

No.1

フェーズフィールドモデルに基づく 二相流界面追跡計算法

Phase-Field Method for Interface-Tracking Simulation of Two-Phase Flows

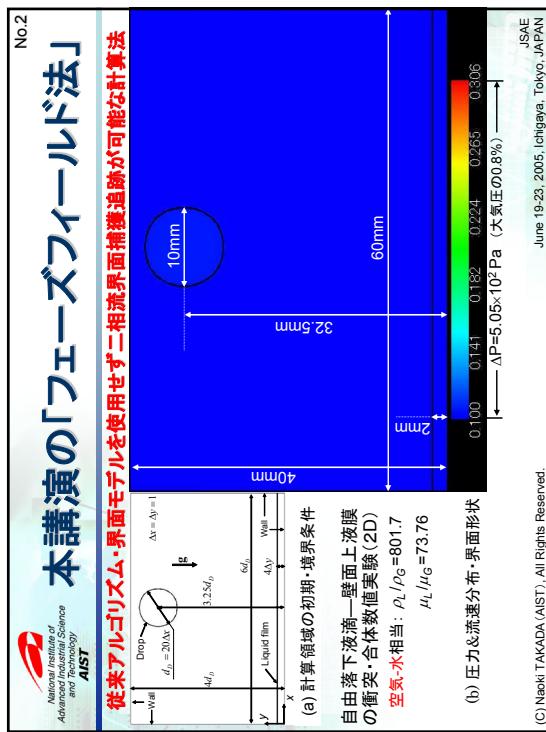
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2005年6月28日(火), 東京・市ヶ谷

JSAE June 19-23, 2005, Ichigaya, Tokyo, JAPAN

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No.3

Outline

Purposes of This Study

- (1) Examination of phase-field model (PFM) for tracking two-phase fluid interface
- (2) Development of PFM-based computational method for interface-tracking simulation of two-phase flows

Contents

1. Basis of PFM
2. Phase-field methods (NS-PFM) combining Navier-Stokes (NS) equations with PFM for,
 - (1) Isothermal fluid at a high density ratio
 - (2) Thermal fluid with phase change
3. Numerical result of two-phase flows
4. Conclusions

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No.5

Phase-Field Model (PFM) Based on van-der-Waals or Cahn-Hilliard Free-Energy Theory

- Two-phase coexistence by a free-energy functional F :
$$F = \int \left[f(\phi) + \frac{\kappa}{2} |\nabla \phi|^2 \right] dx$$

$f(\phi)$: Bulk energy (double-well for ϕ)
 ϕ : Index function to describe interface
 κ : Capillary coefficient

- without imposing topological constraints on interface as phase boundary.
- Interface = Volumetric transition zone with steep & continuous variation of physical properties (density ρ , viscosity μ , etc.)

Surface tension $\sigma = \kappa \int_{-\infty}^{+\infty} \left(\frac{\partial \rho}{\partial x} \right)^2 dx$ (for flat interface perpendicular to x axis)

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No.6

Phase-Field Model (PFM) Method for Two-Phase Flow Simulation

- PFM method does not necessarily require conventional,
 - Interfacial advection-reconstruction algorithms
 - CSF (Continuum Surface Force) model
- Simplified interface-tracking calculation on a fixed spatial grid
- Advantages of PFM method over others,
 - multi-dimensional interface advection
 - heat & mass transfer across interface
- Classical NS-PFM for low density ratio ρ_L/ρ_g (due to numerical instability)
- A new NS-PFM proposed by extending a lattice Boltzmann method (LBM) for high ρ_L/ρ_g up to 1,000
- NS-PFM has lower computational cost & more flexibility than LBM.

