Identification of Base Parameters for Large-scale Kinematic Chains Based on Physical Consistency Approximation by Polyhedral Convex Cones

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Abstract In this paper, we propose the identification method to realize physical consistency and computational stability. As inertial parameters of each link are represented with a finite number of mass points, the physical conditions can be approximated by linear inequalities. The evaluation function is also designed to control the exactness of identification results and the stability of computation.

1 Introduction

For the study of physical ability performed by humanoid robots and humans, it is required to understand the dynamics of their multibody systems. In the field of robotics, the identification technique using the feature found in the equations of motion is developed (Mayeda et al. (1984); Atkeson et al. (1986)), which identifies the unknown inertial parameters, such as mass, center of mass, and inertia of each link. The classical identification problem can be solved with a least squares method. Nevertheless, the obtained result is not necessarily physically consistent (Yoshida and Khalil (2000)). Mata et al. (2005) proposed the method considering physical consistency, and also tested it on the 6-axis industrial manipulator. This method solves the quadratic programming with the non-linear inequality constraints which come from the condition of positive definiteness of the parameters.

For application to legged systems, there exist the following problems when using the method developed for manipulators. First above all, the number of degrees of freedom (DOF) of the system is large, which requires

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large-scale non-linear optimization. And it is difficult to generate the exciting motion because of the kinematics and dynamics complexity of the systems, resulting ill condition problem of optimization. Thus, converged solution is difficult to be obtained by the usual method mentioned above.

In this paper, we propose the identification method to realize physical consistency and computational stability for large-scale systems like legged systems. Inertial parameters of each link are represented with a finite number of mass points. Then the positive definiteness of the parameters can be replaced with linear inequalities, which always satisfy the original conditions, and the performance of approximation can be enhanced as the number of mass points increase. We also design the evaluation function to control the exactness of identification results and the computational stability.

2 Identification methods developed for manipulators

The equations of motion of multibody systems can be written in a linear form with respect to the dynamic parameters. The equations of the robot, composed of $N$ rigid bodies and that has $N_J$ DOF, is given by Eq.(1).

$$ Y \phi = \tau + \sum_{k=1}^{N_c} J_k^T f^\text{ext} k $$

Where, $\phi \in R^{10N}$ is the vector of constant inertial parameters, $Y \in R^{N_J \times 10N}$ is the regressor matrix or regressor, $\tau \in R^{N_J}$ is the vector of joint torques, $N_c$ is the number of contact points with the environment, $f^\text{ext} k \in R^6$ is the vector of external forces exerted to the system at contact $k$, $J_k \in R^{N_J \times 6}$ is the basic Jacobian matrix of the position at contact $k$ and of the orientation of the contact link with respect to generalized coordinates.

Only the minimal set of inertial parameters that describes the dynamics of the system can be identified. This minimal set is called base parameters. It is computed symbolically or numerically from the inertial parameters $\phi$ by eliminating those that have no influence on the model and regrouping some according to the kinematics of the system(Khalil and Bennis (1995)). The minimal identification model given by Eq.(2) is thus obtained.

$$ Y_B \phi_B = f $$

Where, $N_B$ is the number of the base parameters, $Y_B \in R^{N_J \times N_B}$ is called the regressor matrix for the base parameters, and $f \in R^{N_J}$ is equal to the right-hand side of Eq.(1). $\phi_B \in R^{N_B}$ is the vector of the base parameters, and it is a liner combination $\phi_B = Z \phi$, using the composition matrix $Z \in R^{N_B \times 10n}$ which can be computed from the structure of a robot.
For the identification process, we have to compute $Y_B$ and $f$ at every sampling time, measuring generalized coordinates, joint torques, and external forces acted on a robot. Then we arrange $T$ sampled regressors and forces of Eq.(2) at $t = t_1, t_2 \cdots t_T$ lengthwise, and compose the large regressor matrix $Y_{all}$ and the large vector of forces $F_{all}$ as bellow.

