NEW METHODS OF MICROTREMOR EXPLORATION: THE CENTERLESS CIRCULAR ARRAY METHOD AND THE TWO-RADIUS METHOD

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ABSTRACT - We present two new methods of microtremor exploration using circular arrays of seismic sensors, namely the Centerless Circular Array (CCA) method and the Two-Radius (TR) method, both being concrete realizations of a general theory of microtremor exploration methods which we have recently formulated. The CCA method, which uses vertical-motion records, is capable of giving accurate estimates of the phase velocities of Rayleigh waves up to very long wavelength ranges relative to the array radius, while the TR method, which uses horizontal-motion records, allows to infer phase velocities of Love waves by cancelling out the effects of Rayleigh-wave components with a unique mathematical procedure. Analysis of field data demonstrates that the CCA method performs effectively over a broad range of length scales, ranging from a few meters to several kilometers in terms of the radius of the seismic array.

1. Introduction

One method of microtremor exploration, which is practiced widely in the research community, is the spatial autocorrelation (SPAC) method (Aki, 1957), which allows to estimate phase velocities of Rayleigh and Love waves by processing microtremor records from a circular array of seismic sensors deployed on the ground surface. Recently, we have cast, in the wake of Henstridge (1979), a new light on the theoretical rationale of the SPAC method, and have developed on its basis a new, comprehensive theoretical framework for microtremor exploration methods using circular arrays (Cho et al., 2006a), which is so general in nature that it encompasses both Aki's (1957) and Henstridge's (1979) formulations as special cases.

The Centerless Circular Array (CCA) method (Cho et al., 2004, 2006b) is one concrete realization of our general theory that is distinct from the classical SPAC method. The CCA method uses vertical-motion records of microtremors, obtained at different (typically three or five equidistant) locations around a single circle (Figure 1a), to evaluate phase velocities of Rayleigh waves; unlike the classical SPAC method, it does not require records at the center of the circle. Field tests have demonstrated (Cho et al., 2006b) that the CCA method has the potential to resolve waves of much longer wavelengths than the popularly used frequency-wavenumber (f-k) spectral method (Capon, 1969) and the traditional SPAC method; even with just three sensors around the circle, accurate estimates of the phase velocities are available, under optimal conditions, up to wavelengths as long as several ten times the array radius.
The Two-Radius (TR) circular array method (Tada et al., 2006) is another new method of microtremor exploration that is derived from our general theory. The uniqueness of the TR method, which uses horizontal-motion records obtained around two circles of different radii (Figure 2a; again, no center station is needed), lies in cancelling out the effects of Rayleigh-wave components by following a unique mathematical procedure, and yielding estimates of the phase velocities of Love waves in a straightforward way, even when those of Rayleigh waves remain unknown.

In the present paper we first outline the basic algorithms of the CCA and TR methods, and later describe a case of field implementation of the CCA method. The applicability of the CCA method over a broad range of length scales is illustrated by analysis of field data, obtained with circular arrays of different radii ranging from a few to several thousand meters.

2. Method

The present section outlines the algorithms of the CCA method (Cho et al., 2006b) and the TR method (Tada et al., 2006). For the sake of brevity, only the gist of the theory is presented, and the reader is referred to the general theory of Cho et al. (2006a) for more details. The basic assumption is that the field of microtremors is modeled as an ensemble of Rayleigh and Love waves, propagating as plane waves and arriving from different directions with different intensities, and that the Rayleigh and Love wave components, mutually uncorrelated, can each be regarded as a random field which is stationary in both time and space. We define "noise" as components of microtremors that fail to befit the above description of the microtremor "signals," such as non-propagating waves and instrument-specific electronic noise. When we deal with cases where noise is present, we assume for simplicity that noise is also stationary, that noise is uncorrelated with the signals, and that noise components appearing in different sensor records are both mutually incoherent and identical in intensity. (Note that the term "noise" as we use it here refers only to the non-propagating components contained in the field of microtremors, and should not be confused with the entire body of the field of microtremors which is sometimes also called "seismic noise.")

