Introduction to Tree Language Theory

Hitoshi Ohsaki

National Institute of
Advanced Industrial Science and Technology (AIST)

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IV. Modulo Axioms

Set modulo relation

Let $\succcurlyeq$: binary relation on a set S

$\succcurlyeq$ is preorder if it is reflexive† and transitive‡ (†, ‡ : next page)

$\succcurlyeq$ is partial order if it is reflexive and transitive and

[Anti-symmetric] $a \succcurlyeq b$ & $b \succcurlyeq a \Rightarrow a \equiv b$

$\succ$ is defined as $\succcurlyeq - (\succcurlyeq)^{-1}$, called strict part of $\succcurlyeq$

$\succcurlyeq$ is well-founded if it does not admit an infinite sequence

$s_1, s_2, s_3, \cdots$ such that $s_i \succ s_{i+1}$ $(i \geqslant 1)$

$\min_{\succcurlyeq}(S)$ is set of minimal elements in S with respect to a well-founded preorder $\succcurlyeq$

$[\, a\,]_{\succcurlyeq}$ is set $\{\, x \in S \mid x \succcurlyeq a \,\}$

$S \,/ \succcurlyeq$ is set $\{\, [\, a\,]_{\succcurlyeq} \mid \exists a \in \min_{\succcurlyeq}(S) \,\}$

Equivalence classes

Let $\sim$: binary relation on a set S

$\sim$ is equivalence relation if for all a, b, c in S,

[Reflexive] $a \sim a$

[Symmetric] $a \sim b \Rightarrow b \sim a$

[Transitive] $a \sim b$ & $b \sim c \Rightarrow a \sim c$

$(S, \sim)$ is called setiod

$[\,a\,]_\sim$ is set $\{\, x \in S \mid x \sim a \,\}$, equivalence class of a

$S \,/\!\sim$ is set of all equivalence classes $\{\, [\,a\,]_\sim \mid \exists a \in S \,\}$,
quotient set of S by $\sim$

Note

Every equivalence relation $\sim$ is a well-founded preorder, and $\min_\sim(S) = S$
($\because$ the strict part of $\sim$ is empty)

F-algebras

Let F : signature

F-algebra $\mathcal{A} = (\, C, \{\, f_{\mathcal{A}} \mid f \in F \,\}\,)$

C : non-empty set, called carrier set of $\mathcal{A}$

$f_{\mathcal{A}}$: mapping from C^n (n-tuples of C) to C if $\mathrm{ar}_F(f) = n$ (called operation of $\mathcal{A}$)

Let $\mathcal{A} = (\, C, \{\, f_{\mathcal{A}} \mid f \in F \,\}\,)$ and $\mathcal{B} = (\, D, \{\, f_{\mathcal{B}} \mid f \in F \,\}\,)$

F-homomorphism h from $\mathcal{A}$ to $\mathcal{B}$

h : mapping from C to D such that for all $f \in F$,

if $\mathrm{ar}_F(f) = n$, then for all $c_1, \ldots, c_n \in C$,

$$h(\, f_{\mathcal{A}}(c_1, \ldots, c_n)\,) = f_{\mathcal{B}}(\, h(c_1)\,, \ldots, \, h(c_n)\,)$$

Congruence relations

Let $\sim$: binary relation on C

$\sim$ is congruence relation of $\mathcal{A} = (C, \{ f_{\mathcal{A}} \mid f \in F \})$ if

- $\sim$ is equivalence relation,
- for all $f \in F$, $f_{\mathcal{A}}(c_1, \ldots, c_n) \sim f_{\mathcal{A}}(d_1, \ldots, d_n)$
  if $\mathrm{ar}_F(f) = n$ & $c_1 \sim d_1$ & $\cdots$ & $c_n \sim d_n$
  $(c_i, d_i \in C \ , \ 1 \leqslant i \leqslant n)$