$$Y_B \phi_B (t_1^T \cdots Y_{B,t_T}^T)^T \phi_B) = f_{all} (f_{t_1}^T \cdots f_{t_T}^T)^T$$

(3)

After sampling, the parameter $\phi_B$ in Eq.(3) is solved by the least squares method (LSM). Nevertheless, the obtained result is not necessarily physically consistent because of measurement noise and modeling error. Physical consistency means that the inertial matrix of each link $j (1 \leq j \leq N)$ must be positive definite(Yoshida and Khalil (2000)).

$$m_j > 0, \quad I_{Cj} > 0$$

(4)

Where, $I_{Cj} = I_j - m_j (s_j \times [s_j \times]^T)$ is the inertia matrix around center of mass $s_j$ expressed in the frame attached to link $j$. Mata et al. (2005) proposed a identification method to solve the quadratic programming (QP) with the nonlinear inequality constraints Eq.(4), and also tested it on the 6-axis industrial manipulator. The solution satisfies Eq.(4) and minimizes the following evaluation function.

$$f(\phi) = \lambda_\tau |Y_{all}\phi - f_{all}|^2 + \lambda_\tau |Z\phi - \hat{\phi}_B|^2$$

(5)

Where, $Y_{all} \triangleq Y_{Ball} Z$, and $\hat{\phi}_B \triangleq Y_{Ball}^# F_{all}$ is the solution of LSM.

3 Identification for legged systems considering physical consistency and computational stability

QP with the evaluation function Eq.(5) and the nonlinear inequality constraints Eq.(4) can be also applied for legged systems. However, the following problems exist, which make it difficult to obtain the converged solution.

- First above all, the number of DOF of legged systems is generally large, which require large-scale non-linear optimizations.
- As there exist no links fixed to the environment, it is difficult to separate motion planning for identification from control stability. This dynamics constraint leads to decrease the performance of persistent exciting trajectories(Gautier and Khalil (1992)).
- The characteristics of every joint such as joint velocity and range of motion are significantly different, which causes gaps of identification performance among parameters.
In this paper, we approximate both Eq.(4) and Eq.(5) to improve the computational stability under those problems.

First, we place a finite number of mass points in the convex hull of the link object as shown in Figure 1, in order to replace Eq.(4) with linear inequalities.

$$S_{\phi_j}(\frac{1}{2} R^{10})$$ is the set of inertial parameters $$\phi_j$$ of the link $$j$$ satisfying Eq.(4). $$S_{\phi_j}$$ is clearly a convex set, and $$\forall a > 0, (a \phi_j) \in S_{\phi_j}$$ is verified, thus $$S_{\phi_j}$$ is an open set within a convex cone. And we define the function $$\Phi(m,s,I) \in R^{10}$$ to compose the vector of inertial parameters.

$$\Phi(m,s,I) \triangleq \begin{bmatrix} m ms_x ms_y ms_z I_{xx} I_{yy} I_{zz} I_{xy} I_{xz} I_{yz} \end{bmatrix}^T \quad (6)$$

Here, we approximate the parameters $$\phi_j \triangleq \Phi(m_j,s_j,I_j)$$ of the link $$j$$ using $$N_{\rho,j}$$ mass points as follow.

$$\phi_j = P_j \rho_j \quad (\triangleq \sum_{k=1}^{N_{\rho,j}} \Phi(1, j r_k, [j r_k \times][j r_k \times]^T) \rho_{j,k}) \quad (7)$$

Where, $$\rho_{j,k}$$ is the mass of the point $$k(1 \leq k \leq N_{\rho,j}), \rho_j \triangleq [\rho_{j,1} \cdots \rho_{j,N_{\rho,j}}]^T \in R^{N_{\rho,j}}, j r_k \in R^3$$ is the position with respect to the origin of the frame attached to link $$j$$, and $$P_j \in R^{10 \times N_{\rho,j}}$$ is the matrix to compose $$\phi_j$$ of $$\rho_j$$. Hence, $$\phi$$ is represented using all $$N_\rho(= \sum_j N_{\rho,j})$$ mass points as follow.

