

高温超伝導の理論的研究

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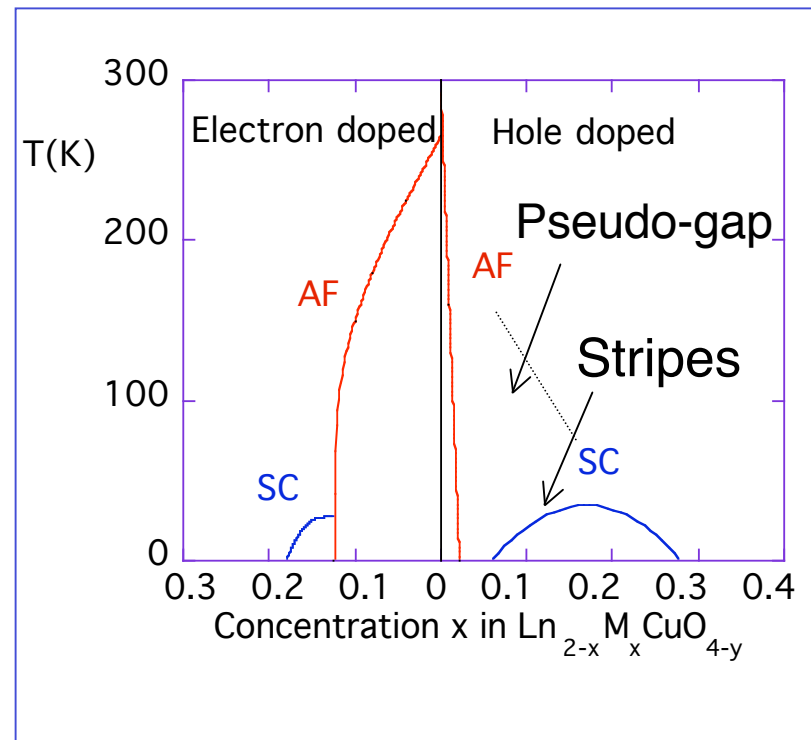
Outline

1. Introduction
2. Superconductivity
3. High Temperature Superconductivity
4. Hubbard Model
5. Variational Monte Carlo method
6. Stripes in high- T_c cuprates
7. Spin-orbit coupling and Lattice distortion
8. Summary

1. Introduction

Key words: Physics from U (Coulomb interactions)

- A possibility of superconductivity
Superconductivity from U
- Competition of AF and SC
- Incommensurate state
Stripes and SC
Compete and Collaborate
- Stripes in the lightly-doped region
- Singular Spectral function



Purpose of Theoretical study

1. Origin of the superconductivity

- Symmetry of Cooper pairs
- Mechanism of attractive interaction

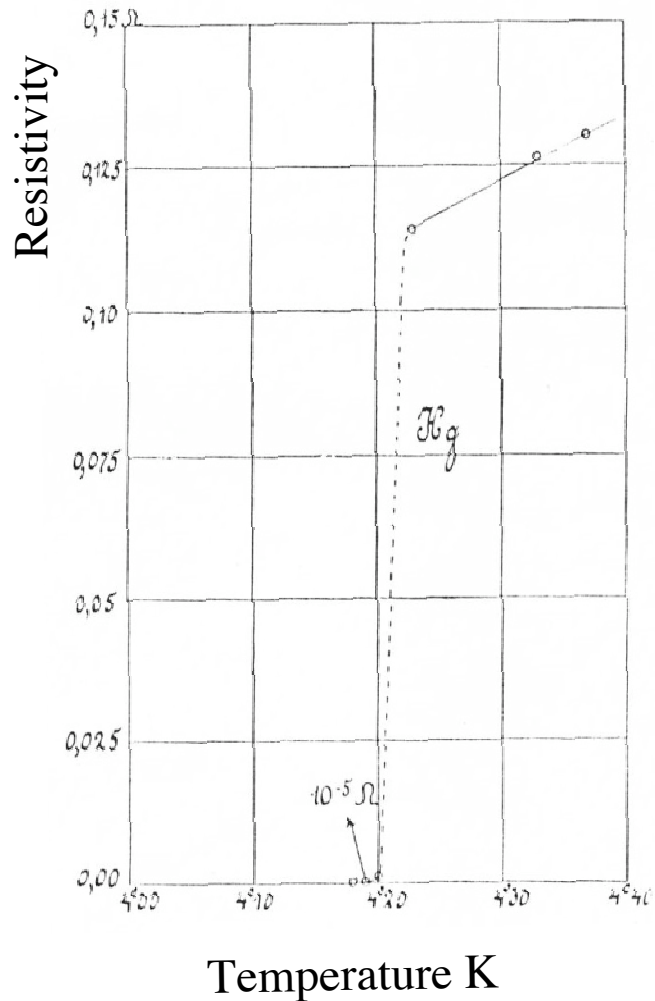
Coulomb interaction U , Exchange interaction J