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No.7

二相流界面追跡計算法の構成 従来法／Phase-Field Method (PFM)

巨視的現象の記述法 流れの基礎方程式	界面現象の記述法 ／表面張力計算法	二相流計算手法 ＝ Conventional NS Method	数值計算手法の 具体例
plain Navier-Stokes (NS) Method	I/F-Tracking Scheme / CSF model	Front-Tracking Level-Set Marker-and-Cell Volume-of-Fluid	Inamuro, T., et al. Seta, T., et al. Swift, M.R., et al.
plain LB Method	Phase-Field Modeling (PFM)	Two-Phase Fluid LBM	Doi, M., et al.(1997) Jacqmin, D. (1999) Jamet, D., et al. (2001)
plain-NS Method		NS-PFM	

* 注：支配方程式の離散化スキーム、計算アルゴリズム、座標系、等は3者とも任意選択可能

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No.8

Navier-Stokes Phase-Field Method (NS-PFM) for Incompressible Isothermal Two-Phase Flow with High Density Ratio ρ_L/ρ_G

Mass & Momentum Conservation Equations for Two-Phase Flow:

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla \cdot \mathbf{P} + \nabla \cdot \left[\mu(\rho) (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \right] + (\rho - \rho_c) \mathbf{G}$$

Pressure tensor : $\mathbf{P} = \left(p - \kappa_S |\nabla \rho|^2 \right) \mathbf{I} + \kappa_S \nabla \rho \otimes \nabla \rho$

Density : $\rho(\phi) = \frac{\rho_L + \rho_G + \rho_L - \rho_G}{2} \sin \left(\frac{(\phi_L + \phi_G)/2}{\phi_L - \phi_G} \pi \right)$

Surface tension parameter : $\sigma \equiv \kappa_S \int_{-\infty}^{+\infty} \left(\frac{\partial \rho}{\partial x} \right)^2 dx$

Viscosity : $\mu(\rho) = \frac{\mu_L - \mu_G}{\rho_L - \rho_G} (\rho - \rho_G) + \mu_G$

Cahn-Hilliard(CH) Equation with Advection of Fluid Interface & Volume:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = -\nabla \cdot [-\Gamma(\phi) \nabla \eta] \quad (R.H.S. = 0 \text{ for a local equilibrium state})$$

Chemical potential : $\eta = \frac{\partial \psi_\phi}{\partial \phi} - \kappa_\phi \nabla^2 \phi$

Mobility : $\Gamma(\phi) = \Gamma \phi > 0$

ψ_ϕ : Bulk free energy = $\phi T \ln [\phi / (1 - B\phi)] - A\phi^2$
 (van-der-Waals)

κ_ϕ : Interface thickness parameter $\eta_\phi \propto \sqrt{\kappa_\phi}$

* Original : LBM by Inamuro, T., et al., J. Comput. Phys., 198(2004), 628.

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No.9

Navier-Stokes Phase-Field Method (NS-PFM) for Thermal Two-Phase Flow with Phase Change

A. Full Set of Navier-Stokes Conservation Equations for Compressible Non-ideal Fluid

ρ : Density \mathbf{u} : Velocity T : Temperature

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$E = \frac{1}{2} \rho \mathbf{u}^2 + \rho(cT - A\rho) + \frac{\kappa_s}{2} |\nabla \rho|^2$$

$$\mathbf{P} = \left(p - \kappa_s \rho \nabla^2 \rho - \frac{\kappa_s}{2} |\nabla \rho|^2 \right) \mathbf{I} + \kappa_s \nabla \rho \otimes \nabla \rho$$

$$\boldsymbol{\tau} = -\frac{2}{3} \mu (\nabla \cdot \mathbf{u}) \mathbf{I} + \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

Numerical solution of : (1) MacCormack scheme (+ explicit artificial viscosity)
 NS eqs.+E.O.S. : (2) κ_s , c , k , μ = constant in whole flow field

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No.10

二相流界面追跡に関する従来計算法とフェーズフィールド法 (PFM)との比較

	従来計算法(NS方程式)	PFM
計算法 (網分類)	MAC VOF Level Set MPS Front Tracking	NS-PFM LBM LGCA
界面厚さ	数値的 (0 or $\approx \Delta X$)	物理的 ($> \Delta X$, 数セル)
表面張力	Continuum Surface Force (CSF) Model (界面厚さと関連なし)	界面自由エネルギー (界面厚さと関連あり)
界面移流・再構成	MAC PLIC MARS CIP DA TVD	化学生テンショル勾配
離散化	Eulerian Semi/Fully-Lagrangian FDM FEM	ALE BEM / BFC