$\mathcal{A}$ with $\sim$ is quotient F-algebra, denoted by $\mathcal{A}/{\sim}$

$C/{\sim}$: carrier set of $\mathcal{A}/{\sim}$

$f_{\mathcal{A}}/{\sim}$: set of operations of $\mathcal{A}/{\sim}$ defined as

$$(f_{\mathcal{A}}/{\sim})([c_1]_{\sim}, \ldots, [c_n]_{\sim}) = [f_{\mathcal{A}}(c_1, \ldots, c_n)]_{\sim}$$

Equational theory

Let F : signature

equation $s \approx t$ is a pair of trees s, t with variables in $\mathcal{T}_{F,V}$

equational theory $\mathcal{E} = (F, E)$

E : finite set of equations

Note

Equations $s \approx t$ and $t \approx s$ are distinguished if $s \not\equiv t$ (equations are orientation sensitive), and thus, $E_1 = \{\mathsf{a} \approx \mathsf{b}\}$ and $E_2 = \{\mathsf{a} \approx \mathsf{b}, \mathsf{b} \approx \mathsf{a}\}$ are not the same.

$\to_{\mathcal{E}}$: binary relation over $\mathcal{T}_{F,V}$ such that $s \to_{\mathcal{E}} t$ if

$\exists\, l \approx r \in E$: $s = C[\,l\theta\,]$ & $t = C[\,r\theta\,]$ for some $C[\,], \theta$

$\leftrightarrow_{\mathcal{E}}$: the smallest symmetric relation containing $\to_{\mathcal{E}}$

$=_{\mathcal{E}}$: the smallest equivalence relation containing $\to_{\mathcal{E}}$

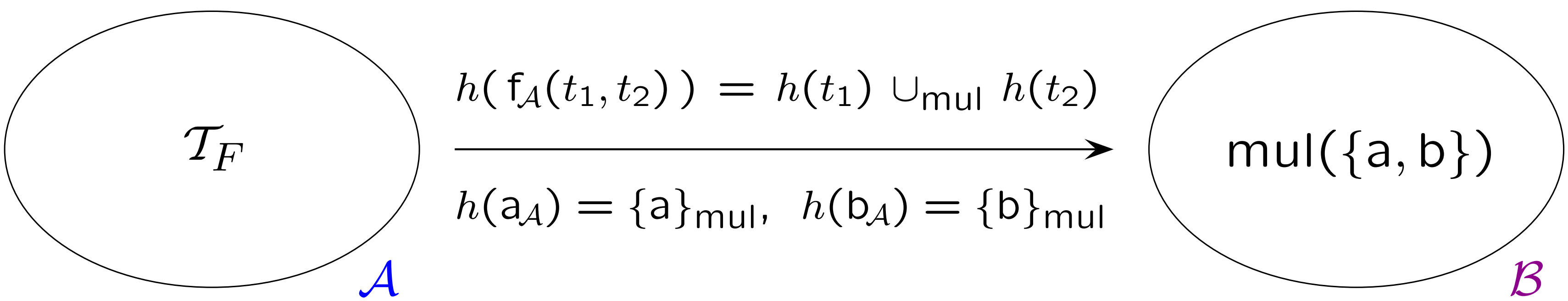
Example : AC-theory and multisets

Let $F = \{ \mathsf{f}, \mathsf{a}, \mathsf{b} \}$ with $\mathsf{ar}_F(\mathsf{f}) = 2$, $\mathsf{ar}_F(\mathsf{a}) = \mathsf{ar}_F(\mathsf{b}) = 0$

$$\mathcal{A} = (\mathcal{T}_F , \{ \mathsf{f}_{\mathcal{A}}(x,y) = \mathsf{f}(x,y) ,\ \mathsf{a}_{\mathcal{A}} = \mathsf{a} ,\ \mathsf{b}_{\mathcal{A}} = \mathsf{b} \})$$

$$\mathcal{B} = (\mathsf{mul}(\{\mathsf{a},\mathsf{b}\}) , \{ \mathsf{f}_{\mathcal{B}}(x,y) = x \cup_{\mathsf{mul}} y ,\ \mathsf{a}_{\mathcal{B}} = \{\mathsf{a}\}_{\mathsf{mul}} ,\ \mathsf{b}_{\mathcal{B}} = \{\mathsf{b}\}_{\mathsf{mul}} \})$$