2. Physics of Anomalous Metallic behavior

- Inhomogeneous electronic states: stripe
- Pseudogap phenomena
- Structural transition LTO, LTT

2. Superconductivity

1911 Kamerlingh Onnes



Elements that become superconducting



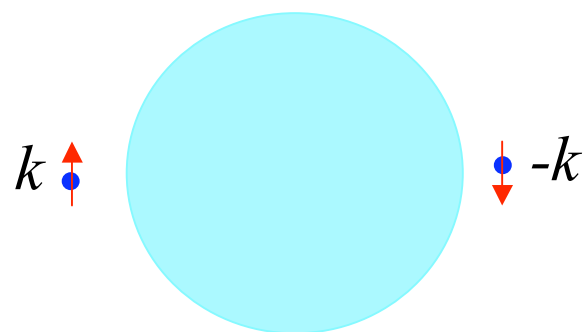
Superconductive at low temperatures



Superconductive under pressure

巨視的量子現象

k と $-k$ の電子がペアをつくり、ゲージ（位相）不変性が破れた状態



BCS理論

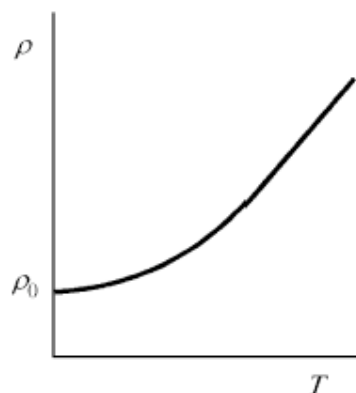
電子には位相という仮想的空間内での回転の自由度があり、勝手な方向を向いている。が、超伝導状態ではすべての電子ペアが同じ方向を向いている。