PFMは新しい二相流計算アルゴリズムを提供する。

「従来法 v.s. PFM」は不十分で両者の最適な取扱選択・融合

LBM: Lattice Boltzmann Method LGCA: Lattice Gas Cellular Automaton
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No.12

Benchmark Test on Interface Advection Using the Cahn-Hilliard (CH) Equation

(1) 1D Linear Transport

$\Delta x = \Delta y = d/32, \Delta t = 1$

(2) 2D Linear Transport

Computational domain

Interface profile at $t=200$.
 (drawn as a contour line)

Gas-phase cell number N/N_0

Dimensionless time $*t=U/d$

Time series of volume of circular-shaped fluid .

Parameters in CH eq.: $A=B=1$, $T=0.293$, $\kappa_\phi=0.1$

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No.11

Benchmark of Transport of Interface Using Cahn-Hilliard (CH) Equations

(1) 1D Linear Transfer on a fixed spatial grid

Generally, $\frac{\partial \phi}{\partial t} + \nabla \cdot [\Gamma(\phi) \mathbf{j}] = 0$ $\mathbf{j} = -\nabla \eta$

CH Equation $\Gamma(\phi) = F_0 \phi$: Mobility $F_0 > 0$: constant

In this study, $\frac{\partial \phi}{\partial t} + \nabla \cdot [\phi \mathbf{u} + \Gamma_0 \mathbf{J}] = 0$ $\mathbf{J} = -\phi \mathbf{j} = -\zeta \nabla \phi + \kappa_2 \phi \nabla (\nabla^2 \phi)$

Case (A) 1D square-wave propagation.

Interface profile at $t=200$.
 (drawn as a contour line)

Gas-phase cell number N/N_0

Dimensionless time $*t=U/d$

Time series of volume of circular-shaped fluid .

Parameters in CH eq.: $A=B=1$, $T=0.293$, $\kappa_\phi=0.1$

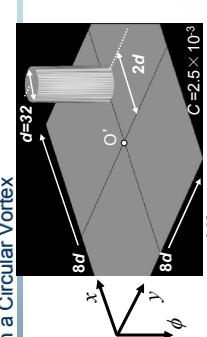
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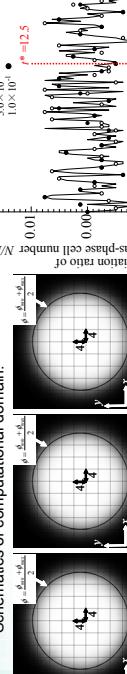
Benchmark Test of Interface Advection Using the Cahn-Hilliard (CH) Equation

(3) 2D Rotation in a Circular Vortex



Courant number $C = U_\phi \Delta t / \Delta x$, $\Delta t = 3.2 \Delta x$, $\Delta x = AY = 1$, $U_\phi = 2\pi / 8d$, $d = 3.2 \Delta x$, $C = 2.5 \times 10^{-3}$, $a = 32$.

Schematics of computational domain.



Interface profile drawn as a contour line of ϕ at $t^*=12.5$. Dimensionless time $t^*=U_\phi/d$. Volume conservation.

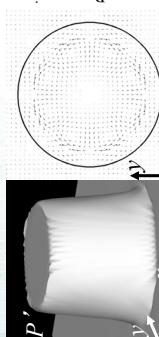
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No.15

Surface Tension Force of NS-PFM

(2D Neutrally-Buoyant Bubble in Liquid)



$\rho_L / \rho_G = 801.7$ (for air-water system), $\mu_L / \mu_G = 73.76$, $\sigma = 4.31 \times 10^{-4}$, $\kappa_2 = 0.1$, $\rho_G = 1.25 \times 10^{-6}$, $\rho_L = 1.0 \times 10^{-3}$, P' , R , P .

Pressure increase inside 2D bubble [$\times 10^{-5}$]

Curvature of interface $1/R$

(a) Pressure (b) Velocity (c) Pressure increase inside bubble.

