Non-Parametric GNSS Integer Ambiguity Estimation via Positional Likelihood Field Marginalization

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***Abstract*— In this paper, we propose a non-parametric method for estimating the posterior distribution of global positioning satellite systems (GNSS) integer ambiguity. It is difficult to estimate the posterior probability of discrete integer ambiguities directly from carrier phase observations due to the unclear domain definition. We thus introduce a positional likelihood field that accumulates the ambiguity function method values in the position space and then estimate the integer ambiguity distributions by marginalizing the likelihood over the entire position. Defining the positional likelihood field in the position space facilitates carrier phase likelihood accumulation. To correctly estimate the posterior distribution, however, a sufficient density of samples is required, which results in a large computational cost. The proposed method enables large-scale sampling by taking advantage of GPU parallel processing. Experimental results demonstrate that the proposed method enables accurate and robust estimation of integer ambiguity distributions, contributing to improved centimeter-level position estimation accuracy. In addition, the histograms provide quantitative evidence of events in urban environments where integer ambiguity is not uniquely determined.**

I. INTRODUCTION

Global positioning satellite systems (GNSS) are commonly used for applications that require position information in outdoor environments. In particular, carrier phase based positioning can provide position information at the centimeter level. These systems are used for kinematic positioning and real time kinematic (RTK)-GNSS. When the carrier phase is used, the state variables to be estimated include position and integer ambiguity, which refers to the number of waves in the carrier wave. Because integer ambiguity is integer constrained, no general analysis method has been found. Therefore, many methods for solving integer ambiguity based on an exploratory approach have been proposed [1] [2].

A commonly used approach is integer least squares (ILS) [3]. ILS requires a non-integer ambiguity as the initial value. The initial value is then converted to an integer using the error function of the least-squares method. ILS typically requires an extensive search of the solution space for integer ambiguities. To solve this problem, methods have been proposed to minimize the computational cost by limiting the search area [4] [5].

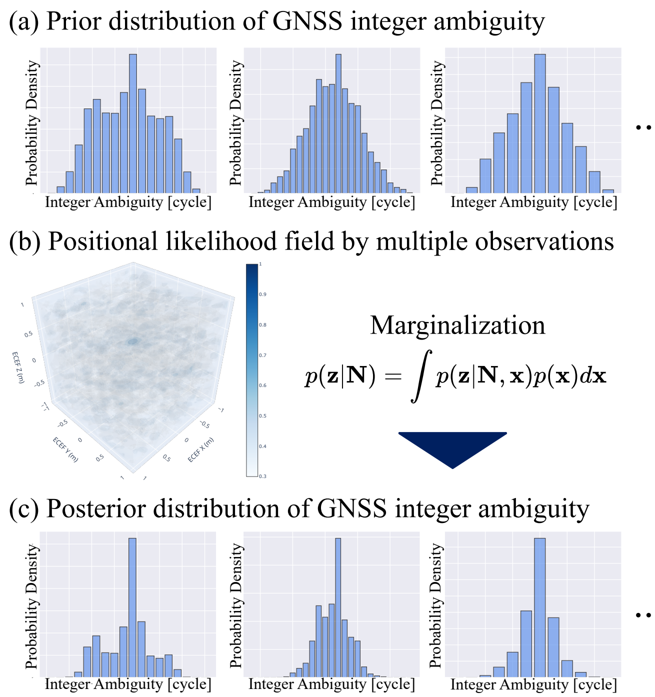

Fig. 1: Overview of proposed method for estimating integer ambiguity with positional likelihood field. (a) Prior distribution of integer ambiguity. (b) Positional likelihood field is constructed using grid based sampling and then marginalized to obtain likelihood distributions in integer ambiguity space. (c) Posterior distribution of integer ambiguity.