対称性の破れ

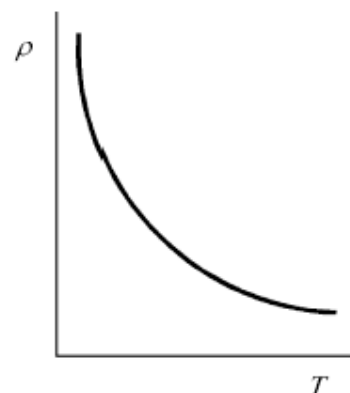
引力相互作用によるフェルミ面の不安定性によって超伝導が引き起こされる

超伝導体の特徴

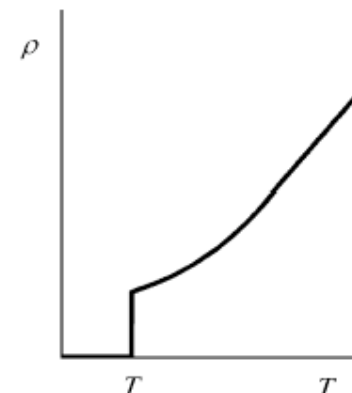
電気抵抗 0



金属

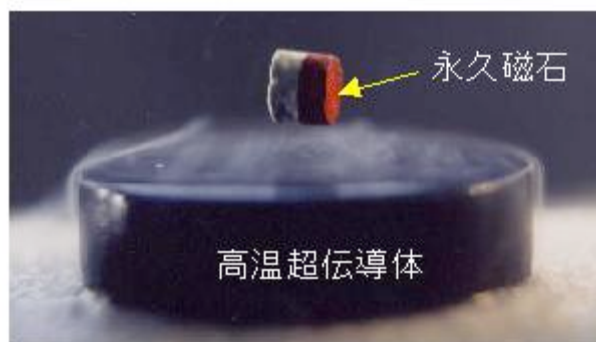


半導体



超伝導体

マイスナー効果



磁場は超伝導体に入り込むことができない

フッシング効果

超伝導体に磁石を近付けておいて冷やすと超伝導体内部に入り込んだ磁場により（第2種超伝導体）、つりあげることができる。

BCS理論

BCS理論 どうして電子対を考えたか (近藤淳「超伝導」(固体物理)より)

超伝導状態: 一つのSlater行列式では表わせない

電子間引力 H_1

$$H = H_0 + H_1 \quad \Psi = \sum_j c_j \Psi_j \quad \begin{array}{l} \Psi_j : \text{Slater行列式} \\ \Psi_0 : \text{Fermi球} \end{array}$$

波動関数 $\Psi = c_0 \Psi_0 + \sum_{j \neq 0} c_j \Psi_j$ 摂動計算 $c_0 \approx 1$
 $c_j \langle \Psi_0 H_1 \Psi_j \rangle < 0$

エネルギー $\langle \Psi H \Psi \rangle = \sum_j c_j^2 \langle \Psi_j H_0 \Psi_j \rangle + \sum_i c_i c_j \langle \Psi_i H_1 \Psi_j \rangle$

もし、すべての $\langle \Psi_i H_1 \Psi_j \rangle < 0$ なら、すべての $c_j > 0$

Ψ_j としてペアーの状態をとるならば、 $c_j > 0$ としてエネルギーを下げるができる。

BCS波動関数

電子対

$$\Psi_i = |k_1 \uparrow - k_1 \downarrow, k_2 \uparrow - k_2 \downarrow, k_3 \uparrow - k_3 \downarrow, \rangle$$

$$\Psi_j = |k'_1 \uparrow - k'_1 \downarrow, k_2 \uparrow - k_2 \downarrow, k_3 \uparrow - k_3 \downarrow, \rangle$$

順番を変えても
符号は変わらない。

$$\langle \Psi_i | H_1 | \Psi_j \rangle = \langle k'_1 - k'_1 | V | k_1 - k_1 \rangle < 0$$

非対角要素を常に負にできる

基底状態はすべての対状態の一次結合で表わされる:

$$\Psi = \sum c_{k_1 k_2} |k_1 \uparrow - k_1 \downarrow, k_2 \uparrow - k_2 \downarrow, \rangle$$

独立対近似（一体近似）をすると

$$\Psi = \sum c_{k_1} c_{k_2} |k_1 \uparrow - k_1 \downarrow, k_2 \uparrow - k_2 \downarrow, \rangle$$