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No.14

Cahn-Hilliard方程式を用いた界面移流問題

ベンチマークのまとめ



(a) 高精度の体積保存性の実現
(b) 自動的な界面形状再構成の実施
…既存アルゴリズムなしに界面形状・厚さを一定に保持

(c) 数値拡散と数値振動の適切な抑制
…標準的な有限体積スケーム、時間進行法のみ使用

二相流界面追跡計算における要求への対応

(1) 形状保存性 … 界面形状の正確な捕獲と輸送
(2) 体積保存性 … 流体体積率の正確な輸送
(3) 流体率の連続性 … 匀配を考慮した界面再構成

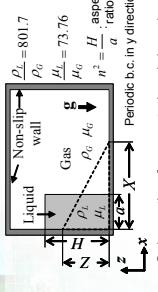
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No.16

Collapse of 2D Liquid Column under Gravity

(1) Initial & Boundary Conditions



Non-slip wall, Gas, ρ_G , μ_G , H , ρ_L , μ_L , a , $n^2 = \frac{H}{a}$, aspect ratio $n^2 = \frac{H}{a}$, Periodic b.c. in y direction.

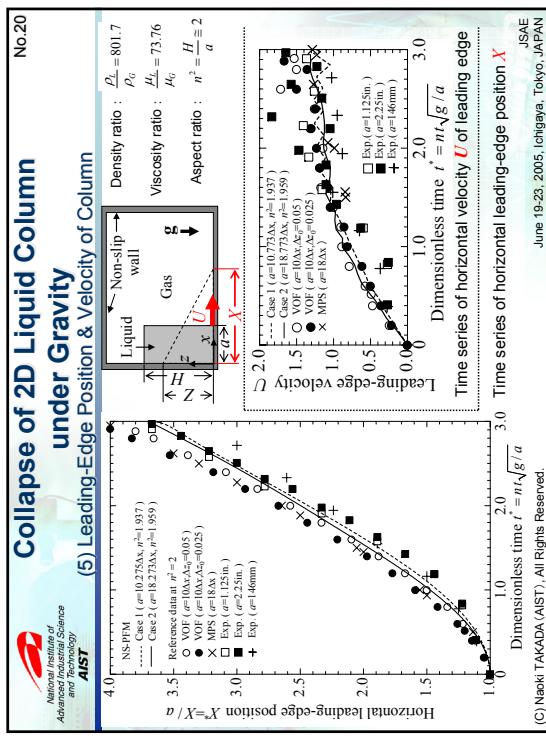
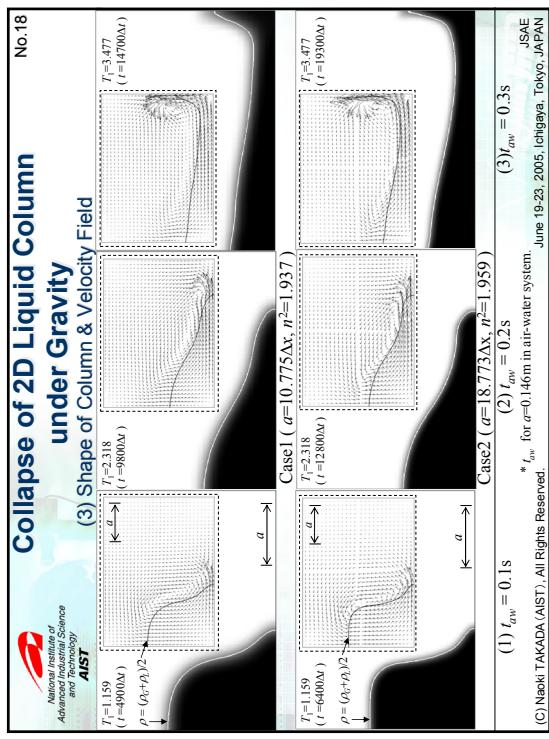
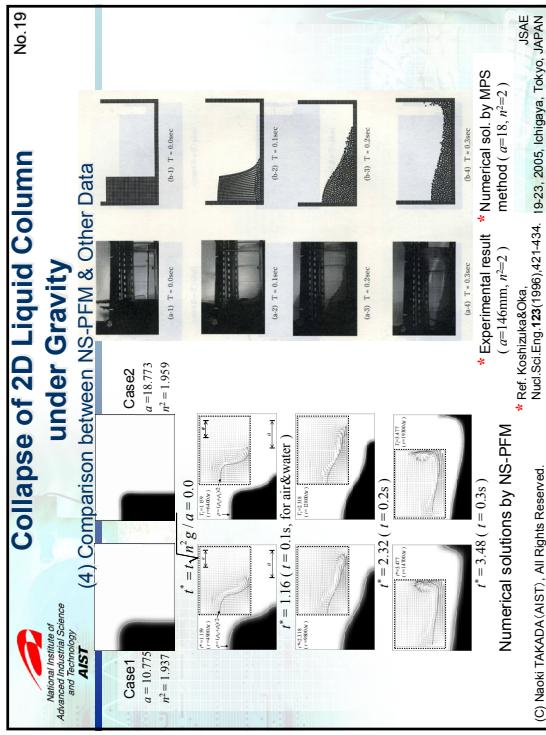
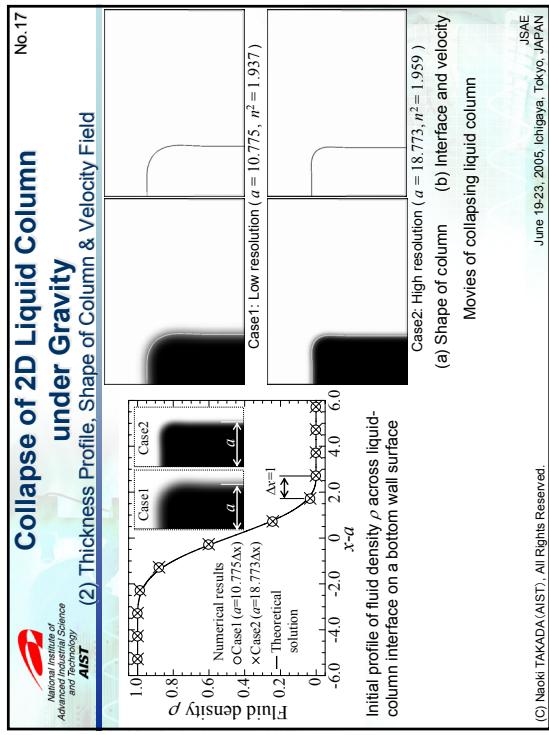
Schematic of computational domain

