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Exact Point Cloud Downsampling for Fast and Accurate Global Trajectory Optimization

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Keypoints

Point cloud downsampling technique that accelerates global registration error minimization

- A sampling result gives the **same quadratic registration error** function as that of the original one at the evaluation point
- **Only 29 residuals** are required to be re-linearized at a minimum
- Drastic reduction of **memory consumption (by 99%)** and **processing time** (**by 87%**) while retaining the accuracy

Problem and Proposal

Global Registration Error Minimization

Directly minimizes registration errors over the entire map (i.e., multi-scan registration)

☺ Accurate loop closing for frames with a very small overlap

☺ Accurate propagation of per-point scan matching uncertainty to pose variables

👍 More accurate than the conventional pose graph optimization

☹ Needs to remember point correspondences for each factor

☹ Needs to re-evaluate registration errors every optimization iteration

👎 Large memory consumption and computation cost

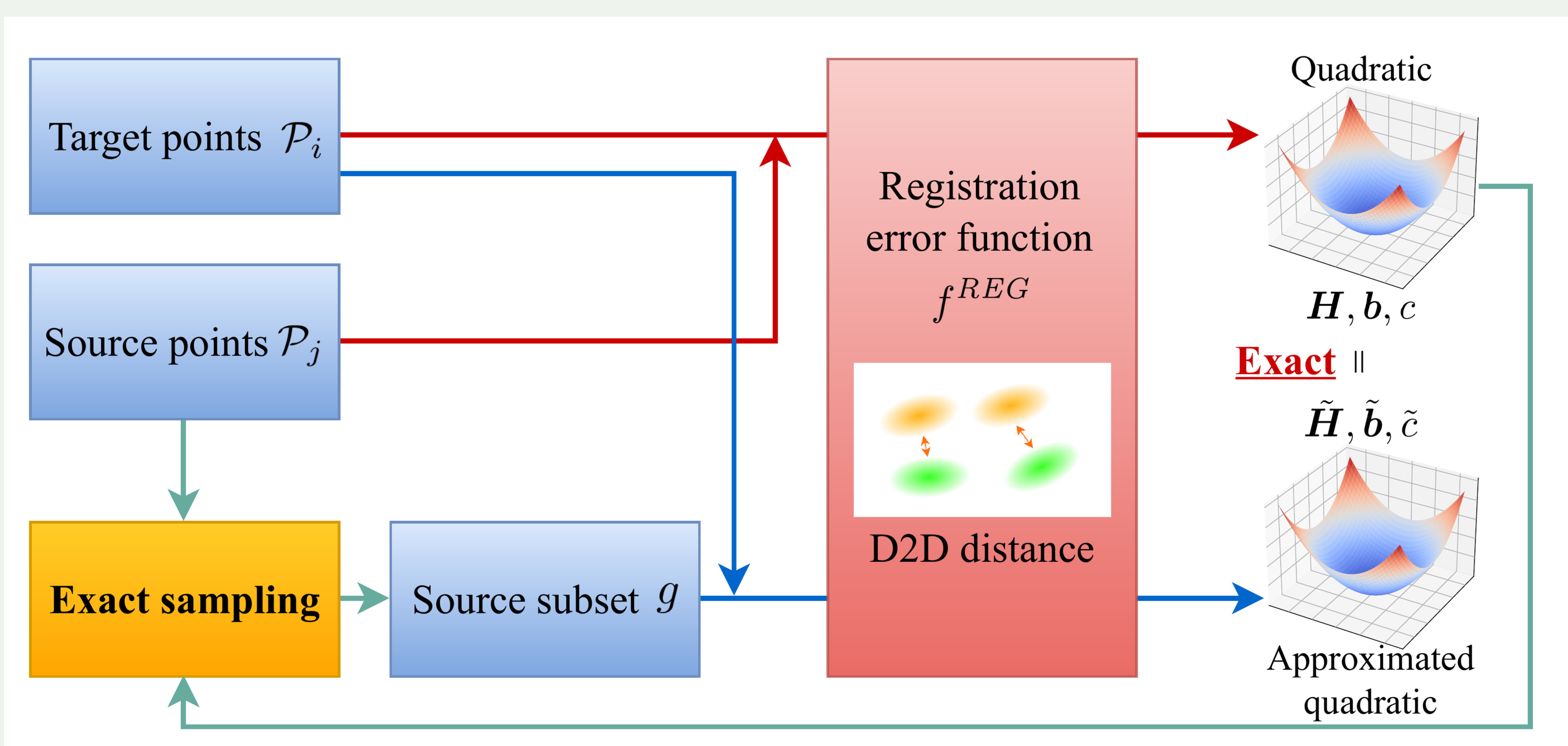
Point Cloud Downsampling

Conventional methods (e.g., random and geometry-aware sampling) introduce approx errors and require a trade-off between processing cost (# of points) and accuracy

