Fluctuation-induced Nambu-Goldstone bosons in a Higgs-Josephson model

Takashi Yanagisawa and Yasumoto Tanaka
Electronics and Photonics Research Institute, National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba Central 2, 1-1-1 Umezono, Tsukuba 305-8568, Japan

We present a new mechanism of fluctuation-induced Nambu-Goldstone bosons in a scalar field theory of Higgs-Josephson systems. We consider a simple scalar field model with $U(1)^n$ rotational symmetry. When there is an interaction which violates the rotational symmetry, the Nambu-Goldstone bosons become massive and massless bosons are concealed. We present a model where the massive boson becomes a massless boson as a result of the perturbative fluctuation. In our model the $\mathbf{Z}_2$-symmetry associated with the chirality is also broken. The transition occurs as a weak first-order transition at the critical point. The ground state at absolute zero will flow into the state with more massless bosons due to fluctuation effects at finite temperature.

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Introduction When global and continuous symmetries are spontaneously broken, gapless excitation modes, called the Nambu-Goldstone bosons, exist and govern the long-distance behaviors of the system[1–3]. When the $U(1)$ rotational symmetry is spontaneously broken, there is a massless Nambu-Goldstone boson. When there is an interaction that violates the $U(1)$ symmetry, we have no massless boson. An interesting question is whether such an interaction will continuously conceal the Nambu-Goldstone bosons when the perturbative corrections are taken into account. We present a model that exhibits a fluctuation induced Nambu-Goldstone boson in this paper. This means that a massless boson appears inspite of an interaction that hides Nambu-Goldstone bosons. We propose the mechanism of fluctuation induced Nambu-Goldstone boson.

We consider a model of an n-component scalar field with Josephson interactions, so called the Higgs-Josephson model[4–7]. Let us consider the action given as

$$\begin{aligned} S &= \frac{1}{k_BT}\int d^dx\sum_j\left(\alpha_j|\phi_j|^2+\frac{\beta_j}{2}|\phi_j|^4\right) \\ &+ \frac{1}{k_BT}\int d^dx\Big[\sum_j K_j|\nabla\phi_j|^2+\sum_{i\neq j}\gamma_{ij}\phi_i^*\phi_j\Big], \end{aligned} \quad (1)$$

where $\phi\equiv(\phi_1,\cdots,\phi_n)$ is a complex n-component scalar field. We write ϕ_j as

$$\phi_j = e^{i\theta_j}|\phi_j| = e^{i\theta_j}\rho_j, \quad (2)$$

where ρ_j $(j=1,\cdots,n)$ are real scalar fields. The last term in the action is the Josephson term. We assume that γ_{ij} are real and $\gamma_{ij}=\gamma_{ji}$. Without this interaction, the phase modes θ_j $(j=1,\cdots,n)$ represent massless modes. Because of this term, we have $n-1$ phase massive modes and one massless mode as shown by expanding $\cos(\theta_i-\theta_j)$ in terms of $\theta_i-\theta_j$. We adopt that β_j is positive so that the action has a minimum. When α_j is negative, ρ_j takes a finite value at the minimum of the potential. We set this value as Δ_j and write $\rho_j=\Delta_j+H_j$. H_j is the Higgs field and represents fluctuation of the scalar field around the minimum Δ_j. We simply assume that $K=K_j$, $\Delta=\Delta_j$ and $\gamma_{ij}=\gamma_{ji}\equiv\gamma$. Then the action for the phase variables θ_j is

$$S[\theta] = \frac{\Lambda^{d-2}}{t}\int d^dx\left(\sum_j(\nabla\theta_j)^2+\lambda\Lambda^2\sum_{i<j}\cos(\theta_i-\theta_j)\right), \quad (3)$$

where $t/\Lambda^{d-2}=k_BT/(K\Delta^2)$ and $\lambda\Lambda^2=\gamma/K$. We have introduced the cutoff Λ so that t and λ are dimensionless parameters. We assume that $\lambda>0$ in this paper. We now focus on θ_j and consider the case $n=3$. Since the potential term is written as

$$V \equiv (\lambda\Lambda^d/t)(\cos(\theta_1-\theta_2)+\cos(\theta_2-\theta_3)+\cos(\theta_3-\theta_1)), \quad (4)$$