$$\phi = P \rho \quad (\triangleq \text{diag}(P_1, \cdots, P_N) [\rho_{j,1} \cdots \rho_{j,N}]^T) \quad (8)$$

Where, $$P \in R^{10N \times N_\rho}$$ is the matrix with $$P_j$$ on the $$j$$-th diagonal sub-matrix. If $$P_j$$ is a full row rank matrix, then $$P$$ is also full row rank, and
there exists $\rho$ to realize any $\phi$. Thus, mass points have to be located to make each $P_j$ full row rank. Then, Eq.(4) can be approximated as $\rho > 0$.

As we mentioned, Eq.(4) means the open set within a convex cone as Figure 1. On the other hand, the inertial matrix of the parameters $\rho_k \Phi_k$ consist only of the mass point $k$ is semi-positive definite, thus $\rho_k > 0$ means a ridge line of the convex cone as Figure 1. It means that $\rho > 0$ approximates the convex cone by the polyhedral convex cone. Thus, if $\rho > 0$ is verified, then Eq.(4) is verified, and Eq.(4) will be well approximated if the number of mass points increases. And the centers of mass satisfying $\rho > 0$ always exist in the convex hull of the mass points, i.e. the link object. The optimization strategy to approximate nonlinear inequality constraints represented as a convex cone by a polyhedral convex cone is often adopted in other fields of robotics, for example in grasp analysis (Kerr and Roth (1986)).

Next, we deal with the following evaluation function instead of Eq.(4).

$$g(\rho) = (Y_{all}P\rho - f_{all})^T W_f (Y_{all}P\rho - f_{all}) + \rho^T W_\rho \rho \quad (\rho > 0) \quad (9)$$

Where, $W_f \in R^{N_j \times N_j}$ and $W_\rho \in R^{N_\rho \times N_\rho}$ are the weight matrices, and (semi-)positive definite. The solutions from LSM satisfy the first term of Eq.(9). However, the Hessian matrix of the usual evaluation function is always semi-positive definite as following reasons. Standard inertial parameters $\phi$ (and of course $\rho$) cannot be structurally identified as mentioned in the section 2. Moreover, the poor motion trajectories cause the ill condition that the rank of $Y_{Ball}$ is nearly equal to zero and identification performance of some parameters declines significantly. Thus, the second term of Eq.(9) is added to evaluate. This term makes the Hessian matrix of the evaluation function a positive definite matrix, and prevents from these ill conditions.

The evaluation function Eq.(5) is the generalized notation which includes both the evaluation of LSM and QP using Eq.(5). If $W_f = \lambda_f E_{N_j} + \lambda_\phi Y_{Ball}^# Y_{Ball}^#$ and $W_\rho = O$, then Eq.(9) is equal to Eq.(5). Thus we can control both the exactness of the solution obtained from LSM and the stability of computation, choosing the weight matrices $W_f$ and $W_\rho$. This problem establishment uses an analogy from inverse kinematics solution with singularity robustness shown by Nakamura and Hanafusa (1986).

### 4 Experimental results

In this section, we show identification results of the humanoid robot shown in Figure 2 using the proposed method. This robot has 38 DOF as Table 1, however, the fingers were not used and maintained in a constant position during experiments, resulting in the use of 32 DOF. We use the identification model for legged systems(Ayusawa et al. (2009)). Its feature is to make use
Table 1. The joint specification and the mounted sensors of the humanoid.