Define F-homomorphism h :



then for all s, t in $\mathcal{T}_F$:

$$h(s) = h(t) \text{ if and only if } [s]_{=_{\mathcal{E}}} = [t]_{=_{\mathcal{E}}}$$

where $\mathcal{E} = (F, E_{\mathsf{AC}})$ and $E_{\mathsf{AC}} = \{ \mathsf{f}(x,y) \approx \mathsf{f}(y,x) ,\ \mathsf{f}(\mathsf{f}(x,y),z) \approx \mathsf{f}(x,\mathsf{f}(y,z)) \}$

$\mathsf{mul}(S)$: set of multisets over S, $\cup_{\mathsf{mul}}$: multiset union, $\{\cdot\}_{\mathsf{mul}}$: multiset notation

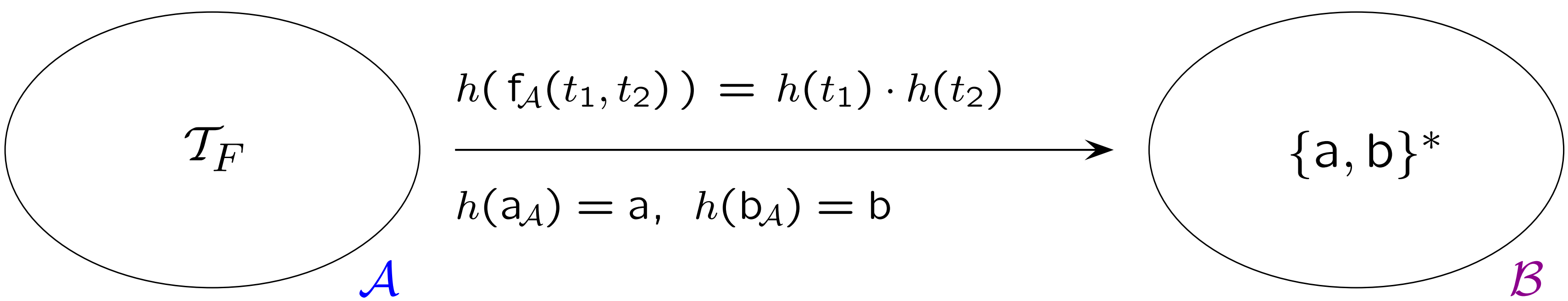
Example : A-theory and words

Let $F = \{\, \mathsf{f}, \mathsf{a}, \mathsf{b} \,\}$ with $\mathsf{ar}_F(\mathsf{f}) = 2$, $\mathsf{ar}_F(\mathsf{a}) = \mathsf{ar}_F(\mathsf{b}) = 0$

$$\mathcal{A} = (\, \mathcal{T}_F \,, \{\, \mathsf{f}_{\mathcal{A}}(x,y) = \mathsf{f}(x,y)\,,\ \mathsf{a}_{\mathcal{A}} = \mathsf{a}\,,\ \mathsf{b}_{\mathcal{A}} = \mathsf{b} \,\}\,)$$

$$\mathcal{B} = (\, \{\mathsf{a},\mathsf{b}\}^* \,, \{\, \mathsf{f}_{\mathcal{B}}(x,y) = x \cdot y\,,\ \mathsf{a}_{\mathcal{B}} = \mathsf{a}\,,\ \mathsf{b}_{\mathcal{B}} = \mathsf{b} \,\}\,)$$

Define F-homomorphism h :



then for all s, t in $\mathcal{T}_F$:

$$h(\,s\,) \;=\; h(\,t\,) \;\text{ if and only if }\; [\,s\,]_{=_{\mathcal{E}}} \;=\; [\,t\,]_{=_{\mathcal{E}}}$$

where $\mathcal{E} = (F, E_{\mathsf{A}})$ and $E_{\mathsf{A}} = \{\, \mathsf{f}(\mathsf{f}(x,y),z) \approx \mathsf{f}(x,\mathsf{f}(y,z)) \,\}$