the mode of the total phase $\theta_1+\theta_2+\theta_3$ remains massless. We do not consider this mode because the coupling to the gauge field turns this mode into a gapped mode (Higgs mechanism). The other $n-1$ modes do not become massive by the coupling to the gauge field. Let us consider the case $\lambda>0$. As is easily shown, the ground state of V has a $2\pi/3$ structure, namely, $\theta_2-\theta_1=2\pi/3$ and $\theta_3-\theta_2=2\pi/3$ as shown in Fig.1(a). The state in Fig.1(b) has also the same energy. Two states are indexed by the chirality $\kappa=1$ for (a) and $\kappa=-1$ for (b), where κ is defined by $\kappa=(2/3\sqrt{3})(\sin(\theta_2-\theta_1)+\sin(\theta_3-\theta_2)+\sin(\theta_1-\theta_3)$[8–15]. We set $\varphi_1=\theta_3-\theta_1$ and $\varphi_2=\theta_1-2\theta_2+\theta_3$ to write the potential density as

$$V = \frac{\lambda\Lambda^d}{t}\left(\cos(\varphi_1)+2\cos\left(\frac{\varphi_1}{2}\right)\cos\left(\frac{\varphi_2}{2}\right)\right). \quad (5)$$

V has a minimum at $\varphi_1=4\pi/3$ and $\varphi_2=0$. We mention here that an S_3 symmetry of the Josephson potential is not lost when we express the potential in terms of φ_1 and φ_2. When V has a minimum at some value of $\varphi_1=\theta_3-\theta_1$, V has also a minimum when $\theta_3-\theta_2$ takes the same value (modulo 2π). When the former has the chirality $\kappa=1$, the latter has $\kappa=-1$. We consider the fluctuation around this minimum. For this purpose, we perform a unitary transformation by defining $\varphi_1=$

$4\pi/3 + \sqrt{2}\eta_1$ and $\varphi_2 = \sqrt{6}\eta_2$:

$$\theta_1 = -\frac{2\pi}{3} - \frac{1}{\sqrt{2}}\eta_1 + \frac{1}{\sqrt{6}}\eta_2 + \frac{1}{\sqrt{3}}\eta_3, \quad (6)$$

$$\theta_2 = -\frac{2}{\sqrt{6}}\eta_2 + \frac{1}{\sqrt{3}}\eta_3, \quad (7)$$

$$\theta_3 = \frac{2\pi}{3} + \frac{1}{\sqrt{2}}\eta_1 + \frac{1}{\sqrt{6}}\eta_2 + \frac{2}{\sqrt{3}}\eta_3. \quad (8)$$

where η_i $(i = 1, 2, 3)$ indicate fluctuation fields. η_3 describes the total phase mode, $\eta_3 = (\theta_1+\theta_2+\theta_3)/\sqrt{3}$, and is not important in this paper because this mode turns out to be a plasma mode by coupling with the long-range Coulomb potential. We obtain $\sum_i(\nabla\theta_i)^2 = \sum_i(\nabla\eta_i)^2$, and then the action $S[\eta] \equiv S[\theta]$ is

$$S[\eta] = \frac{\Lambda^{d-2}}{t}\int d^dx\Big[\sum_j(\nabla\eta_j)^2 + \lambda\Lambda^2\Big(\cos\left(\sqrt{2}\eta_1 + \frac{4\pi}{3}\right) + 2\cos\left(\frac{1}{\sqrt{2}}\eta_1 + \frac{2\pi}{3}\right)\cos\left(\sqrt{\frac{3}{2}}\eta_2\right)\Big)\Big]. \quad (9)$$

The potential term has a minimum at $\eta_1 = \eta_2 = 0$. Both of η_1 and η_2 represent massive modes with mass $3\lambda/(2t)$.