[Reijgwart, RA-L 2020] [Wang, RA-L 2022] [Yokozuka, ICRA2021]

Proposal

To reduce points as few as possible while retaining the result as accurate as possible, we find a subset of input points that yields **the same quadratic error function** as that of the original set via **efficient coreset extraction**



Exact Downsampling

Proposed Approach to Downsampling

Gauss-Newton optimization models an error function in the quadratic form

$$f^{REG}(\mathcal{P}_i, \mathcal{P}_j, \check{\boldsymbol{T}} \boxplus \Delta \boldsymbol{x}) \approx \Delta \boldsymbol{x}^T \boldsymbol{H} \Delta \boldsymbol{x} + 2\boldsymbol{b}^T \Delta \boldsymbol{x} + c,$$

Error function Source / Target points Information matrix / vector Constant

Exact downsampling selects a subset of points that yields the same quadratic function

$$f^{DOWN}(f^{REG}, \mathcal{P}_i, \mathcal{P}_j, \check{\boldsymbol{T}}_i, \check{\boldsymbol{T}}_j) = (\boldsymbol{w}_{ij}, \mathcal{G}_{ij}),$$

Evaluation point Weights / Residual indices

$$\text{s.t.} \quad \boldsymbol{H} = \tilde{\boldsymbol{H}}, \quad \boldsymbol{b} = \tilde{\boldsymbol{b}}, \quad c = \tilde{c},$$

Quadratic function parameters calculated using original set / extracted subset

During optimization, f^{REG} is re-relinearized using the selected subset (i.e., *coreset*)

Coreset Extraction Algorithm

Hessian matrix $\boldsymbol{H}$ can be represented with at most $D^2 + 1$ rows of Jacobian matrix $\boldsymbol{J}$ without approximation via **Caratheodory set extraction**

$$\boldsymbol{H} = \boldsymbol{J}^T \boldsymbol{J} = \sum \boldsymbol{a}_k^T \boldsymbol{a}_k \qquad \frac{1}{N} \sum \text{flatten}(\boldsymbol{a}_k^T \boldsymbol{a}_k) = \frac{1}{N} \sum \boldsymbol{h}_k = \boldsymbol{\mu}^h$$

k-th row of J

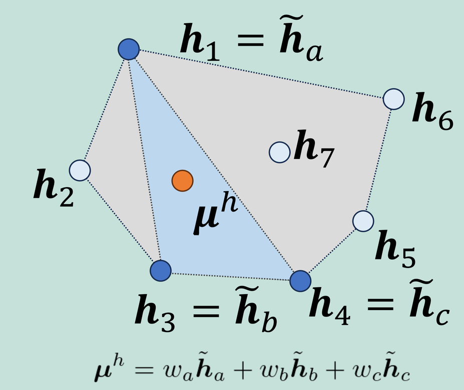
Because $\boldsymbol{\mu}^h$ is in the convex hull of $\boldsymbol{h}_k$, $\boldsymbol{H}$ can be represented by a subset $\tilde{\boldsymbol{J}}$ with at most $D^2 + 1$ rows in $\boldsymbol{J}$ **(Caratheodory set)**

$$\boldsymbol{H} = \tilde{\boldsymbol{J}}^T \tilde{\boldsymbol{W}} \tilde{\boldsymbol{J}}$$

Selected rows (at most $D^2 + 1$ rows) Weights

The classic Caratheodory set extraction algorithm with SVD on hyper-plane takes $O(N^2 D^2)$ time

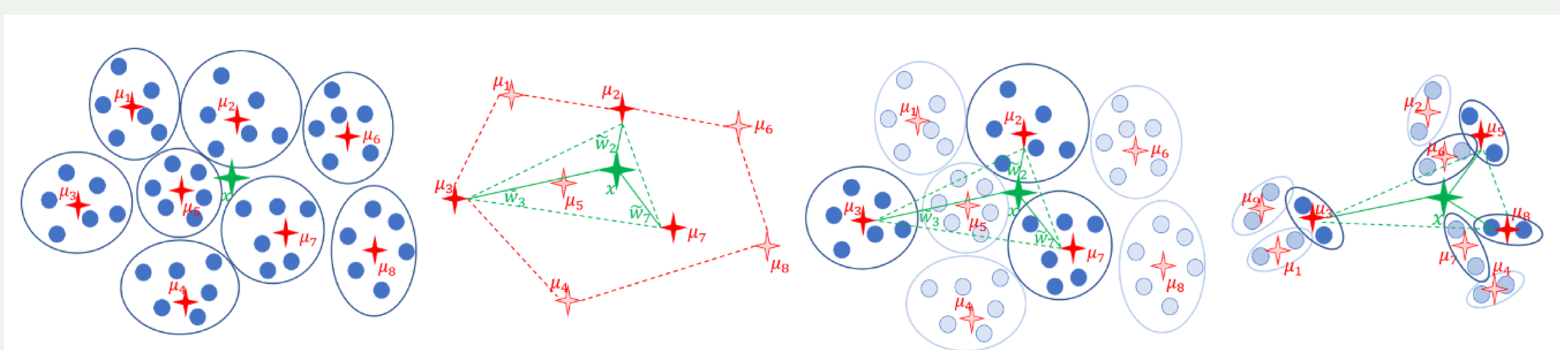
Caratheodory theorem



Every point in the convex hull of a set $S \in \mathbb{R}^N$ can be represented as a combination of $(N+1)$ points in S