2.1. The CCA method

Suppose we deploy a circular seismic array of radius \( r \) in a field of microtremors, whose vertical component we denote by \( Z(t,r,\theta) \). If we synthesize the two complex waveforms

\[
Z_0(t,r) = \int_{-\pi}^{\pi} Z(t,r,\theta) d\theta \\
Z_1(t,r) = \int_{-\pi}^{\pi} Z(t,r,\theta) \exp(i\theta) d\theta
\]

by integrating the seismograms with respect to azimuth, their power-spectral densities, which we denote by \( G_{Z0Z0}(r,r,\omega) \) and \( G_{Z1Z1}(r,r,\omega) \) respectively (\( \omega \) denotes the angular frequency), can each be represented in the following way:

\[
G_{Z0Z0}(r,r,\omega) = 4\pi^2 \sum_{i=1}^{M} f_i^{RV}(\omega) J_0^2(\kappa_i^R(\omega))
\]
\[ G_{Z1Z1}(r, r, \omega) = 4\pi^2 \sum_{i=1}^{M^R} f_i^{RV}(\omega) J^2_i(rk_i^R(\omega)), \]  \tag{4} 

where \( M^R \) is the number of Rayleigh-wave modes present, \( f_i^{RV}(\omega) \) is the intensity of the vertical component of the \( i \)th-mode Rayleigh waves, \( J_0(\cdot) \) and \( J_1(\cdot) \) are the zeroth- and first-order Bessel functions of the first kind respectively, and \( rk_i^R(\omega) \) stands for the wavenumber of the \( i \)th-mode Rayleigh waves. By taking their mutual ratio, we have

\[
\frac{G_{Z0Z0}(r, r, \omega)}{G_{Z1Z1}(r, r, \omega)} = \frac{\sum_{i=1}^{M^R} \alpha_i(\omega) J^2_0(rk_i^R(\omega))}{\sum_{i=1}^{M^R} \alpha_i(\omega) J^2_1(rk_i^R(\omega))}, \tag{5}
\]

where

\[
\alpha_i(\omega) = f_i^{RV}(\omega)/ f^V(\omega), \quad f^V(\omega) = \sum_{i=1}^{M^R} f_i^{RV}(\omega) \tag{6}
\]

is the power partition ratio for the \( i \)th-mode Rayleigh waves relative to the total power of vertical motion.

When the dominance of the fundamental mode can safely be assumed, the expression (5) for the spectral ratio reduces to

\[
\frac{G_{Z0Z0}(r, r, \omega)}{G_{Z1Z1}(r, r, \omega)} = \frac{J^2_0(rk^R(\omega))}{J^2_1(rk^R(\omega))}, \tag{7}
\]

where \( rk^R(\omega) \) stands for the phase velocity of the fundamental-mode Rayleigh waves. The function on the right-hand side is plotted against \( rk^R \) in Figure 1b (where the superscript \( R \) is omitted for simplicity); its value tends to infinity as \( rk^R \rightarrow 0 \). Once the spectral ratio on the left-hand side is known from measurement records, it is possible to estimate \( rk^R \) by inverting equation (7) for each frequency \( \omega \). Since \( r \) is known, one can obtain the wavenumber \( k^R(\omega) \), and finally the phase velocity \( c^R(\omega) = \omega/k^R(\omega) \). The analysis is normally feasible only in the range \( 0 < rk^R < 2.405 \), or in other words, in the range of wavelengths longer than 2.6\( r \), where there exists a one-to-one correspondence between the value and the argument of the function \( J^2_0(\cdot)/J^2_1(\cdot) \).

When the dominance of the fundamental mode cannot be assumed, the value of \( c^R(\omega) \), derived by inverting equation (7), should be looked upon as an apparent phase velocity incorporating the effects of higher-mode Rayleigh waves, and care should be taken, for example, in searching for a soil profile model that is compatible with the dispersion curve of the obtained \( c^R(\omega) \) estimates.

