Whole-body Motion Blending under Physical Constraints using Functional PCA

Soya Shimizu¹, Ko Ayusawa², Eiichi Yoshida² and Gentiane Venture¹,²

Abstract—This paper presents a method for motion synthesis using Functional Principal Component Analysis (Functional PCA) to generate complex humanoid robot motions in a low-dimensional space while considering physical consistency. Since each motion can be expressed by a point in a space called FPC space, this method allows blending different motions. For more complex motion synthesis, we introduce a novel framework to synthesize blended motions by configuring a local FPC space and a global FPC space. This method enables to merge data while considering data features. However, physical consistency was not ensured in our previous work, we here apply optimization under constraints after synthesis. We show the dynamic feasibility and the feature of the synthesized blended motions and also an interesting observation opening to the possibility to generate a variety of motions from a few motion data in a local space at low cost and time.

I. INTRODUCTION

Humanoid robots are expected to be used in many applications thanks to the progress in hardware and software. One example is the application of humanoids for performance evaluation of assistive devices for farmers, construction workers and caregivers. We can compare directly measured joint torques of the humanoid robot with and without such devices in order to assess their performance quantitatively which is impossible to do on the human [1]. This allows overcoming the drawbacks of experiments with humans such as lacking of quantitative measures, poor repeatability and heavy ethical procedures [2]. These particular applications need smooth reproduction of different human motions. For other applications like entertainment and human interaction, a variety of whole-body human-like motions should be generated.

Since humanoid robots have typically more than 30 degrees of freedom (DOF), it is difficult to manually generate stable coordinated whole-body motions. As a solution for this issue, GUI software are usually used. Choregraphe for the humanoid robot NAO is one of them [3], an other example is Choreonoid developed by Nakaoka et al. [4]. Such software have functions such as dynamic simulator, choreography planning, and low level control to ensure the robot balance with correction of the center of mass (COM) position automatically. However, such methods require a skilled animator and it still takes a long time to manually generate long motion sequences or complex whole-body movements. The inverse kinematics approaches are often applied to generate the whole-body motion to achieve tasks in a systematic way [5], [6], [7], [8]. However, it needs another framework to ensure the human-like property of the whole-body motion. An alternative method is retargeting, which is a conversion method from human motion acquired by motion capture to feasible motion for a humanoid robot [9], [10], [11], [12], [13]. Generated motions from retargeting are qualified as more "human-like" compared to manually coded motions by a GUI, though no quantitative metrics of "human-like" has yet been given. Although retargeting methods are practical, they require each single motion to be recorded on the human for a humanoid robot to perform different motions. For this reason, we need a new motion synthesis method to generate various motion more easily using fewer human recorded motions.

Morishima et al. [15] proposed a motion synthesis method using a procedure called smoothing with constraints and Functional PCA [16], [17]. After applying motion smoothing, which is an approximation with a parameterized trajectory, to the human measured motion, Functional PCA converts the smoothed motion into a point in a low-dimensional space called FPC space. A new motion can be generated by blending two (or potentially more) motions in the FPC space. They succeeded in synthesizing left and right lunge motions (one leg is positioned forward with its knee bending and the other leg backward) into a new motion that looks like a squat in simulation. However, this motion was generated from linear synthesized values in FPC space, so we didn’t utilize the effort of Functional PCA. Moreover, in the real humanoid experiment, the robot could not perform the blended motion properly because the synthesized motion did not satisfy physical consistency including stability and self-collision avoidance.

In this paper, we propose a framework allowing to blend motions with maximizing the characteristic of motion subject. The problem of how to consider the data feature by using a low-dimensional point in FPC space for synthesis is solved by establish a local FPC space and a global FPC space. Other problem of how to take physical consistency into account is solved by applying smoothing with constraints and Functional PCA again to already synthesized data. This method makes it possible to choose a subject point in a local FPC space and obtain a new point in the global FPC space. We can generate not only one point (motion) but a group of points (motions) corresponding to different physically consistent motions. If the point group are
distributed as a specific shape such as a curve or a surface and we formulate it, we will be able to combine original motion data at any mixture rate and generate an infinite variety of feasible motions.