$x \cdot y$: concatenation of words x and y

$\mathcal{E}$-equivalence problem

instance : trees s, t in $\mathcal{T}_F$

equational theory $\mathcal{E} = (F, E)$

solution : "yes" if $[s]_{=_\mathcal{E}} = [t]_{=_\mathcal{E}}$; "no" otherwise

This problem is called $\mathcal{E}$-equivalence problem

E.g. consider

E_{A} : $\mathsf{f}(\mathsf{f}(x, y), z) \approx \mathsf{f}(x, \mathsf{f}(y, z))$ E_{I} : $\mathsf{f}(x, x) \approx x$

E_{C} : $\mathsf{f}(x, y) \approx \mathsf{f}(x, y)$ E_{U} : $\mathsf{f}(x, \mathsf{e}) \approx x$

then

the problem for equational theory $\mathcal{E}$ with any combination of the above equations (16 combinations including the empty theory) is decidable

Word problem

instance : trees s, t in $\mathcal{T}_F$

equational theory $\mathcal{E} = (F, E \cup E_{\mathsf{A}})$, where

$$E = \{ s_1 \approx t_1, \cdots, s_n \approx t_n \} \quad (s_i, t_i \in \mathcal{T}_F,\ 1 \leqslant i \leqslant n)$$

solution : "yes" if $[s]_{=_\mathcal{E}} = [t]_{=_\mathcal{E}}$; "no" otherwise

Note

According to the previous example, if $F = \{\mathsf{f}\} \cup F_{(0)}$, it holds that

$[s]_{=_\mathcal{E}} = [t]_{=_\mathcal{E}}$ if and only if

$h(s)$ and $h(t)$ are equivalent under the axioms $h(s_1) \approx h(t_1), \cdots, h(s_n) \approx h(t_n)$

Words u, w are equivalent under axioms $u_i \approx w_i$ $(u_i, w_i \in \Sigma^+,\ 1 \leqslant i \leqslant n)$ if $u \equiv w$ or there exists a sequence $x_1\, x_2\, \cdots\, x_k$ $(k \geqslant 2)$ such that (1) $x_1 = u$, $x_k = w$, (2) $\exists\, u_{\ell_i} \approx w_{\ell_i}$ $(1 \leqslant i < k)$: $x_i = y_i\, u_{\ell_i}\, z_i$ & $x_{i+1} = y_i\, w_{\ell_i}\, z_i$ or $x_i = y_i\, w_{\ell_i}\, z_i$ & $x_{i+1} = y_i\, u_{\ell_i}\, z_i$.

This problem can be reduced from the halting problem of TM (Exercise).

Equational term rewriting systems (ETRS)

ETRS $\mathcal{R} = (\mathcal{E}, R)$

$\mathcal{E}$: equational theory (F, E)

R : finite set of equations over F, called rewrite rules (equation $s \approx t$ in R is denoted by $s \to t$)

$\to_{\mathcal{R}}$: rewrite relation over $\mathcal{T}_{F,V}$ defined as $s \to_{\mathcal{R}} t$ if $\exists\, l \to r \in R$: $s =_{\mathcal{E}} C[\,l\theta\,]$ & $t =_{\mathcal{E}} C[\,r\theta\,]$ for some $C[\,], \theta$

$\to^*_{\mathcal{R}}$: the smallest reflexive and transitive relation containing $\to_{\mathcal{R}}$

$=_{\mathcal{R}}$: the smallest equivalence relation containing $\to_{\mathcal{R}}$

Note

If $\mathcal{E} = (F, \varnothing)$, we call $(\mathcal{E}, R)$ a term rewriting system (TRS). For convenience, $((F, \varnothing), R)$ can be denoted by (F, R).