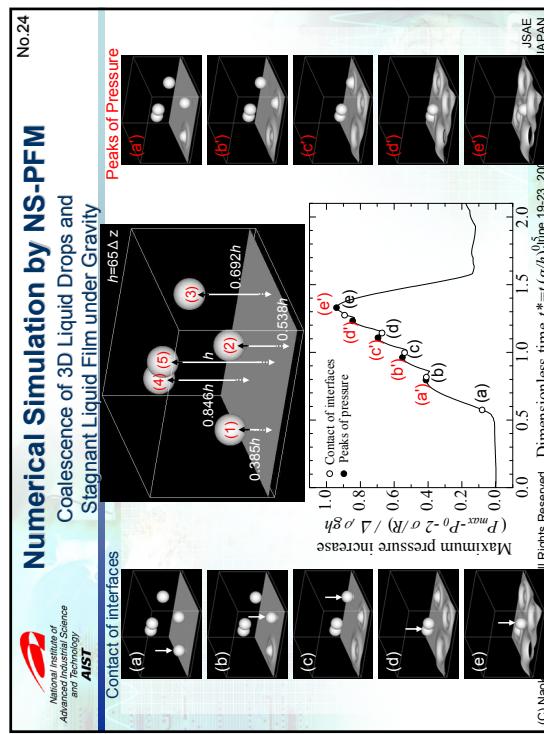
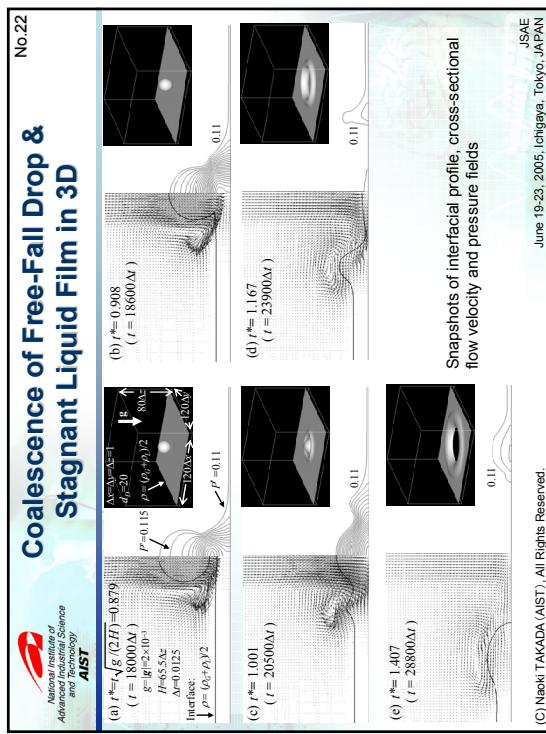
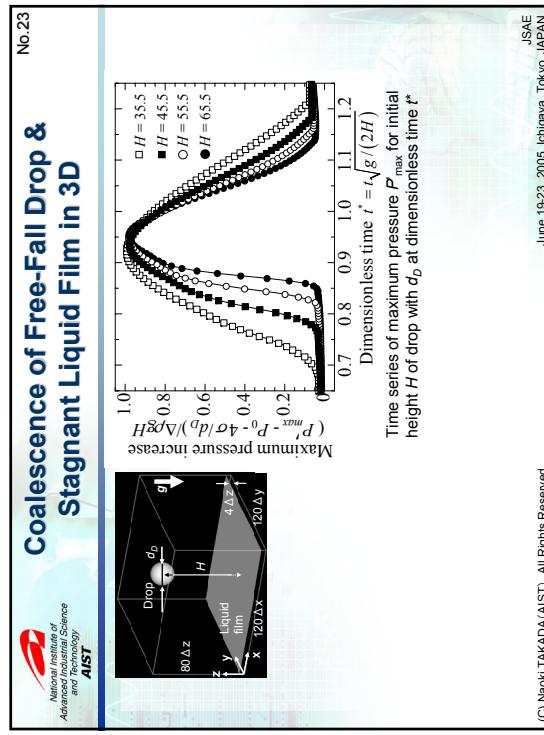
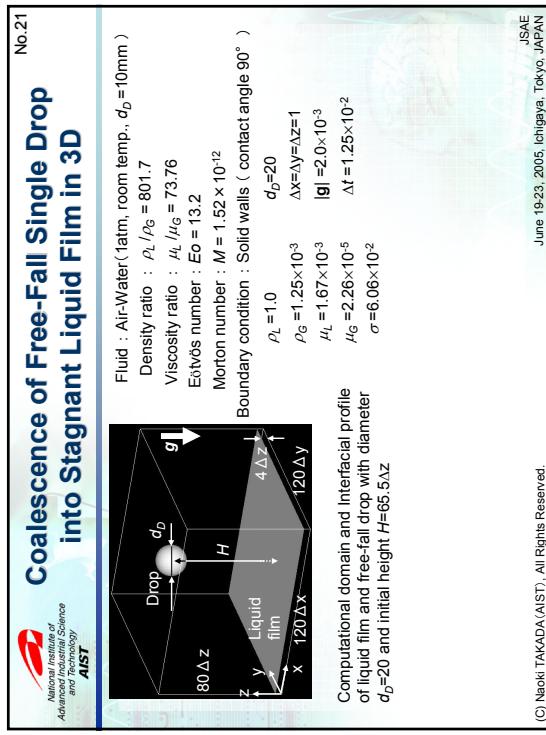
Table1 Parameters in liquid column simulation
($|g|=2\times 10^3$, $\rho_L=1$, $\rho_G=1.25\times 10^3$, $\Delta=0.0125$)

