Virtual Nonholonomic Constraint for Human-Robot Cooperation in 3-D space

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Abstract

In this paper, we discuss a robotic system that assists a human to carry a long object. The operator and the robot grasp each end of the object and carry it cooperatively. We extend the concept of virtual nonholonomic constraint to the motion in a vertical plane, in which the robot wrist behaves like a wheel and the operator can maneuver the object like a wheelbarrow. The operator and the robot can cooperatively lift the object to the desired height and inclination. Furthermore, we combine the vertical motion and the horizontal motion to carry the object to the desired position and posture in 3-D space. The effectiveness of our method is experimentally confirmed.

1. Introduction

Most robots are currently used for simple and repetitive tasks in isolated sites. However, the technologies, which the human and robot can cooperate sharing the same workspace, are expected for expanding robot applications besides factory automation, such as construction works, transportation industry, home and office works. Thus, in this paper we focus on human-robot cooperative handling of an object. The concept of our target system is depicted in Fig.1. The human and the robot hold each end of a long object and the robot assists the handling according to the force applied to the object by the human.

A long or large object is difficult to carry if held at one point even when it is not heavy. Such an object can be carried more easily when two persons hold each end. It would be useful if the cooperative transportation can be achieved by a robot in place of a human assistant.

In this research, we use just the force sensor at the robot wrist and the joint angle sensors as the sensory information for the controller. We utilize neither visual nor auditory information, which requires complicated setups for image processing and speech recognition. We also omit intelligence for understanding human intention and task planning. We aim for a robot system like an extension of simple tools which people commonly use, rather than an autonomous agent that substitutes a human assistant.

In our previous study[1], we dealt with cooperative transportation in a horizontal plane. As a technique for human-robot cooperative transportation of a long or large object, some researchers have studied impedance control, in which the object follows the human force[2]-[6]. However, human force mainly consists of translational components since it is difficult to apply a large torque at an end of a long object. This method causes sideslip of the object in the normal direction when the object turns in the horizontal plane (Fig.2 (a)).

To solve this problem, we proposed a virtual nonholonomic constraint at the robot hand, which simulates a wheel in the axial direction of the object (Fig.2 (b)). The object behaves as if it were carried on a virtual wheel. It does not slip sideways and can be maneuvered like a wheelbarrow. The human operator does not have to apply torque to the object and has only to treat translational force. In addition, controllability in the horizontal plane is guaranteed because it is a nonholonomic system equivalent to a unicycle and the object can reach an arbitrary position and orientation. The constraint is achieved by anisotropic impedance control, where the impedance parameters depend on the direction, defined in the hand coordinate. It was experimentally confirmed that a human could understand the behavior of the robot easily. Fig.3 shows an example of horizontal transportation.

As for the cooperative manipulation of a long object in a vertical plane, we also proposed another control method based on the cooperative behavior of humans[2]. The basic control law is the impedance control in the
horizontal direction, and to raise or lower the robot hand in the vertical direction trying to maintain the object horizontal. However, this method does not allow the rotation of the object in the vertical plane. A different method would be necessary if the object were to change its inclination.

In this paper, we extend the virtual nonholonomic constraint to the movement in a vertical plane so that the object can be raised, lowered and inclined to the desired configuration. Next, cooperative transportation in three-dimensional space is considered combining the movement in the vertical plane and horizontal plane. In this case, the posture of the object might be twisted around the major axis during the transportation when the rotation is defined in the hand coordinate. To avoid this, we have separated the rotation of the object into the rotation in the vertical plane including the object, and the rotation of its projection on the horizontal plane. This method enables to carry the object in 3-D space without twisting around the major axis.

The rest of this paper is organized as follows. First of all, a virtual nonholonomic constraint is defined in a vertical plane. It is shown that the positioning to a desired height and inclination is possible. Next, the movements in a vertical plane and a horizontal plane are combined for cooperative transportation in 3-D space. Finally, the control law is implemented into a robot to demonstrate the effectiveness of our method in human-robot cooperation.

