the size of the volume data is normally very large. Thus the coding method for the volume data is needed.

Since the volume data is the three dimensional one, the rendering method which converts the volume data into the two dimensional image $I(X, Y)$ is needed to display it. The region segmentation is indispensable to render only the desired portions. That is, the structural information is inherently needed in this visualization. The region-oriented coding is, therefore, suitable for the volume data.

Figure 1 shows the outline of the proposed coding method. First, the feature of the each voxel is extracted. The probability density function of the feature vectors is estimated using the finite mixture model. The volume data is segmented using the posteriori probabilities of the feature vectors for each component density function of the model. After the region segmentation, the information of the regions, i.e. the contour, the color and the opacity, are represented. The contours of the regions are represented by the eight directional chain codes in each slice. The colors of the regions are approximated by the second order polynomial. Hence, the colors can be represented as the coefficients of this polynomial. Since the posteriori probabilities can be regarded as the opacities of the voxels, the opacities can be represented as the estimated parameters of the finite mixture model.

The experimental result for the CT data shows the usefulness of the proposed method.

2. VOLUME RENDERING

The volume rendering[4] is one of the visualization methods of the volume data. In this method, the high opacities should be assigned to the portions to be visualized. Thus these portions should be segmented in the first place. After the segmentation, the colors of the voxels calculated by the shading process are projected onto the image plane $I(X, Y)$ through the transparencies added from back to front along the viewing ray (Fig.2). This process can be represented as follows:

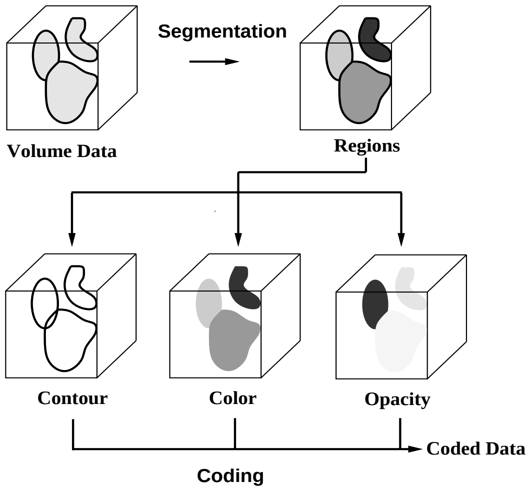


Fig. 1: Outline of the proposed coding method.

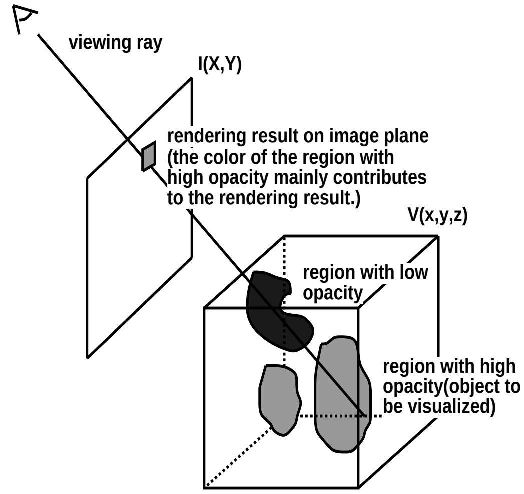


Fig. 2: Volume rendering.

$$I(X,Y) = \sum_{i=1}^{K} \left[c_i \alpha_i \prod_{j=i+1}^{K} (1-\alpha_j) \right] \quad (1)$$

where c_i and α_i are the color and the opacity of the i-th voxel $(i = 1, \ldots, K)$ which intersects the viewing ray. Obviously, the region segmentation is indispensable to visualize the desired portions in the volume data by the volume rendering.

3. REGION SEGMENTATION USING FINITE MIXTURE MODEL

The region segmentation for the volume rendering is carried out using the finite mixture model. The distribution of the feature vectors obtained from the volume data is represented by the following finite mixture model.

$$f(\boldsymbol{x} \,|\boldsymbol{\Theta}) = \sum_{i=1}^{r} \lambda_i \alpha_i (\boldsymbol{x} \,|\boldsymbol{\theta}_i) \quad (2)$$

where $\boldsymbol{x}$ is a feature vector in $\boldsymbol{R}^n$, r is the number of component densities $\alpha_i()$, $\boldsymbol{\Theta} = \{\{\lambda_i\}_{i=1}^r \ \{\boldsymbol{\theta}_i\}_{i=1}^r\}$ is a set of parameters, $\{\lambda_i\}_{i=1}^r$ are the mixing proportions such that $\sum_{i=1}^r \lambda_i = 1, 0 \le \lambda_i \le 1$, and $\boldsymbol{\theta}_i$ is a set of parameters of $\alpha_i()$.