Fluctuation induced Nambu-Goldstone boson The potential V corresponds to the potential of a two-dimensional XY model on the triangular lattice with a frustrated interaction. The ground state has an well known $2\pi/3$-structure. We consider the role of fluctuation and show the existence of fluctuation-induced massless mode. We examine the following free-energy density by neglecting the kinetic term:

$$f = k_BT\frac{\lambda\Lambda^d}{t}\Big[\cos\left(\sqrt{2}\eta_1 + \frac{4\pi}{3}\right) + 2\cos\left(\frac{1}{\sqrt{2}}\eta_1 + \frac{2\pi}{3}\right)\cos\left(\sqrt{\frac{3}{2}}\eta_2\right)\Big]. \quad (10)$$

The partition function is given by

$$Z = \int d\eta_1 d\eta_2 \exp\left(-\frac{F}{k_BT}\right), \quad (11)$$

for the free energy functional F. Using the formula for the modified Bessel function,

$$I_0(z) = \frac{1}{\pi}\int_0^\pi e^{z\cos\varphi}d\varphi, \quad (12)$$

we have, by using $\varphi_2/2 = \sqrt{3/2}\eta_2$,

$$\int_0^{2\pi} d\varphi_2 \exp\Big[-\frac{2\lambda\Lambda^d}{t}\cos\left(\frac{\varphi_1}{2}\right)\cos\left(\frac{\varphi_2}{2}\right)\Big] = 2\pi I_0\left(\frac{2\lambda\Lambda^d}{t}\cos\left(\frac{\varphi_1}{2}\right)\right). \quad (13)$$

We use $I_0(-x) = I_0(x)$ and the asymptotic formula $I_0(z) \sim e^z/\sqrt{2\pi z}$ $(z > 0)$ at low temperature. Then the effective free-energy density for η_1 is

$$\frac{f[\eta_1]}{\Lambda^d} = \epsilon_0\cos\left(\sqrt{2}\eta_1 + \frac{4\pi}{3}\right) - 2\epsilon_0\left|\cos\left(\frac{1}{\sqrt{2}}\eta_1 + \frac{2\pi}{3}\right)\right| + \frac{1}{2}\frac{k_BT}{\Lambda^d}\ln\left(\frac{\lambda\Lambda^d}{\pi t}\left|\cos\left(\frac{1}{\sqrt{2}}\eta_1 + \frac{2\pi}{3}\right)\right|\right), \quad (14)$$

where $\epsilon_0 \equiv k_BT\lambda/t$. We have an effective entropy term being proportional to the temperature T. $f[\eta_1]$ has a minimum at $\eta_1 = 0$ ($\varphi_1 = 4\pi/3$) at absolute zero $T = 0$. In contrast, at finite temperature $T > 0$, the minimum is at $\eta_1 = -\sqrt{2}\pi/6$ and $\varphi_1 = \pi$. This is shown in Fig.2 where we present the potential $f[\eta_1]/\epsilon_0$ as a function of $\varphi \equiv \varphi_1 = 4\pi/3 + \sqrt{2}\eta_1$ for $t = \lambda$ with setting $\Lambda = 1$. At $\varphi = \pi$, η_2 becomes a massless boson because the free-energy density in eq.(10) becomes independent of η_2 with vanishing of the mass term. This is due to the fluctuation of η_2 field at finite temperature. The qualitatively same result is obtained by the Gaussian integration with respect to η_2 after expanding cosine function as $\cos(\sqrt{3/2}\eta_2) = 1 - (3/4)\eta_2^2 + \cdots$ and assuming that $\cos(\eta_1/\sqrt{2} + 2\pi/3) < 0$. (We can use the formula $I_0(z) \sim e^z/\sqrt{2\pi z}$ when $z > 0$ is large. In the limit $z \to 0$, we have a largee entropy coming from the volume of the phase space and thus the minimum is at $\varphi_1 = \pi$ for $T > 0$ when T is higher than a critical value.) This state is shown in Fig.1(c) using a spin analogue where we have two antiferromagnetic spins and one vanishing spin. This means that the η_2-mode is massless and $\varphi_2 = \sqrt{6}\eta_2 = \theta_1 - 2\theta_2 + \theta_3$ can take any value. At the absolute zero, we have the index of chirality $\kappa = \pm 1$ as shown in Figs.1(a) and 1(b). The chirality disappears at finite temperature leading to the emergency of a Nambu-Goldstone boson. This represents a phenomenon that the Nambu-Goldstone boson appears due to a fluctuation effect.