II. MOTION SYNTHESIS FRAMEWORK

The motion synthesis flowchart taking into account physical consistency of the humanoid robot using smoothing with constraints and Functional PCA is illustrated in Fig.1. The process of conversion of human motions into humanoid robot motions is divided into three steps; 1) Retargeting: Acquiring robot motion data through smoothing with constraints from human motion data. 2) Synthesis: Analyzing and synthesizing robot motion data using Functional PCA. 3) Adding physical constraints: Adding conditions to the synthesized motion data using smoothing with constraints.

At step 1, we acquire whole-body humanoid robot motions based on human movements and carry out smoothing to reduce the amount of motion data. First, human motions are obtained by motion capture including upper and lower body motions. These data, expressed in 3D positions of markers in the global coordinate system, are converted into sequences of angles of each joint of the desired humanoid robot based on inverse kinematics according to [6], [8]. Converted data however may not satisfy some conditions for stable humanoid motions. We then apply an approximation called smoothing to those discrete data in order to obtain continuous data by considering physical conditions such as COM position. Here, smoothing uses basis functions called B-spline superposing continuous parameterized functions on the original discrete data [11]. The smoothed continuous data are compressed into a low-dimensional data by Functional PCA for the purpose of motion synthesis through blending at step 2. The compressed data can be plotted in the low-dimensional feature space as points. We compute those points in the local and global feature space separately. The local space corresponds to the motions from limited parts of the body (for example upper body only, right leg only...), whereas the global space is obtained from the whole-body motions. Assuming that motions in different local spaces are superposed, the low-dimensional data allow us to synthesize data possessing different local motion characteristics at the same time. After interpolating points between different local spaces in the global space, we can obtain synthesized motions by reconverting those points back into motion data, hence the use of Functional PCA. However, there is still no guarantee that those interpolated data will satisfy physical consistency. The 3rd step finally takes care of this issue. This operation shifts those interpolated points to physically feasible motions in the FPC space.

We finally confirm that any motions generated from the blended points are dynamically feasible through dynamic simulation of a humanoid robot. The smoothing and Functional PCA inspired by [15] and the improved method are briefly explained in the next section.

III. SYNTHESIS METHOD WITH PHYSICAL CONSISTENCY

This section briefly summarizes the representation of human motion data in a low-dimensional space by using smoothing and Functional PCA as we originally proposed in [15].

A. Smoothing with constraints for physical consistency

Smoothing allows estimating the time trajectory along the sampled time of a given regression model. Data after smoothing are expressed as a set of coefficients of basis functions for smoothing. Since the amount of the data can be reduced from the original dataset, the smoothed data can be handled more easily for the analysis and for further computations.

Smoothed joint angles of the robot are expressed by the following equation:

$$ q_t = \sum_{i=1}^{N_B} c_i b_{i,t} $$

where, $q_t \in \mathbb{R}^{N_J}$ is the vector of joint angle at time instance $t$, $N_J$ is the number of joints. $b_{i,t} \in \mathbb{R}$ is the cubic B-spline function, $N_B$ is the number of spline functions in the basis,
and \( c_i \in \mathbb{R}^{NJ} \) is expressed as a coefficient matrix. This data described by smoothing function is called “functional data”. We can regard smoothing optimization with constraints as in the following equation.

\[
\min_{c_1, \ldots, c_N} \sum_{t=1}^{N_T} |q_t - \tilde{q}_t|^2 \\
\text{subject to } \forall t \ g_t(r_t, q_t) \leq 0
\]

where, \( \tilde{q}_t \) is the acquired joint angle data at time sample \( t \), \( r_t \in SE(3) \) is the position and the orientation of the humanoid robot, and \( N_T \) is set as number of time instances. The function \( g_t \) represents the constraints for obtaining physical consistency required for feasible robot motions, such as the joint limits, the static stability condition for COM, and a geometrical conditions for feasible motion at the time sample \( t \). This optimization problem is solved by a penalty function method [8].