Confluence & termination

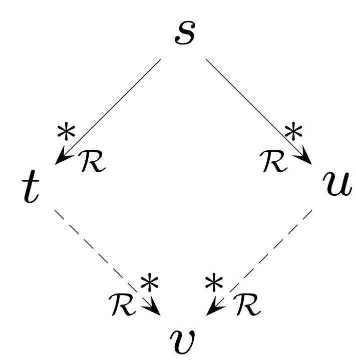
Let $\mathcal{R}$: ETRS over F

$\mathcal{R}$ is confluent if $\forall\, s\ t\ u\ :\ s \to^*_{\mathcal{R}} t\ \ \&\ \ s \to^*_{\mathcal{R}} u$

$\Rightarrow\ \exists\, v\ :\ t \to^*_{\mathcal{R}} v\ \ \&\ \ u \to^*_{\mathcal{R}} v$

$\mathcal{R}$ is locally confluent if $\forall\, s\ t\ u\ :\ s \to_{\mathcal{R}} t\ \ \&\ \ s \to_{\mathcal{R}} u$

$\Rightarrow\ \exists\, v\ :\ t \to^*_{\mathcal{R}} v\ \ \&\ \ u \to^*_{\mathcal{R}} v$



confluence

$\mathcal{R}$ is Church-Rosser if $\forall\, s\ t\ :\ s =_{\mathcal{R}} t\ \Rightarrow\ \exists\, u\ :\ s \to^*_{\mathcal{R}} u\ \ \&\ \ t \to^*_{\mathcal{R}} u$

$\mathcal{R}$ is terminating if $\mathcal{R}$ does not admit any infinite sequence

$t_1, t_2, t_3, \cdots$ such that $t_i \to_{\mathcal{R}} t_{i+1}\ \ (i \geqslant 1)$

Note

$\mathcal{R}$ is confluent if and only if it is Church-Rosser

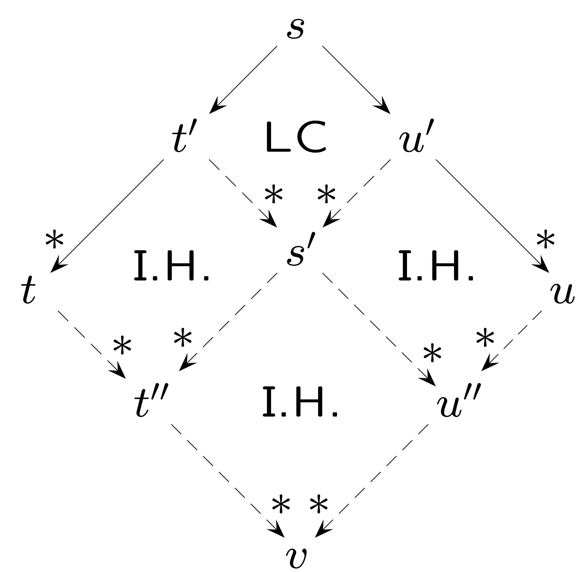
(The "if" is obvious by definition. The "only if" can be shown by the induction on the number of peaks ${}^*_{\mathcal{R}}\!\leftarrow \cdot \to^*_{\mathcal{R}}$ in $=_{\mathcal{R}}$.)

Newman's lemma

Every locally confluent and terminating (E)TRS is confluent.

Proof

Suppose $\mathcal{R} = (\mathcal{E}, R)$ is locally confluent and terminating. Define the binary relation $\succcurlyeq \; = \; \to^*_{\mathcal{R}}$. By definition, $\succcurlyeq$ is a preorder. Moreover, since $\mathcal{R}$ is terminating, $\succcurlyeq$ is well-founded, because $\succcurlyeq - (\succcurlyeq)^{-1}$ is the transitive closure of $\to_{\mathcal{R}}$. Using the induction on trees with $\succcurlyeq$, one can show that $\mathcal{R}$ is confluent : The base case is obvious, because $s \equiv t \equiv u$. For induction step, suppose $s \to^*_{\mathcal{R}} t$ and $s \to^*_{\mathcal{R}} u$. If $s \not\equiv t$ and $s \not\equiv u$ (otherwise, it is obvious), the right figure holds. We note that $s \succ w$ if $u' \succcurlyeq w$ or $t' \succcurlyeq w$, because by assumption, $s \succ t'$ and $s \succ u'$. $\square$



Critical pairs & local confluence

Suppose $l_1 \to r_1, l_2 \to r_2 \in R$ are rewrite rules such that there exists a most general unifier θ for unifying l_1 and a sub-tree l of l_2 (so $l_1\theta \equiv l\theta$). Let $l_2 = C[\,l\,]$, then $\langle C[r_1\theta], r_2\theta \rangle$ is called critical pair. It is known that a TRS $\mathcal{R}$ is local confluent if and only if its critical pairs are joinable ($\exists\, t\colon C[r_1\theta] \to^*_{\mathcal{R}} t \;\&\; r_2\theta \to^*_{\mathcal{R}} t$).