The multipath effect is a serious problem for a GNSS. Observation noise caused by the multipath effect is known to be non normally distributed. Therefore, approaches that assume normally distributed observations must eliminate those affected by the multipath effect. Similar to ILS, the least-squares method assumes a normal distribution and thus its performance degrades in environments where GNSS signals could be perturbed by the multipath effect (e.g., urban environments).

Ambiguity function methods (AFM) enable position estimation using only carrier phase information [6]–[8]. The AFM approach uses a formulation that removes integer ambiguity from the error function and evaluates likelihoods as a function of only receiver positions. Since the carrier phase observations can be aggregated in the position space, the position can be estimated using probability sampling in the position space. However, multiple local optimal solutions appear due to the high frequency of the carrier signal,

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which can lead to sub-optimal solutions. To obtain a globally optimal solution through AFM, it is essential to appropriately sample the position space with a wide search space.

In this study, we propose a method for estimating the probability distribution of integer ambiguity using non-parametric histograms. It is difficult to directly sample integer ambiguity probability given all satellite observations because each satellite has a different basis space for carrier phase observations. To overcome this challenge, we apply AFM to aggregate the carrier phase observations in the position space and sample integer ambiguity probability in the position space. Leveraging GPU parallel processing for large-scale sampling, our method constructs a positional likelihood field, including the globally optimal solution, with AFM. Finally, the probability distribution of integer ambiguity is obtained by marginalizing the positional likelihood field.

Fig. 1 shows an overview of the proposed method. Fig. 1(a) shows the prior distribution of integer ambiguity, which is treated as a uniform distribution with an unknown domain in the initial state. We construct a positional likelihood field using AFM through grid-based sampling (Fig. 1(b)). The posterior probability is estimated using Bayes' theorem by combining the observation likelihood, obtained through the marginalization of the positional likelihood field, with the prior distribution (Fig. 1(c)). As shown in Fig. 1, the proposed method can represent multimodal distributions due to its robust sampling and nonparametric approach. Consequently, the method can be understood as maintaining multiple hypotheses for integer ambiguity, each accompanied by probability information.

In this paper, we investigate the fundamental performance of the proposed method and compare it with several baseline methods. The proposed method enhances the accuracy of integer ambiguity estimation through dense resolution sampling. Furthermore, due to its ability to sample a histogram as wide range of the position space, the method contributes to improving both the frequency and accuracy of integer ambiguity estimation compared to the baseline methods.

II. RELATED WORK

A. Integer Ambiguity Estimation Method using ILS

Integer ambiguity estimation has been widely studied in the context of ambiguity resolution [9]–[11]. Methods that estimate integer values and approaches that solve the problem as a mixed-integer programming problem have been proposed. A common method is to convert non-integer ambiguity to integer values using the ILS method [12] [13]. Typically, the combination of integer ambiguities is strongly correlated due to the geometric configuration of satellites. As a result, the search space in ILS tends to be large.

The commonly used least-squares ambiguity decorrelation adjustment (LAMBDA) method can limit the search area by making it uncorrelated [5]. The LAMBDA method formulates the error function in terms of non-integer ambiguity (float ambiguity). It then searches for the combination of integer ambiguities with the smallest error function within a limited search range. However, since the error function of the LAMBDA method depends on the float ambiguity, the entire estimation fails if the float ambiguity has errors. Methods have been proposed to improve the accuracy of the float ambiguity [14], [15] and to remove satellites with the multipath effect [16], [17]. These methods are limited in their performance improvement due to the assumption of a strong normal distribution derived from the least-squares method and optimization based on a single float ambiguity.

B. Integer Ambiguity Free Method

AFM is an integer ambiguity free approach because it removes integer ambiguity from the error function. AFM is characterized by its ability to consider all combinations of integer ambiguities in the search range. Therefore, sampling on an arbitrary position space can be used to estimate the optimal solution in the space.