Equation (7) holds in noise-free situations. When noise of intensity \( n^V(\omega) \) is present, equations (3) and (4) should be replaced, if the array is composed of \( N \) seismic sensors placed equidistantly around the circle, by

\[
G_{Z0Z0}(r, r, \omega) = 4\pi^2 \left[ \sum_{i=1}^{M^R} f_i^{RV}(\omega) J^2_0(rk_i^R(\omega)) + n^V(\omega)/N \right], \tag{8}
\]

\[
G_{Z1Z1}(r, r, \omega) = 4\pi^2 \left[ \sum_{i=1}^{M^R} f_i^{RV}(\omega) J^2_1(rk_i^R(\omega)) + n^V(\omega)/N \right], \tag{9}
\]
so that equation (7) has to be replaced with

\[
\frac{G_{Z_{0z0}}(r, r, \omega)}{G_{Z_{1z1}}(r, r, \omega)} = \frac{J_0^2 (rk^R (\omega)) + \varepsilon^V (\omega)/N}{J_1^2 (rk^R (\omega)) + \varepsilon^V (\omega)/N},
\]

where

\[
\varepsilon^V (\omega) = n^V (\omega)^2 f^V (\omega)
\]

is the noise-to-signal (NS) ratio. In this case, the spectral ratio on the left-hand side does not tend to infinity as \(rk^R \rightarrow 0\), but instead gets saturated at a finite value (Figure 1c). This means that, if we use the noise-free equation (7) to estimate \(rk^R\) using a value of the spectral ratio which should in fact rather be expressed by the noise-inclusive equation (10), we tend to overestimate the value of \(rk^R\), and hence underestimate the phase velocity \(c^R(\omega)\), in long-wavelength (small \(rk\)) ranges. This biasing effect of incoherent noise on the phase velocity estimates sets a limit on the resolution of the CCA method in long-wavelength ranges. According to our field experiences, the longest limit of the wavelength range, over which reasonable estimates of the phase velocities can be obtained, is in fact principally controlled by the NS ratio of the dataset analyzed, and can be as long as several ten times the array radius \(r\) when the NS ratio is low enough. The shortest resolvable wavelength typically lies at slightly less than \(3r\) (Cho et al., 2006b).

### 2.2. The TR method

Next, let us denote by \(T(t,r,\theta)\) the tangential component of the horizontal seismograms of microtremors to be recorded on a circle of radius \(r\). If we define the waveform

\[
T_0(t,r) = \int_{-\pi}^{\pi} T(t,r,\theta) d\theta
\]

by integrating the seismograms with respect to azimuth, its power-spectral density \(G_{T_0 T_0}(r,r,\omega)\) is given by
\[ G_{T0T0}(r, r, \omega) = 4\pi^2 \sum_{i=1}^{M^L} f_i^L(\omega) J_i^2(rk_i^L(\omega)), \]  
\( (13) \)

where \( M^L, f_i^L(\omega) \) and \( k_i^L(\omega) \) are the Love-wave counterparts of \( M^R, f_i^{RV}(\omega) \) and \( k_i^{R}(\omega) \), respectively. The mutual ratio of such power-spectral densities, calculated on two circles of radii \( r \) and \( \lambda r \) (\( 0 < \lambda < 1 \)), is given by

\[ \frac{G_{T0T0}(r, r, \omega)}{G_{T0T0}(\lambda r, \lambda r, \omega)} = \sum_{i=1}^{M^L} \gamma_i^L(\omega) J_i^2(rk_i^L(\omega)) \],
\( (14) \)

where

\[ \gamma_i^L = f_i^L(\omega)/f^{II}(\omega), \quad f^{II}(\omega) = \sum_{i=1}^{M^F} f_i^{RH}(\omega) + \sum_{i=1}^{M^L} f_i^L(\omega) \]  
\( (15) \)

is the power partition ratio for the \( i \)th-mode Love waves relative to the total power of horizontal motion, both Love and Rayleigh components included (\( f_i^{RH}(\omega) \) here denotes the horizontal-motion counterpart of \( f_i^{RV}(\omega) \)).