Knuth-Bendix procedure

Let E : set of equations

$\succcurlyeq$: well-founded preorder & closed under contexts and substitutions

Given an equational theory $\mathcal{E} = (F, E)$ with $\succcurlyeq$, this procedure attempts to construct a terminating and confluent TRS $\mathcal{R} = (F, R)$ with $=_{\mathcal{R}} \; = \; =_{\mathcal{E}}$. When the procedure successfully terminates, it returns R (set of rewrite rules); otherwise, it fails or runs forever. The procedure `normalize` is non-deterministic function that returns one of the candidates. Termination of $\mathcal{R}$ is a consequence of the compatibility with $\succ$. Confluence of $\mathcal{R}$ follows from the previous observation about critical pairs and Newman's lemma.

procedure main$(E, \succcurlyeq)$

if $\exists\, l \approx r \in E : l \not\succ r \;\&\; r \not\succ l$ **then** break("fail") ;

R_0 := $\{\, l \to r \mid l \approx r$ or $r \approx l \in E$ such that $l \succ r \,\}$;

while $\exists$ critical pair $\langle C[r_1\theta], r_2\theta \rangle$ in R_i is not joinable

do

s := normalize$(R_i,\; C[r_1\theta])$;

t := normalize$(R_i,\; r_2)$;

if $s \succ t$ **then** R_{i+1} := $R_i \cup \{\, s \to t \,\}$;

if $t \succ s$ **then** R_{i+1} := $R_i \cup \{\, t \to s \,\}$;

else break("fail") ;

i++ ;

end ; **return** R_i

procedure normalize(R, t)

if $t \to^*_{\mathcal{R}} s$ & s is not rewritten by $\mathcal{R}$ anymore

then **return** s

Proposition [Knuth & Bendix]

If Knuth-Bendix procedure successfully terminates at the n-th loop, the TRS $\mathcal{R} = (F, R_n)$ is terminating and confluent with $=_{\mathcal{R}} = =_{\mathcal{E}}$.

Proof

Termination and confluence of $\mathcal{R}$ are verified in the previous page. Let $\mathcal{R}_i = (F, R_i)$ where R_i is the set of rewrite rules obtained at the i-th loop, then we show that $=_{\mathcal{R}_i} = =_{\mathcal{E}}$: Use the induction on the number of loops. The base case is obvious, because according to the procedure, every equation in E is oriented by $\succ$, and thus $=_{\mathcal{R}_0} = =_{\mathcal{E}}$. For induction step, suppose $=_{\mathcal{R}_i} = =_{\mathcal{E}}$. If there is no more critical pair in R_i, the computation is done, so $\mathcal{R}_i = \mathcal{R}_{i+1}$. If there is a critical pair $\langle C[r_1\theta], r_2\theta\rangle$, let $s = \texttt{normalize}(C[r_1\theta])$ and $t = \texttt{normalize}(r_2\theta)$, then $s =_{\mathcal{R}_i} C[r_1\theta]$ and $t =_{\mathcal{R}_i} r_2\theta$. This implies $s =_{\mathcal{R}_i} t$, because $C[r_1\theta] =_{\mathcal{R}_i} r_2\theta$. Thus, regardless of the orientation of the equation $s \approx t$, we have $=_{\mathcal{R}_i} = =_{\mathcal{R}_i \cup \{s \to t\}}$ and $=_{\mathcal{R}_i} = =_{\mathcal{R}_i \cup \{t \to s\}}$. Hence, $=_{\mathcal{R}_i} = =_{\mathcal{R}_{i+1}}$, and therefore, by induction hypothesis, $=_{\mathcal{R}_{i+1}} = =_{\mathcal{E}}$. □

As a corollary of this proof, the correctness of the procedure can be preserved, even if the procedure is modified as follows : R_{i+1} := $\{l \to r' \mid l \to r \in R_i \ \& \ r' = \texttt{normalize}(R_{i+1}, r)\} \cup \{s \to t\}$ if $s \succ t$ $(t \to s$ if $t \succ s)$.