However, AFM has many local optima within the search range because it searches for all combinations of integer ambiguities. The local optima appear as sharp periodic peaks, which are derived from the wavelength of the carrier wave. With insufficient sampling, the global optimal solution can easily be missed, leading to incorrect estimation. Sampling must be sufficiently finer than the wavelength. On the other hand, in a multipath environment or when the number of observed satellites is small, a sufficiently large search area is required. Therefore, sampling with AFM requires wide-area and dense sampling, which increases the computational cost.

The multiple update particle filter has been proposed to avoid local optima with less sampling [18]. This method updates and resamples particles multiple times in descending order of the spread of the likelihood distribution. By weighting and resampling particles multiple times, it is possible to gradually shift particles to their true positions. However, the likelihood cannot be uniquely determined due to the multipath effect.

The multipath effect is thus a fundamental problem for any GNSS method and should not be handled by methods that assume a normal distribution. We propose a nonparametric method for estimating integer ambiguity using the carrier phase.

III. METHODOLOGY

A. Problem Formulation of Kinematic Positioning

Kinematic positioning using the carrier phase is formulated as a state estimation problem that solves for the receiver position and integer ambiguity. In this paper, we denote the carrier phase observation as $\boldsymbol{z}_t = \{\phi_t^i \in \mathbb{R}^k|_{i=1,\ldots,k}\}$, the integer ambiguity as $\boldsymbol{N}_t = \{N_t^i \in \mathbb{Z}^k|_{i=1,\ldots,k}\}$, and the receiver position in the Earth-centered, Earth-fixed (ECEF) coordinate system as $\boldsymbol{x}_t = \{x_t \in \mathbb{R}^3\}$. In GNSS positioning, observations at a given time t are obtained according to the number of observation satellites k. Carrier phase observations include ionospheric and tropospheric delay errors, generation and receiver clock errors, and initial phase bias. In general, these errors can be eliminated by computing the

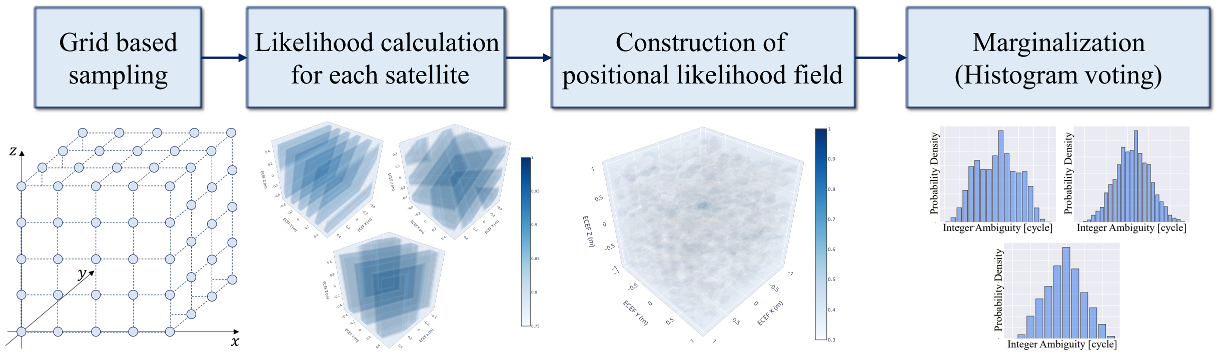

Fig. 2: Process flow of proposed method.

double difference (DD) of the observations. The DD-carrier observation is shown as follows.

$$\phi_t^i = \frac{1}{\lambda} r(\boldsymbol{x}_t) + N_t^i \tag{1}$$

Where λ is the wavelength of the carrier wave and $r(\boldsymbol{x})$ is the geometric distance between the satellite and receiver. As shown in Eq. 1, kinematic positioning is an overdetermined system. Therefore, it is difficult to uniquely solve for position and integer ambiguity from carrier phase observations.