2. Cooperative transportation in a vertical plane

In our previous work [1], we proposed cooperative transportation in a horizontal plane by using virtual nonholonomic constraint. Here we extend this method to the movement in a vertical plane, so that the object can reach desired position and inclination. In this method, the operator can manipulate the object using a similar skill for steering a wheelbarrow on a horizontal plane. Though the object cannot be lifted up vertically, it can be positioned to the desired position along an appropriate trajectory.

A virtual constraint equivalent to a wheel is given to the robot hand (Fig.4). The operator holding point is \( H : (x_h, z_h) \), the robot holding point is \( R : (x_r, z_r) \). The object length is \( L \) and its inclination is \( \theta \). The nonholonomic constraint condition with which the virtual wheel does not slip sideways is represented as,

\[
\dot{x} \sin \theta - \dot{z} \cos \theta = 0 
\]  

(1)

The object can translate only in the axial direction, and can rotate around \( R \). It can also move along a smooth curve continuously changing its direction. The relation between the velocities of the human hand and the robot hand is as follows.

\[
\begin{align*}
\dot{x}_h &= \dot{x}_r - L \dot{\theta} \sin \theta \\
\dot{z}_h &= \dot{z}_r + L \dot{\theta} \cos \theta \\
\end{align*}
\]

(2)

Then, this system is described as,

\[
\begin{bmatrix}
\dot{x} \\
\dot{z} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\cos^2 \theta & \sin \theta \cos \theta & \frac{\dot{x}_h}{L} \\
\sin \theta \cos \theta & \sin^2 \theta & \frac{\dot{z}_h}{L} \\
-\sin \theta & -\cos \theta & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_r \\
\dot{z}_r \\
\dot{\theta}_r
\end{bmatrix}
\]

(3)

Now we prove controllability of this system [7]. Lie bracket of the vector fields is calculated as,

\[
\begin{aligned}
\dot{\theta} &= \dot{x} \sin \theta - \dot{z} \cos \theta \\
\end{aligned}
\]
\[ g_1, g_2 = \frac{\partial g_2}{\partial \theta} g_1 - \frac{\partial g_1}{\partial \theta} g_2 = \begin{bmatrix} \sin \theta & -L \cos \theta \frac{L}{L} & -L \cos \theta \frac{L}{L} \\ \cos \theta & L \sin \theta \frac{L}{L} & L \sin \theta \frac{L}{L} \end{bmatrix} \] (4)

Then,
\[ \det(g_1, g_2, [g_1, g_2]) = \frac{1}{L} \neq 0 \] (5)

Since the matrix \((g_1, g_2, [g_1, g_2])\) is of full rank, the system (3) satisfies the Lie algebra rank condition and the system is controllable. Therefore, it is possible to carry the object to the desired position and inclination without changing the control law or the holding point of the object.

Fig. 5 shows an example of the transportation. Since the robot hand has the nonholonomic constraint, the object cannot rise in a perpendicular direction directly. Instead, the operator can incline the object about the holding point of the robot and can move the object up or down only by pushing or pulling it in the axial direction.

Though the direction of the local movement is restricted, this method does not decrease the degree of freedom of the transportation in the vertical plane.

3. Human-Robot cooperative handling in 3-D space

In this section, we consider the combination of transportation in a horizontal plane and a vertical plane to achieve cooperative transportation in 3-D space.

The constraint in a horizontal plane and a vertical plane has been defined in the hand coordinate frame respectively. If those constraints were simply combined, it could cause the object to twist around the \(X\)-axis while the object rotates around the \(Y\)- and \(Z\)-axes in the hand frame (Fig. 6). Here, we focus on the movement of the object during the composite rotation around the \(Y\)- and \(Z\)-axes.