D.E. Knuth & P.B. Bendix: *Simple Word Problems in Universal Algebra*, Computational Problems in Abstract Algebra, Pergamon Press, pp.263–297, 1970.

Confluence modulo

Let (F, E) : equational theory $\mathcal{E}$

(F, R) : TRS $\mathcal{R}$

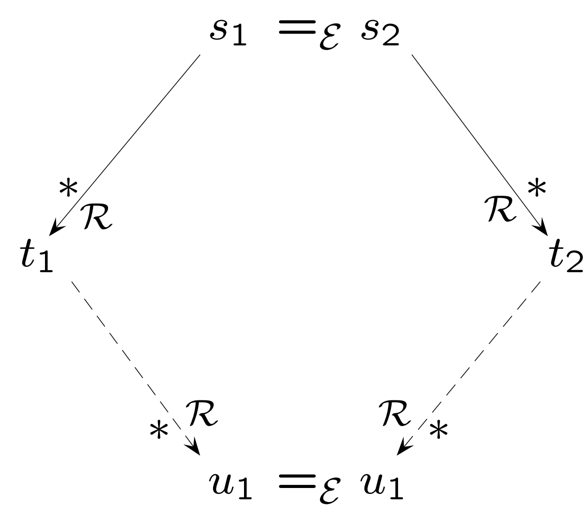
$\mathcal{R}$ is confluent modulo $\mathcal{E}$

if $\forall\, s_1, s_2, t_1, t_2$: $s_1 =_{\mathcal{E}} s_2$ &

$s_1 \to^*_{\mathcal{R}} t_1$ & $s_2 \to^*_{\mathcal{R}} t_2$

$\Rightarrow$ $\exists\, u_1, u_2$: $u_1 =_{\mathcal{E}} u_2$ &

$t_1 \to^*_{\mathcal{R}} u_1$ & $t_2 \to^*_{\mathcal{R}} u_2$



confluence modulo $\mathcal{E}$

Proposition (Huet 1980)

1. If TRS $\mathcal{R}$ is terminating and $\leftarrow_{\mathcal{R}} \cdot \to_{\mathcal{R}} \subseteq \to^*_{\mathcal{R}} \cdot =_{\mathcal{E}} \cdot \leftarrow^*_{\mathcal{R}}$ (called locally confluent modulo $\mathcal{E}$) and $\leftarrow_{\mathcal{R}} \cdot =_{\mathcal{E}} \subseteq \to^*_{\mathcal{R}} \cdot =_{\mathcal{E}} \cdot \leftarrow^*_{\mathcal{R}}$ (called locally coherent with $\mathcal{E}$), then $\mathcal{R}$ is confluent modulo $\mathcal{E}$.
2. If ETRS $(\mathcal{E}, R)$ is terminating and locally confluent modulo $\mathcal{E}$ and $\leftarrow_{\mathcal{R}} \cdot \leftrightarrow_{\mathcal{E}} \subseteq \to^*_{\mathcal{R}} \cdot =_{\mathcal{E}} \cdot \leftarrow^*_{\mathcal{R}}$ (called locally coherent with $\leftrightarrow_{\mathcal{E}}$), then $\mathcal{R}$ is confluent modulo $\mathcal{E}$.