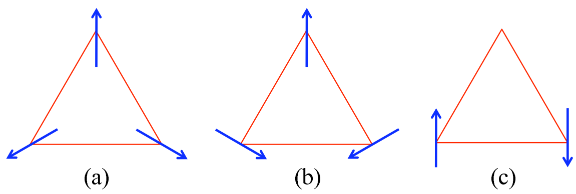

FIG. 1: (a) $2\pi/3$-structure in the ground state with the chirality $\kappa = 1$. (b) Degenerate ground state with the chirality $\kappa = -1$. (c) Spin structure at finite temperature. Two spins are antiferromagnetically aligned and one spin vanishes, that is, the expectation value vanishes: $\langle \mathbf{S}\rangle = 0$. This means that the one spin is rotating freely, indicating the existence of a massless boson.

Phase transition at finite temperature We next consider the kinetic terms of η_j. For this purpose, we use the

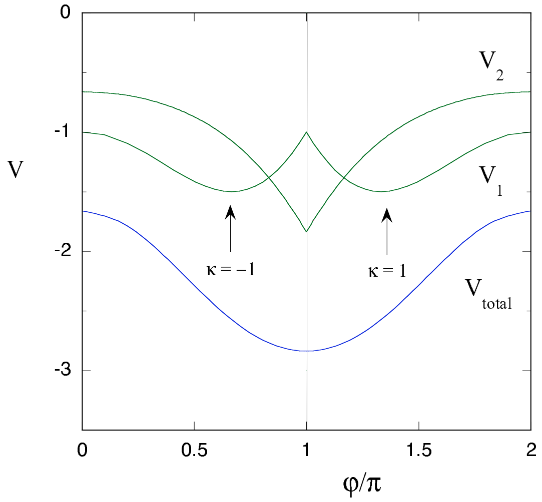

FIG. 2: Potential terms V_1, V_2 and $V_{total} = V_1 + V_2$ as a function of $\varphi \equiv 4\pi/3 + \sqrt{2}\eta_2$ for $t/\lambda = 1$ and $\Lambda = 1$. $V_1 = \cos(\varphi) - 2|\cos(\varphi/2)|$ and $V_2 = 2|\cos(\varphi/2)| - (t/\lambda)\ln(2\pi I_0(2(\lambda/t)|\cos(\varphi/2)|))$. V_1 and V_2 are symmetric with respect to the axis of $\varphi = \pi$. The aymptotic form of $\epsilon_0(V_1 + V_2)$ agrees with eq.(14). The total potential $V_{total} = V_1 + V_2$ has a minimum at $\varphi = \pi$ due to the logarithmic term. Minima of V_1 correspond to the state of chirality $\kappa = 1$ and $\kappa = -1$, respectively.

expansion of cosine term and write the action in the form

$$S = \frac{\Lambda^{d-2}}{t}\int d^dx\Big[\sum_j(\nabla\eta_j)^2 + \lambda\Lambda^2\Big(\cos\left(\sqrt{2}\eta_1 + \frac{4\pi}{3}\right) - 2\Big|\cos\left(\frac{1}{\sqrt{2}}\eta_1 + \frac{2\pi}{3}\right)\Big|\Big) + \frac{3\lambda\Lambda^2}{2}\Big|\cos\left(\frac{1}{\sqrt{2}}\eta_1 + \frac{2\pi}{3}\right)\Big|\eta_2^2\Big]. \quad (15)$$

When $\cos(\varphi_1/2) < 0$, we use $\cos(\sqrt{3/2}\eta_2) = 1-(4/3)\eta_2^2 + \cdots$. Around the minimum at $\varphi_1 = 2\pi/3$ and $\varphi_2 = 2\pi$ (with chirality $\kappa = -1$), we use instead the expansion by defining $\varphi_2 = 2\pi + \sqrt{6}\eta_2$. We integrate out the field η_2 to obtain the effective action, using $\varphi \equiv \varphi_1 = 4\pi/3 + \sqrt{2}\eta_1$,

$$S = \frac{\Lambda^{d-2}}{t}\int d^dx\Big[\frac{1}{2}(\nabla\varphi)^2 + \lambda\Lambda^2\Big(\cos\varphi - 2\Big|\cos\left(\frac{\varphi}{2}\right)\Big|\Big)\Big] + \frac{1}{2}Tr\ln\left(-\frac{\Lambda^{d-2}}{t}\nabla^2 + \frac{3\lambda}{2t}\Lambda^2\Big|\cos\left(\frac{\varphi}{2}\right)\Big|\right). \quad (16)$$