**B. Motion data compression**

Although smoothing reduces the amount of motion data, the resultant coefficient matrix still has over 1000 elements. We thus employ Functional PCA for further data compression, which was proposed by Basse et al. [16] as a statistical method to abstract important information and reduce the dimension of "functional data sets" by analyzing the covariance structure. It works in the following way:

First, the following formula is defined:

\[
\tilde{\mathbf{\xi}}_t = \sum_{i=1}^{N_B} \theta_i h_{i,t}
\]

where, \( \mathbf{\theta}_i \in \mathbb{R}^{NB} \) refers to the vector of parameters of the regression model Eq.(4). \( \mathbf{q}_t^{(n)} \) is the joint angle vector at time sample \( t \) of \( n \)-th data set and can be expressed using Eq.(4). The corresponding scalar value \( f^{(n)} \) called FPC score is defined as follow:

\[
f^{(n)} = \sum_{t=1}^{N_T} \tilde{\mathbf{\xi}}_t^T \mathbf{q}_t^{(n)}
\]

The analysis method of Functional PCA calculates the basis \( \tilde{\mathbf{\xi}} \) to maximize the variance of the FPC scores. By connecting \( \tilde{\mathbf{\xi}}_t^T \), the basis vector \( \mathbf{\xi} \) is defined as follow:

\[
\mathbf{\xi} = [\mathbf{\xi}_1^T \ldots \mathbf{\xi}_{N_T}^T]^T
\]

The solution can be computed by solving an eigenvalue problem [18]. This FPC score has \( N_B \times N_T \) dimensions. We choose \( N_B \) values of it, and plot them into \( N_M \) dimensional feature space called FPC space. After performing the functional PCA, the relationship between the arbitrary FPC scores \( f \) in \( N_M \) dimensional FPC space and the coefficients \( c \) of the B-spline bases can be summarized as follows:

\[
f = \mathbf{M}(c - \mathbf{e})
\]

where, \( \mathbf{M} \in \mathbb{R}^{N_M \times N_B NJ} \) is the conversion matrix which can be computed from \( \tilde{\mathbf{\xi}} \) and \( h_{i,t} \), and \( \mathbf{e} \) indicates the averaged coefficients determined by the training data.

The coefficients \( c \) can be reconstructed from the given PCA scores \( f \) by solving Eq.(7). The coefficients \( c \) can be finally obtained from the given score \( f \) by:

\[
c = \mathbf{M}^+ f + \mathbf{e}
\]

where, \( (\cdot)^+ \) indicates the pseudo inverse of \( (\cdot) \).

**C. Synthesis with Functional PCA**

We believe that the low-dimensional representation in FPC space is advantageous to visualize and blend motions with different characteristics. We here introduce the notion of local and global FPC space as shown in Fig.2. The local FPC space expresses motions of some given parts of the body that can be superposed without significant interference. On the other hand, the global FPC space represents the motion of the whole body for which blending can significantly affect the physical feasibility of the robot.

Interpolations in the same local space results in blended motions. For example, by combining left and right lunge motions, we obtained new squat motions in the local FPC space corresponding to the lower body [15]. However, when it comes to blending motions in different local spaces, the global space should be considered. Fig.2b shows the procedure of motion synthesis from different local spaces. We here introduce the "intersection point" that plays the intermediate role of connecting two different spaces. It is possible to synthesize motions linearly in the global space, and generate the synthesized point shown in Fig.2b. This
point represents whole body motion as mentioned above, so the blended data does not consider the specificities of basis motions for synthesis and the physical feasibility of the robot such as self collision. Proposed method by using local FPC can synthesize without those interference. We will compare the details of the provided method and linear synthesis later in IV-B

We now explain the mathematical interpretation of the motion synthesis using local and global spaces. First, FPC scores in local and global spaces are defined according to Eq.(7) and Eq.(8) as follow:

\[
\begin{align*}
M_G(c - \bar{c}_G) &= f_G \\
\{c = M_{L1}f_{L1} + \bar{c}_{L1} \\
\{c = M_{L2}f_{L2} + \bar{c}_{L2}
\end{align*}
\]

Where, the subscript \(G\) is utilized for global space values: the subscript \(L_1\) is utilized for the 1st motion data group, and \(L_2\) is utilized for the 2nd motion data group.