If the fundamental mode dominates, equation (14) reduces to

\[ \frac{G_{T0T0}(r, r, \omega)}{G_{T0T0}(\lambda r, \lambda r, \omega)} = \frac{J_1^2(rk_1^L(\omega))}{J_1^2(\lambda rk_1^L(\omega))}, \]  
\( (16) \)

where \( k_i^L(\omega) \) stands for the phase velocity of the fundamental-mode Love waves. The function on the right-hand side is graphically shown in Figure 2b (where the superscript \( L \) is omitted for simplicity). Once the spectral ratio on the left-hand side is known from measurement records, it is possible to estimate \( rk_1^L \) by inverting equation (16) for each frequency \( \omega \); since \( r \) is known, one obtains the wavenumber \( k_1^L(\omega) \), and finally the phase velocity \( c_1^L(\omega) = \omega / k_1^L(\omega) \). The analysis is feasible in the range \( 0 < \lambda rk_1^L < 3.83 \), or in the range of wavelengths longer than \( 1.6 \lambda r \), where there is a one-to-one correspondence between the value and the argument of the function \( J_1^2(rk)/J_1^2(\lambda rk) \). When the dominance of the fundamental mode cannot be assumed, the value of \( c_1^L(\omega) \), derived by inverting equation (16), represents an apparent phase velocity that incorporates the effects of higher-mode Love waves.

\[ \text{Figure 2. (a) Seismic array configuration for the TR method. (b) Theoretical spectral ratio curves for different values of } \lambda \text{ (equation (16)).} \]
The spectral ratio function on the right-hand side of equation (16) gets saturated at a finite value as $r k \rightarrow 0$. In long-wavelength (small $r k$) ranges where the curve is nearly flat, slight errors in the spectral ratio estimates can cause large errors in the $r k$ estimates, and hence in the phase velocity estimates. Our field tests have shown that, under real conditions, the TR method typically provides reasonable estimates of the phase velocities of Love waves in wavelength ranges roughly between $2r$ and $6r$ (Tada et al., 2006).

Although the number of seismic sensors required is much larger and the resolvable wavelength range is narrower than with the CCA method, the TR method nevertheless possesses a unique methodological significance, as it provides the distinct possibility to infer phase velocities of Love waves alone, even when those of Rayleigh waves are to remain unknown.

3. Field implementation of the CCA method: an example

With a view to enhancing our understanding of the field applicability of the CCA method, we have conducted array measurements of microtremors in Tsukuba City and its environs in the Kanto Plain, Japan, with our central base located on the premises of the National Institute of Advanced Industrial Science and Technology (AIST). The AIST is situated on a diluvial upland called the Tsukuba Upland, which is flanked by alluvial plains on the northeastern and southwestern sides. On the other side of the alluvial plain to the northeast there rises up Mount Tsukuba, 877 m in altitude.

Drilling and PS-logging records are fairly abundant on and around the AIST premises. According to a deep drilling survey conducted on its premises, weathered granite, which marks the top of the seismic bedrock, begins to appear beneath a thick cover of sedimentary layers at a depth of approximately 570 m, and is replaced by fresh granite at a depth of 630 m. The seismic bedrock tends to deepen gently with increasing distance from Mount Tsukuba, but otherwise, lateral variations in the subsurface structure are thought to remain fairly moderate over the extent of the AIST premises.

Figure 3 right shows a one-dimensional profile model of the soil beneath the AIST, which we have constructed on the basis of available PS-logging data; because of limited space, only the S-wave velocity profile is illustrated. Figure 3 top left demonstrates, in thin black curves, the phase velocity dispersion curves of the four gravest modes of Rayleigh waves, including the fundamental, which have been calculated theoretically on the basis of our soil profile model.