When we neglect the kinetic term $-\nabla^2$, this action is reduced to the previous effective free energy. We adopt that the spatial variation of φ field is very slow so that we can perform the $\mathbf{k}$-summation for $-\nabla^2 = \mathbf{k}^2$. In the two-dimensional case ($d = 2$), the effective free-energy density is obtained as

$$\frac{f[\varphi]}{\Lambda^2} = \frac{1}{2}K\Delta^2\Lambda^{-2}(\nabla\varphi)^2 + \epsilon_0\Big(\cos\varphi - 2\Big|\cos\left(\frac{\varphi}{2}\right)\Big|\Big) + \frac{1}{2}k_BT\frac{c}{4\pi}\ln\left(\frac{c\Lambda^d}{t} + \frac{3\lambda\Lambda^d}{2t}\Big|\cos\left(\frac{\varphi}{2}\right)\Big|\right) + k_BT\frac{3\lambda}{16\pi}\Big|\cos\left(\frac{\varphi}{2}\right)\Big|\ln\Big(1 + \frac{2c}{3\lambda}\Big|\cos\left(\frac{\varphi}{2}\right)\Big|^{-1}\Big), \quad (17)$$

where we have chosen the cutoff k_0 in the momentum space as $k_0^2 = c\Lambda^2$ for a constant c.

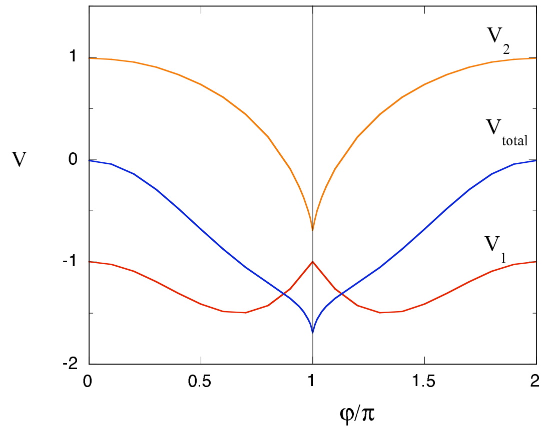

FIG. 3: Potential terms V_1, V_2 and $V_{total} = V_1 + V_2$ as a function of $\varphi \equiv 4\pi/3 + \sqrt{2}\eta_2$ for $t = 8\pi$ and $\lambda/c = 1$ where we set $c = 4\pi$ to compare with V in Fig.2. V_1 is the same as that in Fig.2 and V_2 is $V_2 = (t/2\lambda)(c/4\pi)\ln(c/t + (3\lambda/2t)|\cos(\varphi/2)|) + (3t/16\pi)|\cos(\varphi/2)|\ln(1+(2c/3\lambda)|\cos(\varphi/2)|^{-1})$. The total potential $V_{total} = V_1 + V_2$ has a minimum at $\varphi = \pi$.

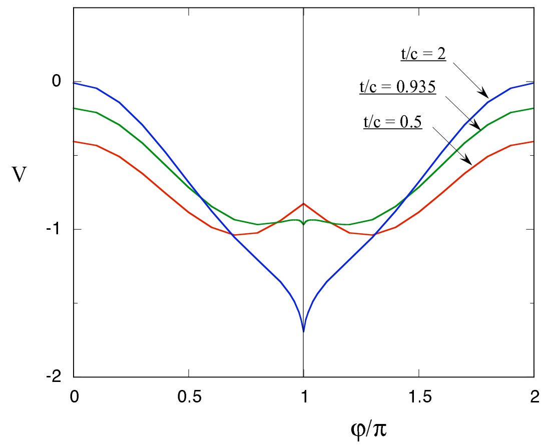

FIG. 4: Potential as a function of φ for $t/c = 0.5$, 0.935 and 2, respectively, where we set $\lambda/c = 1$ and $c = 4\pi$.