Exercise

1. Let $\succcurlyeq \; = \{\, (\mathsf{a},\mathsf{b})\,, (\mathsf{b},\mathsf{c})\,, (\mathsf{c},\mathsf{a})\,\}$, then show that $\succcurlyeq$ is not well-founded. Show, on the other hand, that the transitive closure of $\succcurlyeq$ (the smallest transitive relation containing $\succcurlyeq$) is well-founded.
2. Given a set S equipped with a well-founded preorder $\succcurlyeq$, show that $S = \bigcup_{a \in \min_{\succcurlyeq}(S)} \{\, x \in S \mid x \succcurlyeq a \,\}$.
3. Show that the $\mathcal{E}$-equivalence problem (page 9) is decidable for the classes of E_{I}, E_{U}, E_{ACI}, E_{ACU}, E_{ACIU}.
4. Recall the word problem, which is the question if given words u, w are equivalent under given axioms $u_i \approx w_i$ $(u_i, w_i \in \Sigma^{+},\ 1 \leqslant i \leqslant n)$. Show that the word problem is undecidable. (See page 10)
5. Complete by Knuth-Bendix procedure the equational theory E_{AIU}.

   For defining a strict order (the strict part of a well-founded preorder), use, e.g. the polynomial interpretation $\mathsf{f}(x,y) = 2x+y+1$ and $e = 1$ over positive integers.
6. Let $\mathcal{E} = (F, E)$. Show that if TRS (F, R) is confluent modulo $\mathcal{E}$, then ETRS $(\mathcal{E}, R)$ is confluent.

Appendix : Dauchet's construction

It is undecidable for a given LBA (see page 18, seminar talk 2) whether it halts on any input [Caron 1991]

For each transition rule $\langle \mathsf{p}, \mathsf{a} \rangle \to \langle \mathsf{q}, \mathsf{b}, X \rangle$ of LBA $\mathcal{M}$, define a rewrite rule $s \to t$ whose left-hand side s is

$$\mathsf{f}(\mathsf{g}(x_1, \mathsf{a}), \mathsf{p}, \mathsf{h}(x_2, x_3))$$

To be precise, as a special case (where the tape-head is located at the left- or rightmost position), $\mathsf{f}(\sharp_{\mathsf{L}}, \mathsf{p}, \mathsf{h}(x_2, x_3))$ and $\mathsf{f}(\mathsf{g}(x_1, \mathsf{a}), \mathsf{p}, \sharp_{\mathsf{R}})$ for the LHS have to be considered

and right-hand side t is

$$\mathsf{f}(\mathsf{g}(\mathsf{g}(x_1, \mathsf{b}), x_2), \mathsf{q}, x_3) \text{ if } X = \mathsf{R} \;;\; \mathsf{f}(x_1, \mathsf{q}, \mathsf{h}(\mathsf{b}, \mathsf{h}(x_2, x_3))) \text{ if } X = \mathsf{L}.$$

E.g. for the input a b c, if LBA $\mathcal{M}$ selects the rule $\langle \mathsf{q}_0, \mathsf{a} \rangle \to \langle \mathsf{p}, \mathsf{d}, \mathsf{R} \rangle$, then one can have the reduction from $\mathsf{f}(\mathsf{g}(\sharp_{\mathsf{L}}, \mathsf{a}), \mathsf{q}_0, \mathsf{h}(\mathsf{b}, \mathsf{h}(\mathsf{c}, \sharp_{\mathsf{R}}))$ to $\mathsf{f}(\mathsf{g}(\mathsf{g}(\sharp_{\mathsf{L}}, \mathsf{d}), \mathsf{b}), \mathsf{p}, \mathsf{h}(\mathsf{c}, \sharp_{\mathsf{R}}))$. For the TRS $\mathcal{R}_{\mathcal{M}}$ associated to $\mathcal{M}$, one can show that it is terminating if and only if $\mathcal{M}$ halts on any input. However, since the latter problem is undecidable, termination problem for a (sub-)class of TRS is undecidable.

[1] M. Dauchet: *Simulation of Turing Machine by a Regular Rewrite Rule*, Theoretical Computer Science (TCS) 103, pp.409–420, Elsevier, 1992.

National Institute of Advanced Industrial Science and Technology (AIST) – Senri-site, AIST Kansai.

Office: Shin-Senri Nishi 1–2–14 (MSK bldg. 5th floor), Toyonaka, Osaka 560–0083, Japan

URL: **http://staff.aist.go.jp/hitoshi.ohsaki/**