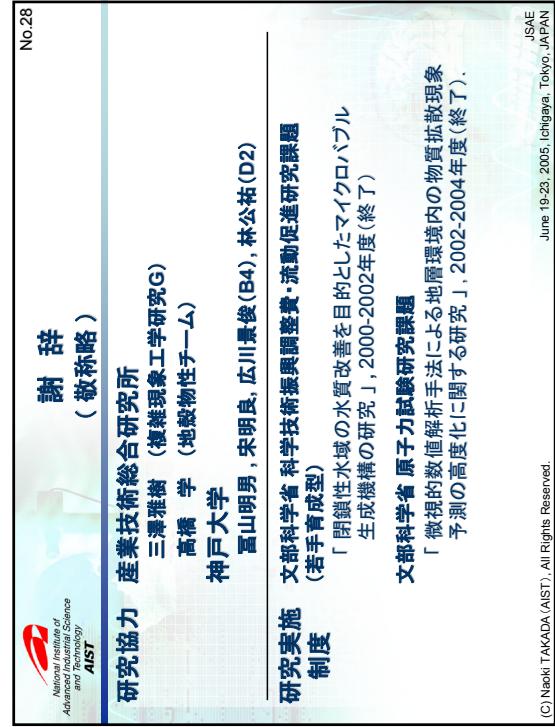
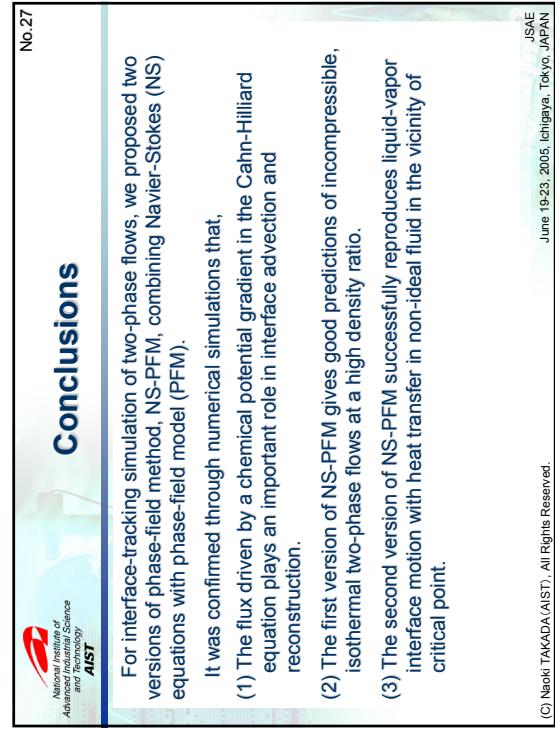
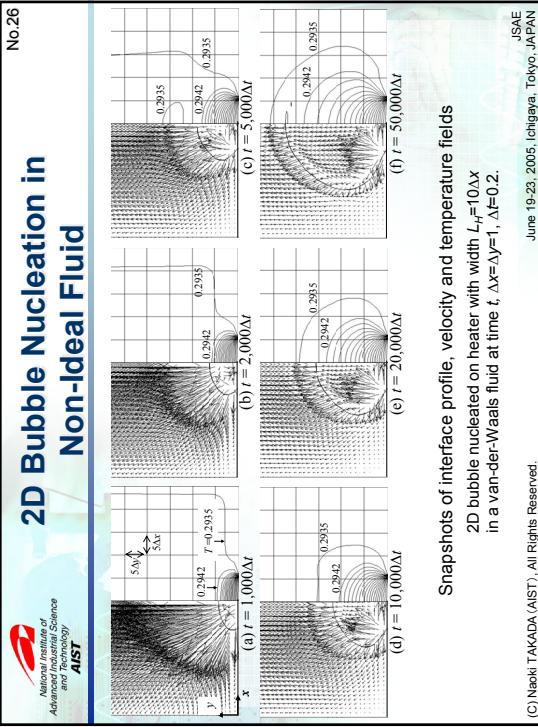
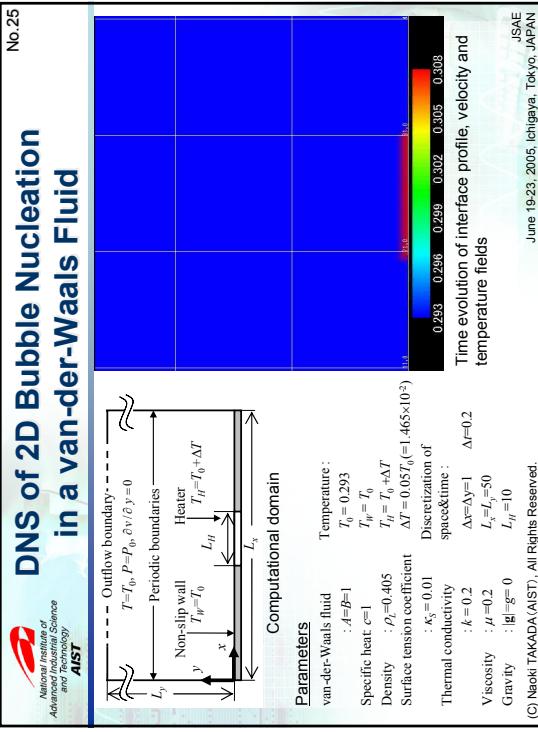
Case#	Aspect ratio $n^2=H/a$ ($\Delta=1$)	Width of column a (m)	Surface tension σ	Viscosities μ_L (liquid)
1	2	10X	1.46×10^1	7.11×10^7
2	2	18X	1.46×10^1	2.30×10^7
				3.47×10^7
				2.56×10^8