The spatial fluctuation softens the thermal fluctuation effect and there is a finite critical temperature where the minimum at $\varphi = 4\pi/3$ disappears and simultaneously the chirality vanishes. We show the potential term as a function of φ for $t = 2c$ and $\lambda = c$ with $c = 4\pi$ in Fig.3 where we subtracted the term $k_BT/2\ln\Lambda^d$ which is independent of φ (or equivalently we set $\Lambda = 1$). We have a minimum at $\varphi = \pi$ when t is large as shown in Fig.3. The critical temperature t_c is scaled by λ/c:

$$t_c = t_c(\lambda/c). \tag{18}$$

t_c is estimated by the equation $V(\varphi = 4\pi/3) = V(\varphi = \pi)$, which gives

$$\frac{k_BT_c}{K\Delta^2} = t_c = \frac{\lambda}{\frac{c}{4\pi}\ln\left(1+\frac{3\lambda}{4c}\right) + \frac{3\lambda}{16\pi}\ln\left(1+\frac{4c}{3\lambda}\right)}. \tag{19}$$

For small $\lambda \to 0$, t_c is small: $t_c \simeq 16\pi/(3\ln(1/3\lambda))$. When λ is large, $\lambda \gg 1$, t_c is also large $t_c \simeq 4\pi\lambda/c\ln(3\lambda/4c)$. In Fig.4 we show the potential for $t/c = 0.5,\ 0.935,\ 2$, and $\lambda/c = 1$ with $c = 4\pi$. When t is small, the potential has a minimum at $\varphi = 4\pi/3$ or at $\varphi = 2\pi/3$ indicating that the ground state has the $2\pi/3$ structure with the chirality ± 1. In contrast, when t is large, we have a minimum $\varphi = \pi$. There is a transition at finite temperature $t = t_c$. This is a first-order transition since we have the double-minimum potential in the range $\pi \le \varphi \le 2\pi$. This should be called a weak first-order transition because the change of $V_{total}(\varphi = \pi)$ is slow as t is varied near the critical temperature. The minimum point of φ changes gradually from $4\pi/3$ and changes suddenly to π at the critical temperature. For $t > t_c$, the η_2-mode represents a massless boson. We show t_c as a function of λ/c in Fig.5.

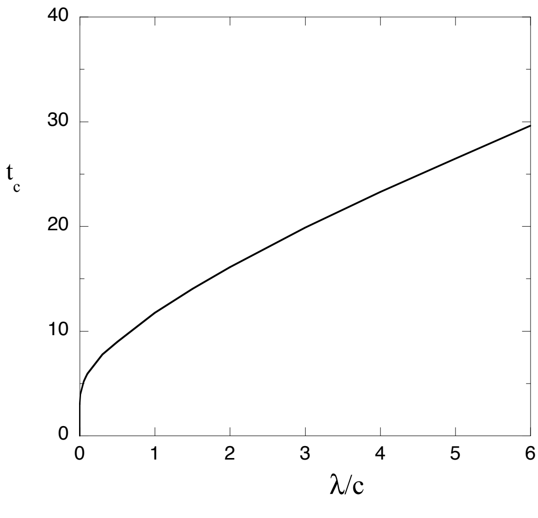

FIG. 5: t_c as a function of λ/c with $c = 4\pi$. t_c is an increasing function of λ/c.

We discuss a relation to the classical XY model on a two-dimensional triangular lattice. The ground state of the 2D XY model has the $2\pi/3$-structure to minimize the energy. There is a transition of the chirality at finite temperature. The critical temperature T_c is of the order of the exchange coupling J because $\lambda/t = J/k_BT$ in this case. The Kosterlitz-Thouless (KT) transition also occurs in the XY model on the 2D triangular lattice. The critical temperature of the KT transition T_{KT} is determined by the renormalization group equation. In general, T_{KT} is different from the critical temperature of the chiral transition $T_{chiral} \equiv K\Delta^2 t_c$.