The object can rotate in \(XY\)- and \(XZ\)-planes in the hand coordinate around the robot hand. Suppose that the object rotates from the origin around the \(Y\)- and \(Z\)-axes by turns. It results in different postures depending on the order of rotations, even when the \(X\)-axis coincides after the rotations. First, we assume that the object initially rotates around the \(Z\)-axis by \(\alpha\), and then rotates around the \(Y\)-axis by \(\beta\). This rotation is represented by the following matrix.

\[
R_{(\alpha, \beta)} = \begin{bmatrix}
\cos \beta \cos \gamma & -\cos \beta \sin \gamma & \sin \beta \\
\sin \alpha \sin \beta \cos \gamma + \cos \alpha \sin \gamma & \sin \alpha \sin \beta \sin \gamma - \cos \alpha \cos \gamma & \sin \alpha \cos \beta \\
\cos \alpha \sin \beta \cos \gamma - \sin \alpha \sin \gamma & \cos \alpha \sin \beta \sin \gamma + \sin \alpha \cos \gamma & \cos \alpha \cos \beta 
\end{bmatrix}
\] (6)

Using the \(ZYX\) Euler angles,

\[
A = \alpha, B = \beta, C = 0
\] (7)

where \(A, B\) and \(C\) represent sequential rotations from the base posture around \(Z\)-, \(Y\)-, \(X\)-axes, respectively. We can see that the rotation around \(X\)-axis does not occur. Next, we assume that the object first rotates around the \(Y\)-axis by \(\beta\), and then rotates around the \(Z\)-axis by \(\gamma\). The rotation matrix is shown as follows.

\[
R_{(\beta, \gamma)} = \begin{bmatrix}
c\beta \gamma & -c\beta \gamma & s\beta \\
s\gamma & c\gamma & 0 \\
-s\beta \gamma & s\beta \gamma & \gamma & c\beta
\end{bmatrix}
\] (8)

This rotation is converted to \(ZYX\) Euler angles as follows.

\[
A = A \tan 2(s\gamma, c\beta \gamma) \\
B = A \tan 2(s\beta \gamma, \sqrt{(c\beta \gamma)^2 + (s\gamma)^2}) \\
C = A \tan 2(s\beta \gamma, c\beta)
\] (9)

To maintain the rotation angle \(C\) to be zero, the rotation angle \(\beta = 0\) and/or \(\gamma = 0\). To avoid the twist around the \(X\)-axis, the operator should not rotate the object around the \(Z\)-axis unless the object is horizontal. The transportation task becomes difficult if the operator is always obliged to consider this condition.

Now, we propose a technique which does not cause...
such twist about the X-axis. Instead of the rotation around the Z-axis in the hand frame, the horizontal rotation is made around the P-axis perpendicular to the xy-plane in an absolute frame (Fig.7). The projection of the object on the xy-plane has a virtual nonholonomic constraint with regard to the horizontal motion. The constraint for the vertical motion is defined in the hand frame, as is in Section 2.

Position \((x, y, z)\) and posture \((\theta_P, \theta_Y)\) of the object can be operated by using this technique. These angles \((\theta_P, \theta_Y)\) correspond to the Z-rotation and Y-rotation of the ZYX Euler angles, respectively.

Then, we prove the controllability of the proposed method. The operator holds the object at \(H: (x, y, z)\). The position and posture of the robot hand is \(R: (x, y, z, \theta_P, \theta_Y)\). The length of the object is \(L\). The condition of no sideslip about the robot hand in the xy-plane and the XZ-plane is

\[
\begin{align*}
\dot{x}_r &\sin \theta_P - \dot{y}_r \cos \theta_P = 0 \quad (10) \\
\dot{x}_r &\sin \theta_Y - \dot{z}_r \cos \theta_Y = 0 \quad (11)
\end{align*}
\]