We use an efficient Caraheodory set extraction algorithm [Maalouf, 2019], which takes $O(ND)$ time, and extend it to:

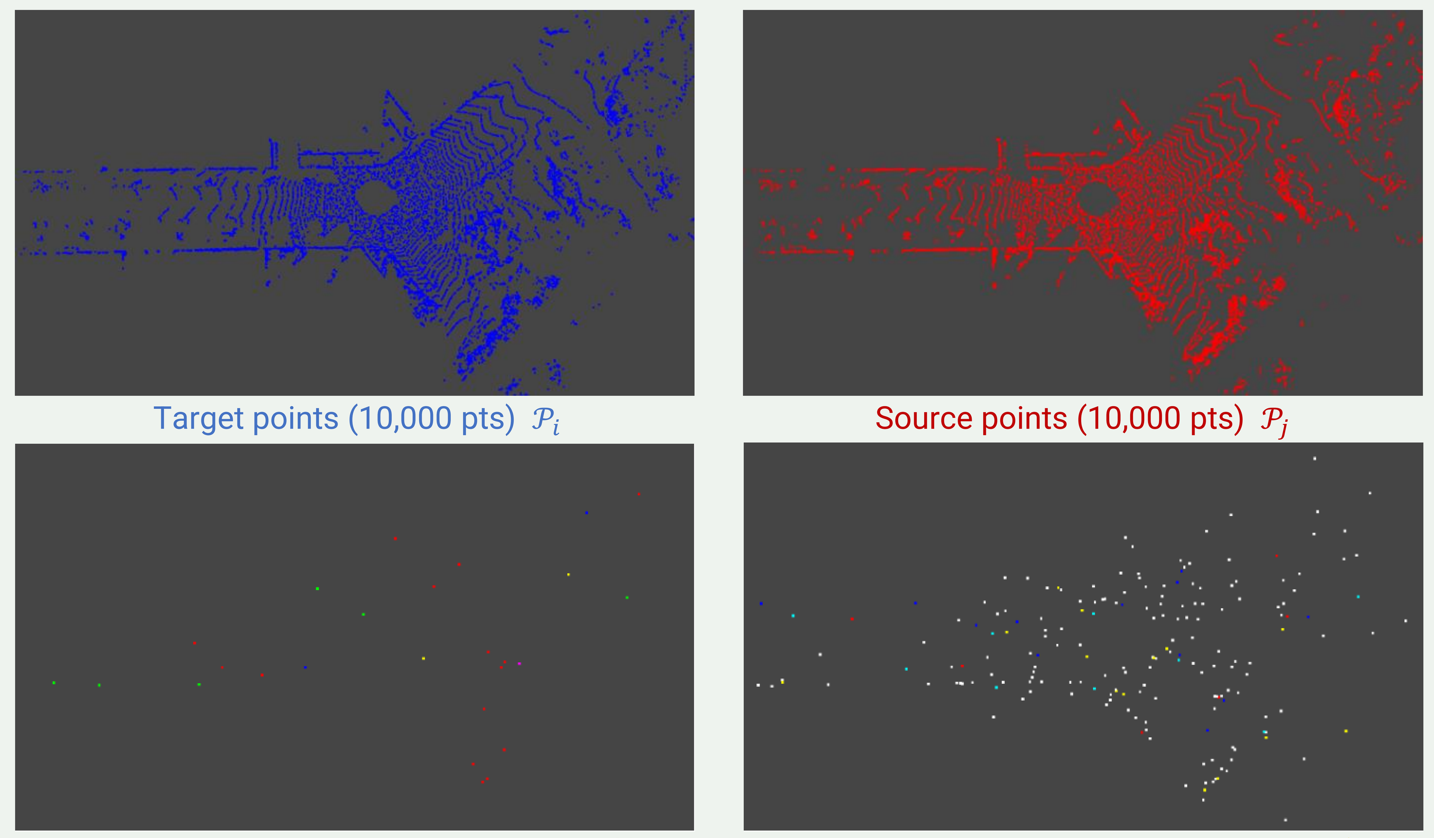
- reconstruct $\boldsymbol{b}$ and c in addition to $\boldsymbol{H}$ to recover the original quadratic function
- control accuracy-vs-speed by changing the number of data to be selected
- speed up the algorithm by eliminating non-upper-triangular elements of $\boldsymbol{H}$