B. Histogram Filter

Our objective is to obtain the posterior distribution $p(\boldsymbol{N}_t|\boldsymbol{z}_t)$ of the integer ambiguity from the given observations. The posterior distribution is obtained by applying Bayes Theorem.

$$p(\boldsymbol{N}_t|\boldsymbol{z}_t) \propto p(\boldsymbol{z}_t|\boldsymbol{N}_t)p(\boldsymbol{N}_t|\boldsymbol{z}_{t-1}) \tag{2}$$

Note that although Eq. 2 consists of only the observation $\boldsymbol{z}_t$ and the integer ambiguity $\boldsymbol{N}_t$, obtaining the likelihood $p(\boldsymbol{z}_t|\boldsymbol{N}_t)$ directly is challenging. This is because integer ambiguity lives in satellite-specific domain and difficult to formulate an expression for inter-satellite interference. Therefore, we focused on AFM's ability to construct a likelihood distribution in the positional space, and then estimate the likelihood $p(\boldsymbol{z}_t|\boldsymbol{N}_t)$ of integer ambiguity in the position space by marginalizing the likelihood in the positional space.

Fig. 2 shows the sequence of processing steps in the proposed method. The proposed method discretizes the positional space in a regular grid (i.e., histogram). The likelihood of integer ambiguity for each satellite is mapped to the positional cells using AFM. The likelihoods of all satellites are multiplied to construct a positional likelihood field that takes into account all observations. The desired likelihood $p(\boldsymbol{z}_t|\boldsymbol{N}_t)$ is obtained by marginalizing the positional likelihood field according to the integer ambiguity. The marginalization is performed by accumulating the likelihood values of the cells corresponding to integer ambiguities. Finally, Bayes' rule in Eq. 2 is used to obtain the posterior distribution of integer ambiguity.

C. Positional Likelihood Filed Marginalization

Conventional methods [18] use AFM to express the observation likelihood using only position from carrier phase observations. The proposed method constructs a positional likelihood field by calculating the observation likelihood on the grid as shown in Fig. 2. The observation likelihood is calculated using AFM as follows:

$$\psi(\phi_t^i, \boldsymbol{x}_t) = \mathrm{round}\left(\phi_t^i - \frac{1}{\lambda} r(\boldsymbol{x}_t)\right) - \left(\phi_t^i - \frac{1}{\lambda} r(\boldsymbol{x}_t)\right) \tag{3}$$

The proposed method calculates the likelihood of each sample using a gaussian kernel with the following equation.

$$p(\phi_t^i|\boldsymbol{x}_t) = \frac{1}{\sqrt{2\pi\sigma_\phi^2}} \exp\left(-\frac{1}{2}\frac{\psi(\phi_t^i, \boldsymbol{x}_t)^2}{\sigma_\phi^2}\right) \tag{4}$$

Where σ_ϕ^2 is the noise parameter of the gaussian kernel. The likelihood of all satellites is expressed as the product of the individual likelihoods of each satellite.

$$p(\boldsymbol{z}_t|\boldsymbol{x}_t) = \prod_{i=1}^{k} p(\phi_t^i|\boldsymbol{x}_t) \tag{5}$$

Using Eq. 5, we can construct the positional likelihood field by computing the observation likelihood for all position samples on the grid. This positional likelihood field indicates the likelihood at position $\boldsymbol{x}$. Integer ambiguity N^i corresponding to position $\boldsymbol{x}$ is obtained as follows:

$$N_t^i = \mathrm{round}\left(\phi_t^i - \frac{1}{\lambda} r(\boldsymbol{x}_t)\right) \tag{6}$$

These relationships allow us to map the positional likelihood field to the observation likelihood $p(\boldsymbol{z}_t|\boldsymbol{N}_t)$ mediated by position $\boldsymbol{x}$. In other words, by appropriately marginalizing the positional likelihood field, we can estimate the required observation likelihood $p(\boldsymbol{z}_t|\boldsymbol{N}_t)$. This idea can be expressed as follows:

$$p(\boldsymbol{z}_t|\boldsymbol{N}_t) = \int p(\boldsymbol{z}_t|\boldsymbol{x}_t)p(\boldsymbol{x}_t)d\boldsymbol{x}_t \tag{7}$$

Eq. 7 shows that the desired observation likelihood $p(\boldsymbol{z}_t|\boldsymbol{N}_t)$ can be obtained by sampling the positions. $p(\boldsymbol{x}_t)$ shows the distribution of the samples, which is assumed to be uniform distribution in the proposed method.