Rayleigh waves of different modes are expected to be present in the field of microtremors in proportions that are different for different frequencies. In frequency ranges where the presence of higher modes cannot be neglected, the apparent phase velocities $c^R(\omega)$, to be obtained with the CCA method by inverting equation (7), can deviate significantly from the phase velocities of the fundamental mode. We have evaluated, after Tokimatsu et al. (1992) and Arai and Tokimatsu (2005), the power-partition ratios $\alpha_i(\omega)$ for different modes of Rayleigh waves by assuming that they are proportional to $|A_i^R(\omega)/k_i^R(\omega)|^2$ (plotted in Figure 3 bottom left), where $A_i^R(\omega)$ is the medium response factor relevant to a harmonic, vertical point force acting on the ground surface, and $k_i^R(\omega)$ stands for the wavenumber of the $i$th-mode Rayleigh waves. The apparent phase velocities $c^R(\omega)$, calculated by equating the right-hand sides of equations (5) and (7), are shown in a thick gray curve in Figure 3 top left. The figure shows that higher modes are expected to dominate over the fundamental mode in the neighborhood of 0.25 Hz, and also roughly between 1 and 4 Hz.
Figure 3. Top left: Theoretical phase velocity dispersion curves of the four gravest modes of Rayleigh waves (thin black curves), together with the dispersion curve of apparent phase velocities including the effects of all modes. Bottom left: Theoretically calculated $|A/k|^2$ values, or indices of the ratios of power partition among different modes. Right: One-dimensional model of the S-wave velocity profile beneath the AIST. Part of the available PS-logging data, on which basis the model was constructed, are also shown for reference.

We measured, between August 2005 and February 2006, vertical components of microtremors using circular arrays of seismic sensors, with varying radii of 2.5, 25, 50, 100, 200, 370, 2650 and 7350 meters. All arrays between 25 and 200 m in radius were composed of six sensors, five of them at equal distances around the circumference and the other at its center. The 2.5 m array and the arrays sized 370 m or greater were composed of four sensors, three of them forming an equilateral triangle and the fourth at its center. Note that, although the center station is not required by the CCA method, its presence nevertheless provides the valuable possibility to evaluate NS ratios of the array seismograms, according to a new algorithm which we have recently invented (Cho et al., 2006b) but are not going to review in the present paper because of space limitations.

For the 2.5 m array, we operated, over a duration of 20 min, VSE-15D servo velocimeters in conjunction with an SPC-51 data recorder, both apparatuses manufactured by Tokyo Sokushin Corporation. The outputs of the sensors were synchronized by cable transmission to the single recorder, and were digitized into 16-bit data at a sampling rate of 1000 Hz. For the arrays sized between 25 and 200 m, we used GPL-6A3P portable recording systems, manufactured by Akashi Corporation, which are composed of a built-in accelerometer and a built-in data logger, and make use of the Global Positioning System for automatic time correction. The ground motion, recorded over a duration of 1 hour for each circle radius, was preamplified and digitized into 24-bit data at a sampling rate of 50 Hz. For the arrays sized 370 m or greater, we employed CMG-3T velocimeters, manufactured by Güralp Systems, in conjunction with LS-7000 data loggers of Hakusan Corporation. The sensors were operated over varying durations between 13 and 17 hours, including the time needed for the sensors to stabilize, and the outputs were digitized into 24-bit data at a sampling rate of 100 Hz.
Power-spectral densities were estimated with the techniques of both segment averaging and smoothing in the frequency domain (Bendat and Piersol, 1971). We extracted a large number of segments from the original seismograms so that consecutive segments mutually overlapped by half their duration, and tapered each segment with a cosine window. The duration of a single segment was set at 2.048 sec for the 2.5 m array, between 5.12 and 20.48 sec for the arrays sized between 25 and 200 m, and 327.68 sec for the arrays sized 370 m or greater. We calculated the root mean square of the data contained in each segment, and discarded the segments for which it deviated significantly from normal values. We also calculated, with Cho et al.’s (2006b) method, NS ratios of the data contained in different parts (worth several segments) of the time-series seismograms, and discarded the parts for which the NS ratios were anomalously large.

Power-spectral densities, calculated for the individual data segments by Fast Fourier Transform, were averaged over all segments, and were smoothed with a Parzen window of varying bandwidths between 0.008 and 0.1 Hz. Care was taken in the choice of the bandwidth of the smoothing window, because broadening the bandwidth tends to result in the underestimation of the spectral ratios and the phase velocities (Cho et al., 2006b). Finally, the power-spectral density estimates were substituted into equation (7), and a grid-search method was employed to solve equation (7) for the unknown argument $R_k(\omega)$.