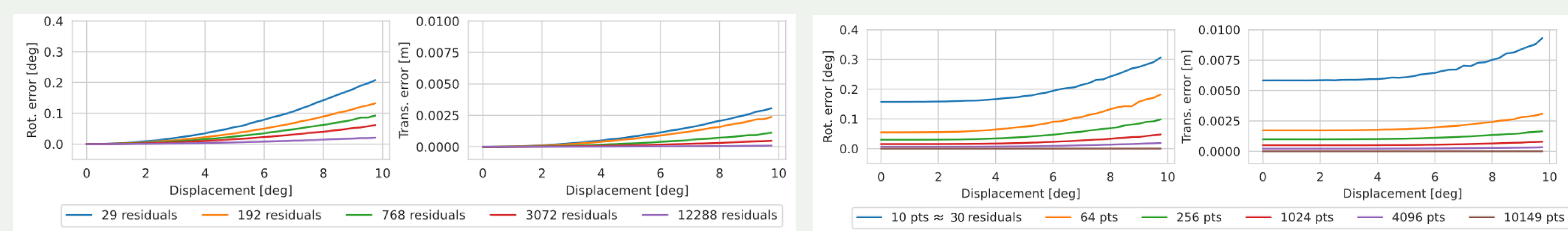
[Maalouf, NeurIPS 2019]

Experimental Results

Approximation Accuracy



$\mathcal{P}_j$, $\tilde{\mathcal{P}}_j^{29}$, $\tilde{\mathcal{P}}_j^{512}$ all yield the **same quadratic registration error function** for $\mathcal{P}_i$ at the evaluation point



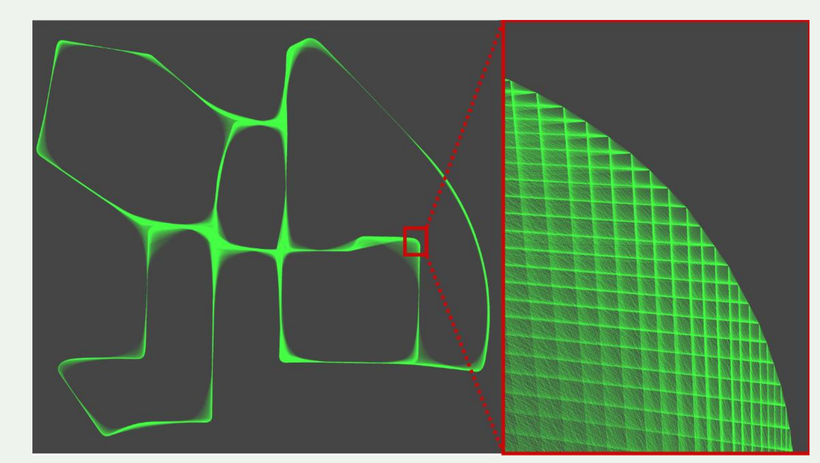
(b) Exact sampling (a) Random sampling

Better **non-linearity approximation accuracy under displacements**

Application to Global Trajectory Optimization

Evaluation on KITTI dataset

- 4,540 pose variables
- 585,417 registration error factors
- Re-linearize registration errors of 10,000 points for each factor at every optimization iteration



Optimization time and memory consumption

- No downsampling **13 hours / 22 GB**
- Exact downsampling **1.7 hours / 0.25 GB**

Method	Sequence 00	02	05	08
Pose graph optimization (Identity)	1.6257 ± 0.7529	23.9856 ± 8.2752	1.5149 ± 0.6760	9.3636 ± 3.2890
Pose graph optimization (Hessian)	1.3777 ± 0.6051	9.3406 ± 3.1490	1.6249 ± 0.8489	5.0532 ± 2.5991
Pose graph optimization (Hessian + Dense)	1.1846 ± 0.5625	9.3393 ± 3.1486	1.6240 ± 0.8488	5.0520 ± 2.5993
Registration error minimization + Exact sampling (M=29)	0.9553 ± 0.4650	8.9679 ± 3.0856	0.2917 ± 0.1060	4.4394 ± 2.5294

Large accuracy gain compared to pose graph optimization

Method	ATE [m]	Optimization time [h]
Exact sampling (M=29)	0.9553 ± 0.4650	1.67
Exact sampling (M=256)	0.9549 ± 0.4646	1.88
Exact sampling (M=1024)	0.9549 ± 0.4647	2.36
Original points (10,000 points)	0.9549 ± 0.4647	13.21

Small accuracy drop with downsampling

More Information

https://github.com/koide3/caratheodory2

https://staff.aist.go.jp/k.koide

https://twitter.com/k_koide3