To include the global optimal solution within the position likelihood field, wide-area and dense sampling is required. For example, if one axis is divided into 100 segments of 2 m each (i.e., with a resolution of 0.02 m), there would be 1 million (100^3) sampling points in the position space. The large computational cost of processing 1 million sampling points on a CPU makes this impractical. To overcome this problem, the proposed method takes advantage of the parallel processing power of GPU. The processing per sample is very simple, as all that is required is the AFM calculation. Therefore, it is expected that all samples can be processed quickly by parallel processing on GPU.

D. State Transition

Based on Eq. 2, a prior distribution is required to obtain integer ambiguity. The proposed method obtains the prior distribution as follows:

$$p(\boldsymbol{N}_t|\boldsymbol{z}_{t-1}) = \int p(\boldsymbol{N}_t|\boldsymbol{N}_{t-1})p(\boldsymbol{N}_{t-1}|\boldsymbol{z}_{t-1})d\boldsymbol{N}_{t-1} \tag{8}$$

Eq. 8 represents the state prediction model, where $p(\boldsymbol{N}_t|\boldsymbol{N}_{t-1})$ denotes the state transition probability. The state transition probability follows a constant model, as the integer ambiguity remains unchanged unless the satellite signal is interrupted.

The proposed method initializes the estimated distribution when satellite observations are lost (i.e., the integer ambiguity changes significantly) or the anomaly flag (loss of lock indicator: LLI) is output by the receiver.

E. Positioning via Maximum a Posterior

We first extract representative values of integer ambiguity $\tilde{\boldsymbol{N}}_t$ using the most basic maximum a posterior as follows:

$$\tilde{\boldsymbol{N}}_t = \arg\max_{\boldsymbol{N}_t} p(\boldsymbol{N}_t|\boldsymbol{z}_t) \tag{9}$$

When $\tilde{\boldsymbol{N}}_t$ is obtained, the position can be estimated using Eq. 1. The proposed method employs the Gauss-Newton method to obtain the position $\tilde{\boldsymbol{x}}_t$ according to the following objective function.

$$\tilde{\boldsymbol{x}}_t = \arg\min_{\boldsymbol{x}_t} \sum_{i=1}^{k} \rho \left\{ \phi_t^i - \left(\frac{1}{\lambda} r(\boldsymbol{x}_t) + \tilde{N}_t^i \right) \right\}^2 \tag{10}$$

Where ρ is Huber's robust kernel, and the proposed method removes simple outlier.

IV. EVALUATION

A. Static Test

The proposed method is tested in static setup for basic investigation. The experiment used data (500 epochs) from base stations located in Tsukuba City, Ibaraki Prefecture, Japan. The proposed method calculated the position error for various sampling resolutions and values of Gaussian kernel noise parameter σ_ϕ^2. In addition, we investigated how the computational cost of the proposed method changes with the number of samples. The parameters used in the tests are shown in Table I. The sampling grid origin of the proposed method is set to reference position of base station at Tsukuba station.