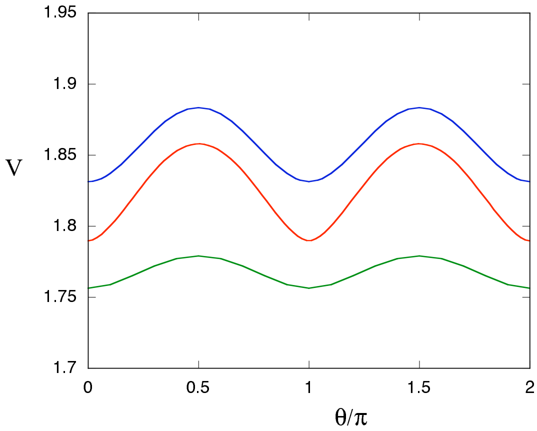

FIG. 6: Effective potential as a function of θ for $n = 4$. From the top, we set $\lambda = 1$ and $a = 1.2$, $\lambda = 1$ and $a = 1.001$ and $\lambda = 0.5$ and $a = 1.2$, respectively, The potential has mimima at $\theta = m\pi$ for integer m.

The similar phenomenon occurs for an $n = 4$ theory with the potential

$$\begin{aligned} V &= \frac{\lambda\Lambda^d}{t}\Big[\cos(\theta_1-\theta_2) + a\cos(\theta_1-\theta_3) + \cos(\theta_1-\theta_4) \\ &+ \cos(\theta_2-\theta_3) + a\cos(\theta_2-\theta_4) + \cos(\theta_3-\theta_4)\Big], \end{aligned} \tag{20}$$

where $a \ge 1$ is a constant. This model has a close relation with the 2D antiferromagnetic XY model on a square lattice[16, 17]. One of the ground state is given by $(\theta_1, \theta_2, \theta_3, \theta_4) = (0, \theta, \pi, \theta+\pi)$ where real θ is arbitrary and the ground state is degenerate with respect to θ. We define $\varphi_1 = \theta_1 - \theta_2 - \theta_3 + \theta_4 = \eta_1$, $\varphi_2 = \theta_1 + \theta_2 - \theta_3 - \theta_4 = \eta_2 - 2\pi$, $\varphi_3 = \theta_1 - \theta_2 + \theta_3 - \theta_4 = \eta_3 - 2\theta$, and the total phase $\Phi = \theta_1 + \theta_2 + \theta_3 + \theta_4$. Then the potential becomes

$$V = \frac{\lambda\Lambda^d}{t}\Big[-2a + \frac{1}{4}(a - \cos\theta)\eta_1^2 + \frac{1}{4}(a + \cos\theta)\eta_2^2 + \cdots\Big], \tag{21}$$

where $\cdots$ indicates higher order terms. The η_3-mode becomes massless and the ground state energy $-2a$ is independent of θ. This is the $n-3$ series state[18] which we call the type I. When $a = 1$, η_1- or η_2-mode is massless in the case $\theta = 0$ or π. This is the $n-2$ series state. The effective potential V_{eff} is obtained by integrating out the η_1 and η_2 variables in a similar way to the case $n = 3$ in two dimensions:

$$\begin{aligned}\frac{V_{eff}}{k_B T\Lambda^2} &= \frac{1}{2}\ln\left((4\pi+\lambda a)^2-\lambda^2\cos^2\theta\right)\\ &+ \frac{1}{8\pi}\lambda a\ln\left(\frac{(4\pi+\lambda a)^2-\lambda^2\cos^2\theta}{\lambda^2(a^2-\cos^2\theta)}\right)\\ &+ \frac{\cos\theta}{8\pi}\ln\left(\frac{4\pi+\lambda(a+\cos\theta)}{\lambda(a+\cos\theta)}\frac{\lambda(a-\cos\theta)}{4\pi+\lambda(a-\cos\theta)}\right),\end{aligned} \tag{22}$$

where we used the cutoff k_0 in the momentum integral satisfying $k_0^2/(4\pi)=\Lambda^2$. The potential is shown in Fig.6 for several parameters where the ground state is at $\theta=m\pi$ for an integer m. This indicates that a Nambu-Goldstone boson emerges for $a=1$ as a result of fluctuation of the $U(1)$ phase variables. We can regard the sign of $\sin\theta$ as a kind of chirality. The emergence of new massless boson is accompanied by the vanishing of chirality.