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No.29

**高効率・高密度比二相流界面追跡計算法
(ナビエ・ストークス・フェーズフィールド法: NS-PFM)
の基礎開発と適用**

**格子ボルツマン法 (LBM) の発展型計算法 = 計算流体力学 (CFD) + 非平衡系熱力学
の基礎開発と適用**

マルチスケール＆マルチフィジックス

—利点—

- (1) 従来計算法より簡素＆高効率 (2) LBMより柔軟な解析適応能力
- 得意分野 = 環境流体力学 —
- (1) 流滴・気泡クラスター運動
- (2) 多孔体内二相流・毛管現象・地下水 / Diesel油粒子回収Filter / 化学分析チップ (u-TAS)
- (3) その他: 相変化・溶解(界面を通過する熱物質輸送計算)

—新規性 —

計算技術面:

- (1) NS-PFM適用の気液二相流計算・国内初
- (2) 高密度比対応可能なNS-PFM・国内外初

学術面:

- 界面追跡での自由エネルギー理論の重要性を指摘・国内初
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No.30

**本講演に関する主要な参考文献
フェーズフィールドモデルと格子ボルツマン法**

National Institute of Advanced Industrial Science and Technology
AIST

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Information

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DSFD2005 (August 22-26, Kyoto Univ.)
URL: <http://fd.kuaero.kyoto-u.ac.jp/DSFD2005/index.html>

ICMIMES2005 (July 26-29, Hong Kong Polytechnic Univ.)
URL: <http://www.icmimes.org/>

格子ボルツマン法の基礎と応用に関する研究分科会
(LBM研究会, A-TS05-19, JSME 流体工学部門)
URL: <http://www.jsme.or.jp/>

JSME 18th 計算力学講演会・OS「格子ガス・ボルツマン法」
(2005年11月19-21日 筑波大学, 講演集集中)
URL: <http://www.jsme.or.jp/cmd/cmc2005/>

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Phase-Field Method for Interface-Tracking Simulation
of Two-Phase Flows
URL: http://staff.aist.go.jp/naoki-takada/phase_field_cfd.htm

Lattice Boltzmann Method (LBM)
URL: http://staff.aist.go.jp/naoki-takada/lbm_study.htm

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