The multivariate t distribution is used as a component.

$$\alpha(\boldsymbol{x} \,|\boldsymbol{\theta}) = \frac{\left|\boldsymbol{V}^{-1}\right| \Gamma\{(\nu+n)/2\}}{\{\Gamma(1/2)\}^n \Gamma(\nu/2) \nu^{n/2}} \cdot \left(1 + d_m^2/\nu\right)^{-(\nu+n)/2} \quad (3)$$

$$d_m^2 = (\boldsymbol{x}-\boldsymbol{m})^t \boldsymbol{V}^{-1} (\boldsymbol{x}-\boldsymbol{m}) \quad (4)$$

where $\boldsymbol{\theta} = \{\boldsymbol{m}, \boldsymbol{V}, \nu\}$, and $\boldsymbol{m}$ $(n \times 1)$, $\boldsymbol{V}$ $(n \times n)$ and ν are the location, the scatter matrix and the degree of freedom, respectively. This distribution is identical to the normal distribution when $\nu \to \infty$, and can represent more heavy tailed distribution by changing ν. Furthermore, the maximum likelihood parameter estimation of this distribution by the EM algorithm[5][6] is a kind of the robust estimation techniques[7]. Thus the segmentation using this finite mixture model can reduce the effect of the outliers in parameter estimation process like as the robust clustering[8]. The EM algorithm and the quasi-Newton method are used in parameter estimation [7].

A following posteriori probability is calculated using the estimated parameter, and it is used as the opacity of each voxel.

$$P(S_i \,|\boldsymbol{x}) = \lambda_i \alpha_i (\boldsymbol{x} \,|\boldsymbol{\theta}_i) / f(\boldsymbol{x} \,|\boldsymbol{\Theta}) \quad (5)$$

where S_i denotes the portion corresponding to the i-th component. The components corresponding to the portions to be coded are selected, and the opacities of all voxels are calculated using them. The region segmentation is carried out by extracting the voxels which have larger opacity than the given threshold.

4. REPRESENTATION OF SHAPE, COLOR AND OPACITY

After the region segmentation, the information of the regions are represented as follows.

The shape of the regions is represented by the chain code. First, the three dimensional connected components are extracted. And then, the contours of each

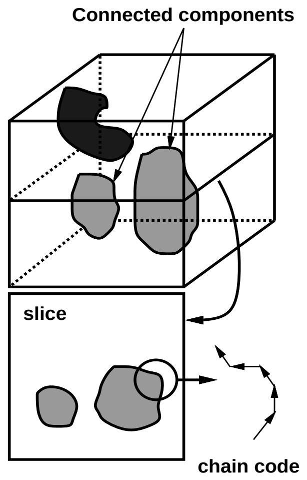


Fig. 3: Region boundary representation by the eight directional chain code.

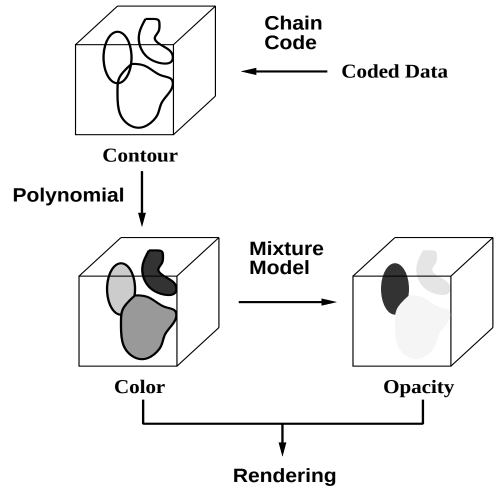


Fig. 4: Decoding process.

connected component are represented by the eight directional chain code on each slice(Fig.3). Thus the shape is represented by the slice numbers, the start points of the chains and the chain code numbers.

The colors of each connected components are approximated by the second order polynomial as follows:

$$C = WP \tag{6}$$

$$C = [\boldsymbol{c}_1, \ldots, \boldsymbol{c}_N] \quad (p \times N) \tag{7}$$

$$\boldsymbol{c}_i = (c_{i1}, \ldots, c_{ip})^t \tag{8}$$

$$W = [\boldsymbol{w}_1^t, \ldots, \boldsymbol{w}_p^t] \quad (p \times m) \tag{9}$$

$$\boldsymbol{w}_j = (w_{j1}, \ldots, w_{jm})^t, j = 1, \ldots, p \tag{10}$$

$$P = [\boldsymbol{p}_1, \ldots, \boldsymbol{p}_N] \quad (m \times N) \tag{11}$$

$$\boldsymbol{p}_i = (1, x_i, y_i, z_i, x_i y_i, y_i z_i, z_i x_i, x_i x_i, y_i y_i, z_i z_i)^t, i = 1, \ldots, N \tag{12}$$

where N is the number of voxels in the connected component, p is the number of dimensions of the color and (x_i, y_i, z_i) are the coordinates of each voxel. The colors are represented by the matrix W constructed by m coefficients of the polynomial.

$$W = CP^+ \tag{13}$$

where P^+ is the Moore-Penrose generalized inverse matrix of P.

The opacities are represented by the estimated parameters of the finite mixture model. Given these parameters, the opacities can be calculated by (5).