We can generalize our argument to an n-component scalar field with Josephson couplings. The potential

$$V=\frac{\lambda\Lambda^d}{t}\sum_{i<j}\cos(\theta_i-\theta_j), \tag{23}$$

has a series of massless bosons; there are two types of ground states called the type I and II[18]. In the ground state I one has $n-3$ massless bosons and in the ground state II one has $n-2$ massless bosons. (The $n-2$ series exists only for even n.) Two ground states I and II are degenerate for the potential V. However, the ground state II becomes more stable than the state I due to fluctuation effect. Thus, when we are in the ground state I first, the fluctuation effect leads us to the state II with increasing the number of Nambu-Goldstone bosons.

Order to order transition by disorder The chiral transition considered in this paper is a transition from the $2\pi/3$-structure in Fig.1(a) (or (b)) to the antiferromagnetic state in Fig.1(c). We can say that the ordered state with a massless boson in Fig.1(c) is induced by disorder, namely, thermal fluctuation. We call this an order to order transition by disorder. We discuss here the fluctuation effect on the induced Nambu-Goldstone boson. For this purpose, we write $\varphi_1=\pi+\phi_1$ so that ϕ_1 indicates the fluctuation mode in the neighborhood of π. The action is written as

$$\begin{aligned}S &= \frac{\Lambda^{d-2}}{t}\int d^dx\Big[\frac{1}{2}(\nabla\phi_1)^2+\frac{1}{2}\lambda\Lambda^2\phi_1^2+\frac{1}{6}(\nabla\varphi_2)^2\\ &\qquad -\lambda\Lambda^2\phi_1\cos\left(\frac{\varphi_2}{2}\right)\Big].\end{aligned} \tag{24}$$

The φ_2-mode is obviously a massless mode, but there is an interaction with ϕ_1. This interaction will generate an effective potential of φ_2 that is proportional to $\cos^2(\varphi_2/2)=(\cos(\varphi_2)+1)/2$. Then, the effective action for φ_2 is given by the sine-Gordon model:

$$S_{\varphi_2}=\frac{\Lambda^{d-2}}{t}\int d^dx\Big[\frac{1}{6}(\nabla\varphi_2)^2-\frac{\lambda}{4}\Lambda^2\cos(\varphi_2)\Big]. \tag{25}$$

The low-energy property is determined by the values of λ and t as indicated by the renormalization group equations[26, 27] near two dimensions. The critical value of t, denoted by t_{2c}, is $t_{2c}=8\pi/3$. We assume that $t>t_c>t_{2c}$. When λ is small, λ is renormalized to 0 following the renormalization flow. This indicates that the φ_2-mode remains massless for small λ. When λ is large, λ is renormalized to be a large value, showing that the potential term dominates the behavior of φ_2-mode and then that φ_2 takes the value near 0. In this case the massless φ_2-mode becomes massive, that is, a gapped mode again. Basically φ_2-mode may remain massless because the Josephson coupling λ is small in real superconductors.

Summary We have proposed the mechanism of fluctuation induced Nambu-Goldstone bosons. In an n-component scalar field theory with frustrated Josephson interactions, massless bosons appear due to fluctuations at finite temperature. In the 3-component theory discussed in the paper, a massless boson appears and the chirality vanishes as the temperature is increased, that is, the Z_2-symmetry breaking is driven by the chirality. This shows that nature prefers massless bosons. In fact, in an n-component model, the ground state at absolute zero will flow into the state with more massless bosons as the temperature is increased from $(n-3)$-state to $(n-2)$-state.

The excitation modes in our model has an analogy to the vibration modes of a molecule CH_2. Two modes, the scissoring mode and the rocking mode, are important in determining the excitation spectra of CH_2[28, 29]. The modes shown by $\varphi_1=\theta_3-\theta_1$ and $\varphi_2=\theta_1-2\theta_2+\theta_3$ represent the scissoring and rocking modes, respectively. In our model, the rocking mode plays a significant role. The fluctuation effect of the rocking mode becomes large as the temperature is increased and gives rise to the phase transition. The model presented in the paper appears as an effective free energy in multi-band superconductors[13–15, 19–23]. Low energy excitation states are important in superconductors, the existence of massless modes have been pointed out[18, 21, 24, 25]. In this paper we have presented a new mechanism of the emergence of Nambu-Goldstone bosons.

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