The robot and human holding points are related as,

\[
\begin{bmatrix}
x_h \\
y_h \\
z_h
\end{bmatrix} = \begin{bmatrix}
\cos \theta_P \cos \theta_Y \\
\sin \theta_P \cos \theta_Y \\
-\sin \theta_Y
\end{bmatrix} L + \begin{bmatrix}
x_r \\
y_r \\
z_r
\end{bmatrix} \quad (12)
\]

Differentiating eq.(11), the relation of the velocities is as follows,

\[
\begin{bmatrix}
\dot{x}_h \\
\dot{y}_h \\
\dot{z}_h
\end{bmatrix} = \begin{bmatrix}
-\sin \theta_P \cos \theta_Y & -\cos \theta_P \sin \theta_Y & 0 \\
\cos \theta_P \cos \theta_Y & -\sin \theta_P \sin \theta_Y & 0 \\
0 & -\cos \theta_Y & 0
\end{bmatrix} \begin{bmatrix}
\dot{x}_r \\
\dot{y}_r \\
\dot{z}_r
\end{bmatrix} L + \begin{bmatrix}
\dot{x}_r \\
\dot{y}_r \\
\dot{z}_r
\end{bmatrix} \quad (13)
\]

The state equation of this system is obtained from eqs.(10), (11), and (13).

\[
\begin{bmatrix}
\dot{x}_r \\
\dot{y}_r \\
\dot{z}_r \\
\dot{\theta}_P \\
\dot{\theta}_Y
\end{bmatrix} = R_T \begin{bmatrix}
x_h \\
y_h \\
z_h \\
\dot{x}_h \\
\dot{y}_h \\
\dot{z}_h
\end{bmatrix} \quad (14)
\]

\[
= g_1 \dot{x}_h + g_2 \dot{y}_h + g_3 \dot{z}_h
\]

Where, the matrix \(R_T\) is \(5 \times 3\) and the vectors \(g_1, g_2\) and \(g_3\) are \(5 \times 1\). This is a drift-free affine system with the configuration of the object as the state variable and the translational velocities of \(H\) as control inputs.

Now, we prove this system is controllable and can reach any arbitrary state[7]. Lie bracket of the vector fields \(g_1, g_2\) and \(g_3\) is calculated as,

\[
\begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix} = \frac{\partial g_2}{\partial q} g_1 - \frac{\partial g_1}{\partial q} g_2
\]

Then,

\[
\text{rank}(g_1, g_2, g_3) = 5 \quad (16)
\]

As the matrix \((g_1, g_2, g_3)\) is of full rank from eq. (16), the system (14) satisfies Lie
algebra rank condition and is controllable. The object can reach any position and posture, treating only the translational velocity of the operator's hand.

Then, we explain the control law to implement the proposed method to the robot arm. The robot arm is assumed to have a force sensor at the wrist. First of all, we assign the following impedance characteristics to each axis considering the coordinates in Fig.7.

\[
\begin{align*}
 f_x &= m_x V_x + b_x V_x \\
 f_y &= m_y V_y + b_y V_y \\
 f_z &= m_z V_z + b_z V_z \\
 \tau_x &= i_x \dot{\theta}_x + c_x \theta_x \\
 \tau_y &= i_y \dot{\theta}_y + c_y \theta_y \\
 \tau_z &= i_z \dot{\theta}_z + c_z \theta_z
\end{align*}
\]

(17)

\( f_x, f_y, f_z, \tau_x, \tau_y, \tau_z \) are the force, \( V_x, V_y, V_z, \dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z \) are the velocity, \( V_x, V_y, V_z, \dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z \) are the acceleration for each axis. To achieve the constraint condition of eq. (10) and (11), the velocity and the acceleration in the \( Y \)- and \( Z \)-directions should be suppressed. Large inertia and viscous friction parameters are given to those directions. With regard to the translation of the \( X \)-direction and the rotation around the \( Y \) - and \( P \)-axes, a nearly free movement is preferable to reduce the reaction force to the operator. The inertia and viscosity parameters in those components should be set in small values.