The apparent phase velocities $c_R(\omega)$ of Rayleigh waves, thus inferred using seismograms from arrays of different radii, are summarized in a single panel and compared with the theoretical values in Figure 4. For the sake of visibility, the inferred dispersion curves are shown only in those frequency ranges where the agreement with the theoretical curve is good and where the CCA method is thought to have sufficient resolution. The fine overall agreement between the inferred and theoretical dispersion curves demonstrates that the CCA method can work effectively, not just with seismic arrays sized 5 to 600 m as we have reported in our earlier publications (Cho et al., 2004, 2006b), but over an even broader range of length scales, extending well into the order of several kilometers on the larger-scale side.

![Figure 4. Phase velocities of Rayleigh waves inferred with the CCA method. The thick, gray curve stands for the theoretical dispersion curve of apparent phase velocities which include the effects of higher modes.](image)

It should be noted that the local bulges, present in the neighborhood of 0.2 Hz in the phase velocity dispersion curves inferred with the 370 m and 2650 m arrays, appear to be related to the local peak which is evident near 0.25 Hz in the theoretical dispersion curve; in fact, a bulge of a similar shape was also recognized in the phase velocity dispersion curve inferred with the SPAC method using the records of the 2650 m array. This local
peak in the apparent phase velocity, which is related to the dominance of higher-mode Rayleigh waves, corresponds to a wavelength of approximately 20 km, and is thought to contain information on the soil properties at depths of similar orders. It is worth emphasizing that with the CCA method, which has superior resolution capabilities in long-wavelength ranges, implications for soil properties at such great depths can be obtained with a seismic array as small as just several hundred meters in radius.

Figure 4 also demonstrates that the resolvable frequency range of the CCA method is not equally broad for all array radii. In Figure 5 left, the analysis results of the CCA method have been redrawn in terms of the apparent wavelengths of Rayleigh waves, $2\pi/k^R(\omega)$, normalized by the array radius $r$. The maximum value of the apparent wavelength, obtained for a certain fixed radius of the seismic array, does not necessarily give the upper limit of the wavelength range in which reasonable estimates of phase velocities are available with the CCA method for that array radius, but careful inspection of Figure 4 and Figure 5 left reveals that the upper limit of the resolvable wavelength range falls somewhat short of $10r$ for both the 2650 and 7350 m arrays, while it lies as far up as $40r$-$60r$ for all arrays sized 370 m or smaller. (With the 2.5 m array, the inferred phase velocities fall discernibly below the theoretical values in wavelength ranges upward of $40r$, but they still keep running parallel to the theoretical dispersion curve as far up as $100r$, or possibly even more.)

Figure 5 right shows the NS ratios of the array seismograms, inferred with the technique of Cho et al. (2006b), for the individual array radii. The remarkable differences in NS ratios, well in excess of two orders of magnitude between the "noisiest" (7350 m) and the least "noisy" (2.5 and 370 m) array seismograms, appears to account for the remarkable variability in the upper limit of the resolvable wavelength ranges relative to the array radius. We might conjecture, incidentally, that the large NS ratio for the 7350 m array seismograms, and the correspondingly limited width of the resolvable frequency range, may partly be due to the breakdown of the approximation of horizontal soil layering, which the theory of the CCA method tacitly assumes.

4. Conclusions

We have reviewed, in the present paper, two new methods of microtremor exploration which we have recently developed. The CCA method, which uses vertical-motion
seismograms from a circular array of sensors minus the center station, has the capacity to yield reasonable estimates of the phase velocities of Rayleigh waves, up to wavelength ranges as long as several ten times the array radius if the noise-to-signal ratios are sufficiently low. The TR method, which uses horizontal-motion seismograms from an array of sensors placed around two circles of different radii, provides the unique possibility of evaluating phase velocities of Love waves independently of those of Rayleigh waves, although its frequency range of resolvability is not necessarily as broad as with other established methods. In the latter half of the present paper, a field example illustrates the performance of the CCA method that remains valid over a broad range of length scales, ranging from a few meters up to several kilometers in array radius. These two new methods, with their unique respective features, are expected not just to add a new dimension to the range of available analysis methods but also to broaden significantly the horizon of microtremor exploration techniques in general.

References