<table>
<thead>
<tr>
<th>Number of joint</th>
<th>Sensor Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>waist</td>
<td>gyro/acceleration sensor</td>
</tr>
<tr>
<td>neck</td>
<td>(in the upper body link)</td>
</tr>
<tr>
<td>(each leg)</td>
<td>6-axis force sensors</td>
</tr>
<tr>
<td>(each arm)</td>
<td>(in both feet)</td>
</tr>
<tr>
<td>(each hand)</td>
<td>encoders (in each joint)</td>
</tr>
</tbody>
</table>

Table 2. Standard inertial parameters of 6 links estimated from the proposed method. (L1: upper trunk, L2: lower trunk, L3: left upper leg, L4: left foot, L5: right upper arm, L6: right hand)

<table>
<thead>
<tr>
<th>Link</th>
<th>( m ) [kg]</th>
<th>( s_{BB}^{x} )</th>
<th>( s_{BB}^{y} )</th>
<th>( s_{BB}^{z} )</th>
<th>( I_{cxx} )</th>
<th>( I_{cyy} )</th>
<th>( I_{czz} )</th>
<th>( I_{cxy} )</th>
<th>( I_{cyz} )</th>
<th>( I_{czx} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>5.05</td>
<td>0.09</td>
<td>0.04</td>
<td>0.70</td>
<td>0.26</td>
<td>0.19</td>
<td>0.16</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L2</td>
<td>4.06</td>
<td>0.03</td>
<td>0.56</td>
<td>-0.03</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L3</td>
<td>2.53</td>
<td>0.12</td>
<td>0.13</td>
<td>-0.30</td>
<td>0.08</td>
<td>0.06</td>
<td>0.06</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>L4</td>
<td>1.07</td>
<td>0.05</td>
<td>-0.12</td>
<td>-0.48</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L5</td>
<td>1.59</td>
<td>0.15</td>
<td>0.34</td>
<td>0.32</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L6</td>
<td>1.07</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.40</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

of only the six equations of motion of the base-link, and thus making the joint-torque measurement unnecessary. And we generated several types of walking motions and upper body motions to be used for identification.

The bounding box (BB) of each link object is treated as the convex hull, and we put 27 (= 3³) equally-spaced points in BB. The matrix \( W_f \) is equivalent to the weight matrix of LSM. Thus, we used the weight matrix which minimizes variances of the estimated errors of input forces. And we choose \( W_\rho = 0.001E_{N_{e,i}} \) to prevent from the ill condition of the optimization.

The standard inertial parameters estimated from the proposed method are given in Table 2. Table 2 shows the mass \( m \) [kg], the center of mass \( s_{BB} \) from the center of BB which is normalized by the size of BB, and the inertias \( I_{cij} \) [kg-m²] (around the center of mass) of some links. All the results satisfy the physical conditions of Eq.(4), and the centers of mass also exist in BB because all the elements of \( s_{BB} \) are within ±1. On the other hand, we could not obtain the converged solution from the QP with Eq.(4) and Eq.(5), and the results obtained from standard LSM using the same weight matrix \( W_f \) are not physically consistent. Hence, the proposed
method shows both physical consistency and computational stability. It should be paid attention that the obtained values of Table 2 themselves are meaningless, because only base parameters can be identified from the dynamics model. Thus, they have meanings when the base parameters are composed of them, and the base parameters show physical consistency if the standard parameters of Table 2 are physically consistency.

Figure 3 shows the comparison of the total external forces acted on the base-link in the walking motion that is not used during the identification procedure. The black thin lines show the external forces $F$ measured by force sensors, the gray thick lines mean the estimated ones $\hat{F}_{LS}$ from LSM, and the black dashed thin lines are the estimated ones $\hat{F}$ of the proposed method. As seen from the figures, the proposed method slightly reduces the accuracy with respect to LSM.

5 Conclusion

In this paper, the identification method for large-scale systems like legged system considering physical consistency and computational stability has
been proposed. Inertial parameters of each link are represented with a finite number of mass points, which can replace the physical conditions with linear inequalities. The performance of approximation can be enhanced as mass points increase, and the physical conditions concerning centers of mass can be also considered. The bounds of centers of mass can give physical consistency of standard inertial parameters, even though only base parameters are obtained in usual identification. We also design the evaluation function to control the exactness of identification results and the stability of computation. The method has been tested on a humanoid robot. The optimization has been performed successfully, and all the results show the physical consistency. The results are compared to ones obtained from LSM, and the proposed method slightly reduce the accuracy with respect to LSM.

Bibliography