The acceleration and the angular acceleration of the robot arm is,

\[
\begin{align*}
 \ddot{V}_x &= (f_x - b_x V_x) / m_x \\
 \dot{V}_y &= (f_y - b_y V_y) / m_y \\
 \ddot{V}_z &= (f_z - b_z V_z) / m_z \\
 \dot{\theta}_x &= (\tau_x - c_x \dot{\theta}_x) / i_x \\
 \dot{\theta}_y &= (\tau_y - c_y \dot{\theta}_y) / i_y \\
 \dot{\theta}_z &= (\tau_z - c_z \dot{\theta}_z) / i_z
\end{align*}
\]

(18)

from eq. (17). Integrating eq.(18), reference velocity of the robot hand is obtained. The reference velocity is converted into the joint angular velocity and given to the servo system of each joint. Thus, the control system of the virtual nonholonomic constraint can be realized.

4. Experiments

We verified that the human and robot can actually manipulate an object in 3-D space using the method in the previous section. We use a 7-axis industrial robot arm (PA-10, MHI) that has a 6-axis force/torque sensor at the wrist. A personal computer (CPU: Pentium 550Mz) is used for the controller. The sampling period is 2ms. The object is an aluminum pipe (length: 0.75m, mass: 0.7kg). The robot holds one end and the operator holds another end of the object. The operator and the robot transport the object cooperatively so that the object can reach the target position. Fig.8 shows a picture of the experiment.

We conducted experiments by the method of the rotation around the \( P \)- and \( Z \)-axes, and compared their behaviors. The impedance parameters are listed in Table 1. The impedance parameter of the constrained direction is about 20 times as large as that of the unconstrained direction. The initial position of the robot and the target position are \((x, y, z, \theta_x, \theta_y, \theta_z) = (-0.47, -0.47, 0.37, 0, 0, 0), (0.35, 0.2, 0.63, 0, 0, \pi / 2)\) in the absolute coordinate, respectively.

Fig.9 and Fig.10 show the experimental results by the rotation around the \( Z \) - and \( P \)-axes, respectively. Fig.11 shows the rotation angle around \( X \)-axis. By the method of the rotation around the \( Z \)-axis, a twist around the \( X \)-axis occurs when the object reaches the target position and posture. It is difficult to avoid the twist for the operator. On the other hand, by the method of the rotation around \( P \)-axis, the twist around the \( X \)-axis does not arise. The operator can easily understand the object behavior and can transport it to the target position and posture.

5. Conclusions

We extended a method for the human-robot cooperative transportation using the virtual nonholonomic constraint into the movement in a vertical plane. In this method, a virtual constraint equivalent to a wheel is assigned to the robot hand. Though the operator and robot cannot move the object in the perpendicular direction directly, he/she can transport it to an arbitrary position and inclination in the vertical plane. Then, we combined the constraints in a horizontal plane and a vertical plane so that the object can be carried to the desired position and posture in 3-D space. We experimentally verified the effectiveness of our method.

References


Table 1 Impedance parameters

<table>
<thead>
<tr>
<th>( m_x ) [Kg]</th>
<th>( b_x ) [Kg/s]</th>
<th>( m_y ) [Kg]</th>
<th>( b_y ) [Kg/s]</th>
<th>( m_z ) [Kg]</th>
<th>( b_z ) [Kg/s]</th>
<th>( i_y ) [Kgm²]</th>
<th>( c_y ) [Kgm²/s]</th>
<th>( i_p ) [Kgm²/s]</th>
<th>( c_p ) [Kgm²/s]</th>
<th>( i_z ) [Kgm²/s]</th>
<th>( c_z ) [Kgm²/s]</th>
</tr>
</thead>
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Fig.9 Experimental result (Rotation around Z-axis)

Fig.10 Experimental result (Rotation around P-axis)

Fig.11 Rotation angle around X-axis