Let us formulate the relationship between the PCA scores represented in the three spaces: the two local spaces and the global space. The local values such as \(c_{L1}\), \(c_{L2}\), \(M_{L1}\), and \(M_{L2}\) are converted into global ones by substituting Eq.(9) to Eq.(10) through the following formula:

\[
\begin{align*}
\{f_G = S_{G/L1}f_{L1} + \bar{f}_{G/L1} \\
\{f_G = S_{G/L2}f_{L2} + \bar{f}_{G/L2}
\end{align*}
\]

where,

\[
\begin{align*}
S_{G/L1} &\triangleq M_GM_{L1} \\
S_{G/L2} &\triangleq M_GM_{L2} \\
\bar{f}_{G/L1} &\triangleq M_G(\bar{c}_{L1} - \bar{c}_G) \\
\bar{f}_{G/L2} &\triangleq M_G(\bar{c}_{L2} - \bar{c}_G)
\end{align*}
\]

Using these values enables to convert each local FPC scores into the corresponding global FPC score: \(f_G^(*)\).

Since \(f_G\) in Eq.(11) implies the intersection point of both local FPC spaces, the solution of the simultaneous equations of Eq.(11) can provide the synthesized data which considers both upper and lower body motion characteristics. Furthermore, the coefficient values \(c\) corresponding to \(f_G\) can be reconstructed from:

\[
c = M_G^*f_G + \bar{c}_G
\]

Finally, the joint angle trajectories \(q\) can be computed from the obtained coefficients according to Eq.(1).

This time, we showed the synthesis method using only two local spaces. Adding more local spaces of motion data, we can perform a broader variety of synthesized motions.

**D. Physically Consistent Motion Synthesis**

Although the motion synthesis method presented in III-C allows generating synthesized motions using Eq.(1) and Eq.(14), its feasibility is not necessarily guaranteed with respect to physical consistency. For example for lower body motions, mixed motions from lunge motions with different foot positions [15] were not feasible, which leads to the robot easily falling down. We solve this issue by applying again the smoothing under the constraints to the synthesized motions. The physically consistent solution is obtained by solving the following optimization problem:

\[
\begin{align*}
\min_{f(1)\cdots f(N_R)} \sum_{j=1}^{N_r} \sum_{l=1}^{N_t} |q_l - \hat{q}_l|^2 \\
+ \sum_{r=1}^{N_r} \sum_{j=1}^{N_c} \lambda_j \max(g_{j,r}, 0)
\end{align*}
\]

where, \(\lambda_j\) represents the penalty weight of each constraint \(g_{j,r}\), and \(N_r\) is the number of constraints. The weight is designed according to the allowable amount of the constraint violation.

We will demonstrate this analysis with examples of whole body motions synthesis of a humanoid robot in the next section.
IV. VALIDATION OF PHYSICALLY CONSISTENT SYNTHESIZED MOTIONS

A. Synthesize upper and lower body motions in FPC space

In this paper, we adopt two local spaces corresponding to the upper body and the lower body as a typical case. Other examples of the local spaces could be left and right, or right leg and left leg... These cases of the synthesis of robot motions are studied in this section. We recorded the upper and lower body motions of human subjects with a motion capture system (Raptor-12 cameras provided by Motion Analysis). The captured data was converted into the joint angles data of the humanoid robot according to inverse kinematics computation[6]. Then, the converted data was utilized for FPCA.

Through the capture process, we acquired 16 motion data in total. All the captured motions were converted to the motion of humanoid robot HRP-4 [19] by using the retargeting method shown in [14]. Fig.3 shows all the captured motions in a 3D dynamic simulator called Choreonoid [4]. There are eight upper body motion data in total. Four kinds of upper body motions were acquired twice respectively. Each motion moves the COM significantly and may result in an unstable robot motion (Fig.3a, Fig.3b, Fig.3c, Fig.3d). Other eight data contains lower body motions. Left and right lunges were captured four times respectively for a lower motion, by changing the longitudinal and lateral distance between the feet (Fig.3e, Fig.3f). In the dynamics simulation using Choreonoid, all 16 motions of the robot could be performed without falling down after the retargeting step. We applied Functional PCA to those motion data and converted them into low dimensional FPC space for synthesis. The two local spaces were generated from the upper body motions and the lower ones, respectively, while the global space were generated using both of them.