TABLE I: Experimental parameters for static test

Grid Resolution [m]	Noise Parameter σ_ϕ^2 [m^2]
0.02	0.001 / 0.005 / 0.01
0.05	0.001 / 0.005 / 0.01
0.10	0.001 / 0.005 / 0.01
0.15	0.001 / 0.005 / 0.01

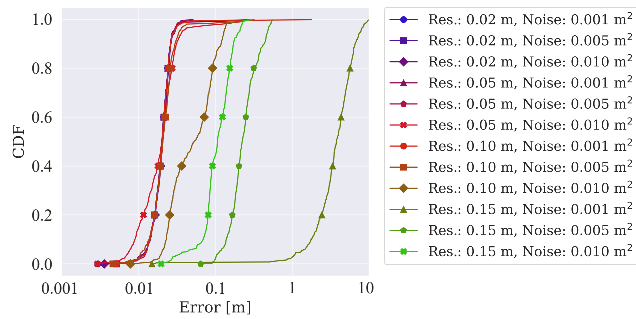

Fig. 3: Results of static test for various combinations of parameter values.

Fig. 3 shows the cumulative distribution function (CDF) of the position error for the proposed method. For the majority of combinations of resolution and noise values, more than 95% of the estimation errors are less than 0.1 m, demonstrating that the correct integer ambiguity is estimated by the proposed method. For a resolution of 0.15 m and a combination of a resolution of 0.1 m and a noise parameter σ_ϕ^2 of 0.01 m^2, lower positional accuracy is achieved. The results indicate that a finer grid resolution generally leads to better accuracy.

Fig. 4 shows estimate integer ambiguity histograms for a specific satellite (G03) with different grid resolutions. The noise parameter σ_ϕ^2 was set to 0.005 m^2. When the resolution is high (0.02 m), the integer ambiguity is uniquely determined. On the other hand, when the resolution is coarse, the estimated histograms exhibit ambiguous distributions implying the integer ambiguity is not uniquely determined. We consider that the histograms with course resolutions failed to capture the sharp peaks of the AFM caused by the high frequency of the carrier phase. We emphasize that, as seen in Fig. 4, our method yields the estimated integer ambiguity distributions in the form of non-parametric histograms that can inherently represent complex multi-modal distributions. As a consequence, we can easily identify satellites affected by outliers by observing the dispersed histograms.

We evaluate the computational cost of the proposed method. We created two implementations of the proposed method respectively using CPU (Intel Core i9-14900KF) and GPU (NVIDIA GeForce RTX 4080), and measured the processing times for histogram sizes of 100^3 to 300^3. Table 2 shows the measured processing times. The processing time of the proposed method increases at the rate of the third power when the number of samples per axis is increased. The CPU

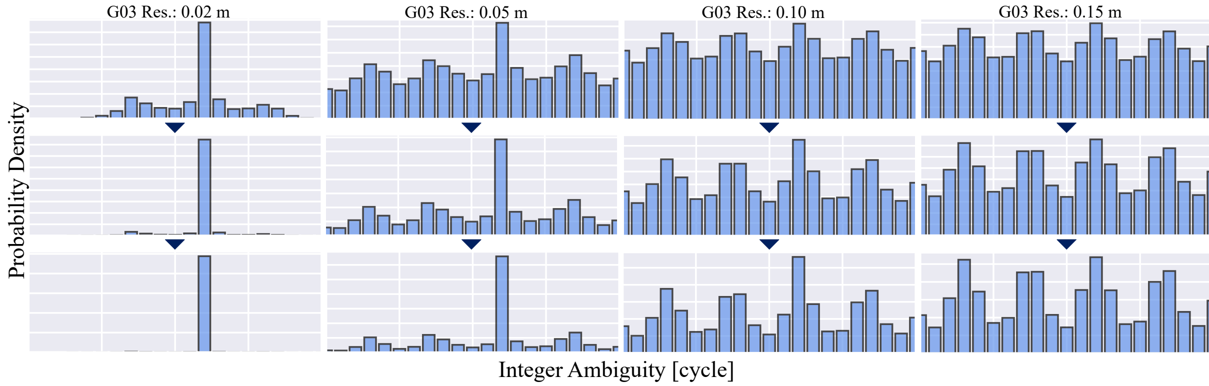

Fig. 4: Histograms of integer ambiguity for various sampling resolutions. Time progresses from the top to the bottom.