Each information is used in the decoding process as shown in Fig.4.

5. EXPERIMENTAL RESULT

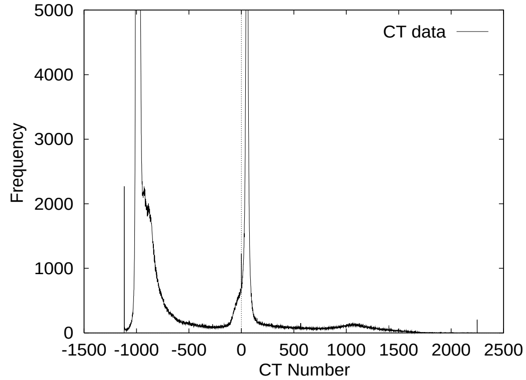
An experimental result for the CT data (128 × 128 × 128,16 [bit/voxel]) is shown. The feature used in the segmentation was the CT number of each voxel. The probability density function of the CT number was estimated by the finite mixture model with eight components(Fig.5).

The components 1,2 and 3 corresponded to the air, 4 and 5 corresponded to the soft tissue and 6,7 and 8 corresponded to the skull. In this experiment, the skull was selected as a coding object. Hence, the opacities of all voxels were calculated using the components 6,7 and 8. Then voxels which had larger opacity than the threshold 0.1 were segmented.

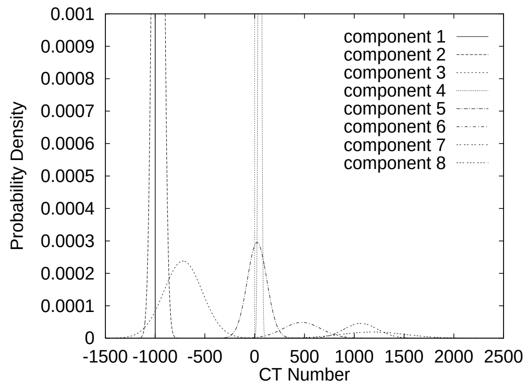
The skull was correctly segmented by the finite mixture model(Fig.6 (a) and (b)). To render the decoded data, the opacities were calculated by (5) using the parameters of the finite mixture model included in the coded data. The rendering results of the decoded data showed that the major features of the skull were sufficiently reconstructed although the detailed features were lost(Fig.6 (c) and (d)). The compression ratio of the skull was 8.7:1 (1.84[bit/voxel]). The size of the volume data was reduced to 1/17.4 of the original size by the region segmentation. As a result, the total reduction of the size of the volume data was 1/151.4.

6. CONCLUSIONS

The volume data coding based on the region segmentation was proposed. The finite mixture model with the t distribution as a component was used to



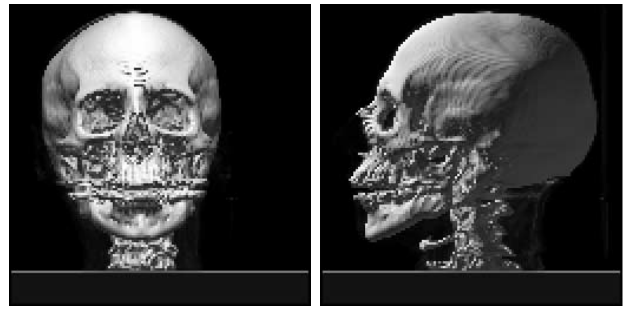
(a)



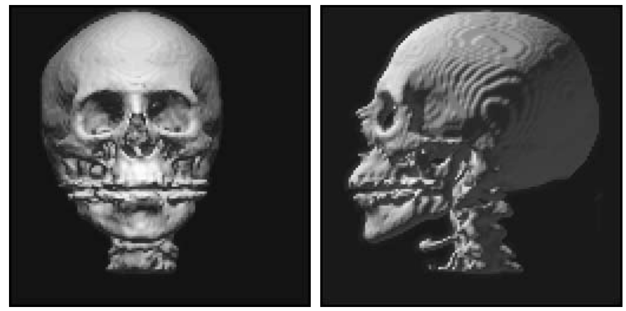
(b)

Fig. 5: Estimation result of the probability density function for the histogram of the CT number by the finite mixture model with eight components. (a) histogram of the CT number. (b) estimated probability density function using the finite mixture model.

segment the volume data. The posteriori probabilities could be regarded as the opacities of the voxels. Thus the opacities could be represented as the estimated parameters of the finite mixture model. The shape and the colors of the regions were represented by the chain codes and the coefficients of the polynomial, respectively. The experimental results for the skull of the CT data showed that the major features of the skull could be sufficiently reconstructed from the coded data, and the total reduction of the size of the volume data by the region segmentation and the coding was 1/151.4.



(a) (b)



(c) (d)

Fig. 6: Experimental result for CT data. (a),(b) rendering result of the original data(front and side view). (c),(d) rendering result of the decoded data(front and side view).

7. REFERENCES

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