Top of Fig.4 shows the point groups of the corresponding dataset in each low-dimensional FPC space. Applying FPCA to each dataset of lower body motions and upper body motions generates local spaces as indicated in Fig.4a and Fig.4b respectively. Point groups of each motion type in those local spaces have unique forms. In the local space of lower body motion in Fig.4a, 1st PC score represents length of both feet in a linear direction, and 2nd PC score means those width. We could not confirm remarkable characteristics of 3rd PC score and every PC score of upper body. It is expected that the point around the local origin is synthesized data of other points, so we can blend local body parts such as legs, hands, arms, and so on. Fig.4c shows all the 16 motion data represented in the global FPC space. These 6 point groups obtained from several iterations of smoothing with constraints as presented in III-D. We can see that the point groups are not distributed regularly and there are some large gaps. As these gaps are located at nearly the same position, we can conclude that there is a singularity or a solution hard to converge towards in the optimization problem.

B. Validation of Synthesized Motions

In accordance with III-C, we synthesized upper body motions and lower body motions by solving Eq.(11). In this...
research, one arm wave motion (Fig.3c) and a squat motion were chosen as blended motions. Squat motion is the origin of the local space of lower body motion. However, generated data could not satisfy physical consistency, so those data were subject to smoothing with constraints again as presented in the paper.

Fig.5 shows the result of the synthesized motion in the simulation environment Choreonoid. Fig.5a is the result of the provided method in III-C, and Fig.5b is the result of linear synthesis shown in [15]. In this case, linear synthesis means that we calculated the average of two FPC scores in the global FPC space; squat motion and arm wave motion. Although both simulation model could perform the synthesized movement without losing stability, comparing with the result of the proposed method shown, the arms of linear synthesis result are not straighten. Fig.6 shows the third pictures from the left in the first row of Fig.5a and Fig.5b. The proposed method shown in Fig.6a bends robot’s knee because of squat motion. On the other hand, both legs of linear synthesis in Fig.6b are not bended well. In other word, the linear
synthesis method shown in Fig. 5b does not duplicate the synthesis subject motions because of some interference, so it is assumed that our proposed method using local FPC is more efficient for motion synthesis than the linear synthesis method.

We carry out the experiment using the real humanoid robot. We use the humanoid robot HRP-4 [19]. The robot has 37 DOFs, and its toe has 1 DOF for human-like movements. In our research, the joint angle of its toe is fixed at zero degree for the stability. We generated the synthesized motion by the proposed method, and examined the trajectory by using the real robot. The snapshots of the synthesized motion performed by the actual robot are shown in Fig. 5c. As can be seen from the figure, the robot could carry out the synthesized motion successfully, i.e. without falling down. Those results show that the proposed framework can provide the physical consistent motions and is useful for the motion synthesis for actual robots.

V. CONCLUSION

In this paper, we presented a method for generating physically consistent motions by blending of motion data using motion smoothing and Functional PCA. The contribution is that we have shown an improved synthesis method of feasible motions by setting the local FPC space and the global FPC space, improving the drawback of the previously proposed method. The feasibility of the synthesized motions have been validated by actual experiments as well as dynamic simulations of a humanoid robot HRP-4. Other robots can be used similarly by simply replacing in the retargeting part, the robot model.

Future work includes extension of the proposed method to a wide variety of humanoid whole-body motions. Since the example presented in this paper deals with upper and lower body motions, and this method can be applied to more complicated motion synthesis by increasing the local space, we will investigate if the same kind of spaces can be identified for other complex cases including combined different time instance motions. Study on the mathematical analysis of these topological spaces is another future topic.

REFERENCES