TABLE II: Results of processing time for CPU and GPU

Implementation	Histogram size	Processing Time [ms]
CPU	100^3	1179.48 $\pm$ 37.55
	200^3	18160.29 $\pm$ 954.26
	300^3	56978.55 $\pm$ 4784.3
GPU	100^3	66.08 $\pm$ 9.95
	200^3	222.16 $\pm$ 19.87
	300^3	669.96 $\pm$ 60.11

TABLE III: Mean errors of float solution.

Sequences	Mean error [m]
Suburban	0.81 $\pm$ 0.47
Urban 1	2.35 $\pm$ 4.52
Urban 2	6.42 $\pm$ 20.0
Urban 3	1.42 $\pm$ 2.46

implementation has a considerable processing time. On the other hand, the GPU implementation can process a histogram with 100^3 bins at 10 Hz and one with 300^3 bins at 1 Hz. These results indicate that the proposed method is simple on a per sample basis and can thus fully utilize the parallel processing power of the GPU.

B. Kinematic Test

In this section, the proposed method is evaluated using real-world environmental data recorded with an on-vehicle setup. The evaluation uses publicly available datasets [19] [20]. The test environments were a suburban area (Suburban) and three urban areas (Urban 1, Urban 2, Urban 3). As a baseline, we applied methods to solve the integer ambiguity as well as the proposed method. Two baseline methods are selected, one is RTKLIB [21] and the other is Demo5 [22], which is an improved version of RTKLIB.

The test conditions for the proposed method were as follows. The grid resolution is set to 0.02 m and the number of samplings is 300 per axis. The proposed method thus searched an area of 6 m per axis centered on the float solution. In these experiments, the Gaussian kernel noise parameter σ_ϕ^2 in the proposal method was set to 0.001 m^2. The center of the sampling position space for the proposed method was the float solution estimated by RTKLIB. The

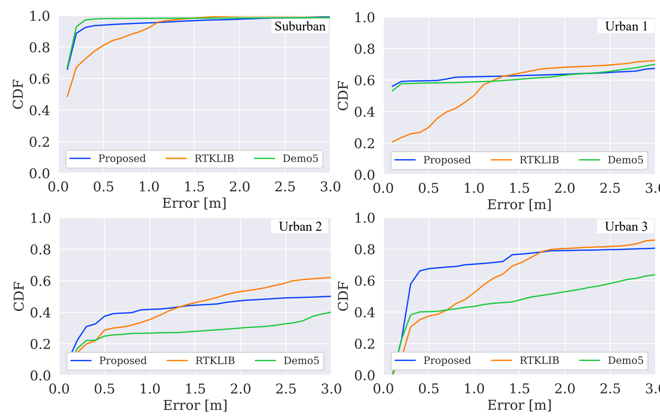

Fig. 5: Kinematic test results for suburban and urban environments.

TABLE IV: Cumulative error whitin 0.5 m.

Sequence	Cumulative frequency [%]		
	RTKLIB [21]	Demo5 [22]	Proposed
Suburban	81.2	**98.1**	94.0
Urban 1	29.9	58.2	**60.0**
Urban 2	28.7	24.9	**37.5**
Urban 3	37.4	40.2	**67.6**

baseline methods used the same conditions as those for the proposed method to search for integer ambiguity around the float solution. Table 3 summarizes the mean errors of the float solution for each sequence.

Fig. 5 shows the CDF of the position estimation error, and Table 4 summarizes the cumulative error within 0.5 m for each method. In this test, we focus on errors within 0.5 m, for which the integer ambiguity can be judged to be approximately correctly solved. In the suburban environment test, for the proposed method, more than 94% of the errors are within 0.5 m; this value is slightly less than that for Demo5 (98.1%). In the urban environment test, the proposed method is superior to the two baseline methods.

The above results are summarized as follows.

- When the accuracy of the float solution is high, the performance of the proposed method is equivalent to that of conventional methods, which can also easily find

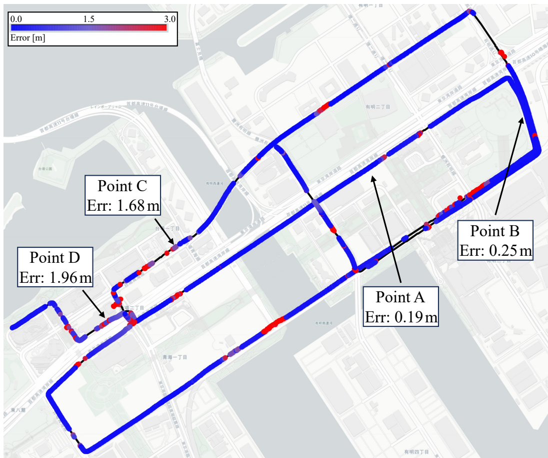

Fig. 6: Driving route of Urban 3 and position error results of proposed method. Four points were selected for the histogram investigation. The error for each point is shown.

Fig. 7: Histogram of estimated integer ambiguity.

the solution despite their limited search space.

- On the other hand, when the float solution deteriorates, existing methods struggle to find correct solutions from observations affected by the multipath effect, whereas the proposed method can find the solutions owing to its wide search range and non-parametric sampling.

Fig. 5 shows that the proposed method is inferior to the baseline methods when the error is 1.5 m or larger. The baseline methods, but not the proposed method, use a pseudorange in addition to the carrier phase. The benefit of a pseudorange appears only at the decimeter level and the proposed method has difficulty estimating positions at the meter level. The extension of the proposed method to include a pseudorange will be considered in a future study.

We further investigate the histogram of the integer ambiguity estimated using the proposed method. Fig. 6 shows the driving path of Urban 3. The error of the proposed method is shown as a heat map on the path. The histograms of integer ambiguity were investigated at four locations. The selected points are those with small and large errors. We focus on satellites G10, G25, and G29, which were received at all four points.

Fig. 7 shows the estimated posterior distributions of the integer ambiguities at each point. Points A and B in Fig. 7 have small positional errors. These points show that the integer ambiguity is uniquely determined in this environment. The probability of two integer ambiguity candidates is high at point A, for satellite G25. This is thought to be due to the half-cycle slip, in which the carrier phase measurement is shifted by half a wavelength. Points C and D in Fig. 7 have large errors due to surrounding buildings. These results indicate that there are several candidates for integer ambiguity and that it is difficult to obtain a unique solution. We consider that this is due to the inclusion of noise caused by the multipath effect in the positional likelihood field.

It is worth emphasizing that existing methods can only express the estimated distributions in the form of Gaussian distributions (mean and covariance) and thus have difficulty in detailed analysis, such as that demonstrated with the proposed method. With the Gaussian distribution representation, it is difficult, if not impossible, to properly propagate and represent the uncertainty of the integer ambiguity distributions. Even under conditions with the multipath effect, the proposed method can inherently propagate the uncertainty of ambiguity probability distributions while retaining their nonlinear multi-modal characteristics. In the future, we will attempt to use these probability distributions to constrain the multipath effect.

V. CONCLUSIONS

In this study, we proposed a nonparametric approach for estimating the probability distribution of integer ambiguity. It is difficult to sample and integrate integer ambiguities between satellites because integer ambiguity measurements are defined in satellite-specific domains. We applied AFM to sampling on the position space to estimate the posterior distribution of integer ambiguity. The evaluation results indicate that the estimated integer ambiguity can be determined uniquely when there is minimal disturbance, while maintaining its nonlinear and multimodal characteristics under the influence of the multipath effect. The proposed method outperforms baseline methods in urban environments, primarily due to the higher frequency of correct integer ambiguity estimation, which leads to improved positional accuracy.

In the future, we will apply the estimated probability distribution of integer ambiguity to find a method for accurately determine integer ambiguity even in a multipath environment.

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