N電子項のみを取り出すとして

$$\Phi = \prod_k (u_k + v_k |k \uparrow - k \downarrow \rangle)$$

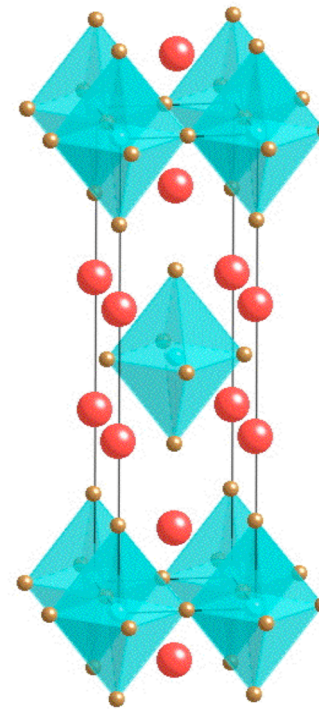
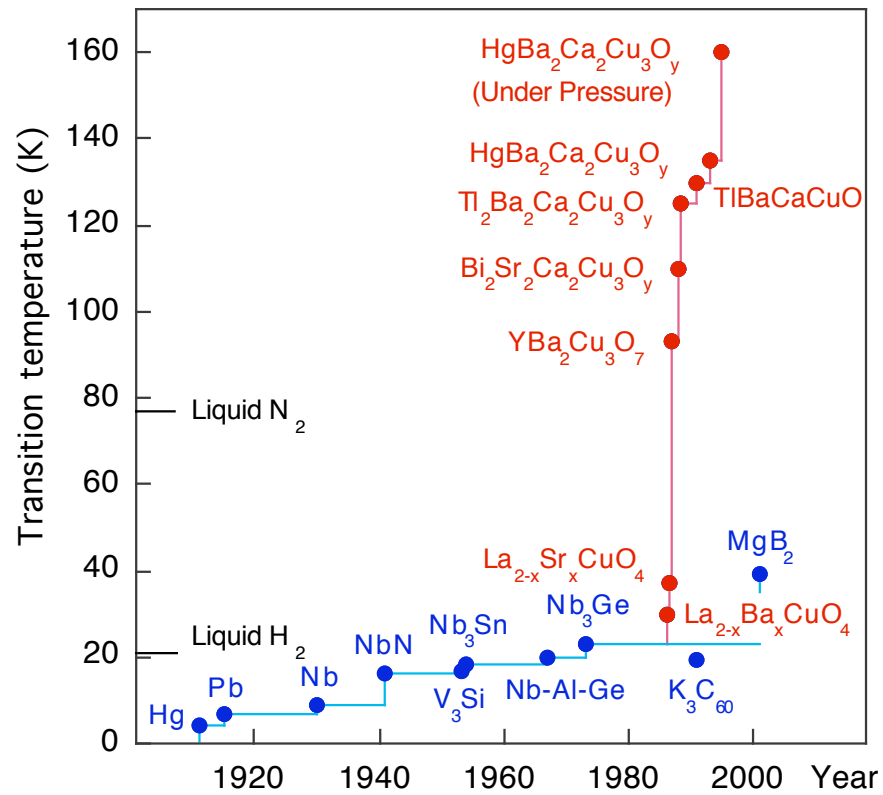
BCSの波動関数

$$\frac{\sqrt{(N - \langle N \rangle)^2}}{\langle N \rangle} \propto \frac{1}{\sqrt{\langle N \rangle}}$$

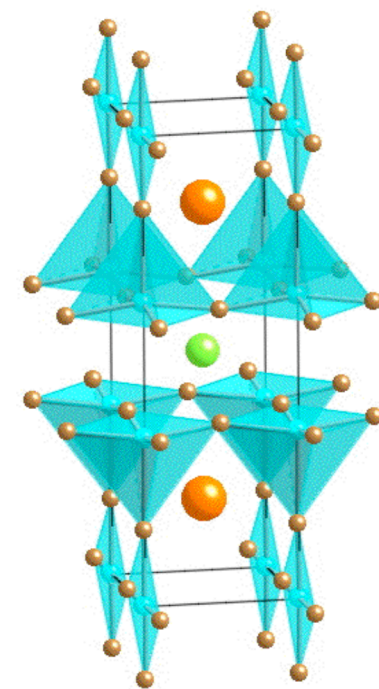
粒子数のゆらぎは小さい

3. 高温超伝導

超伝導臨界温度



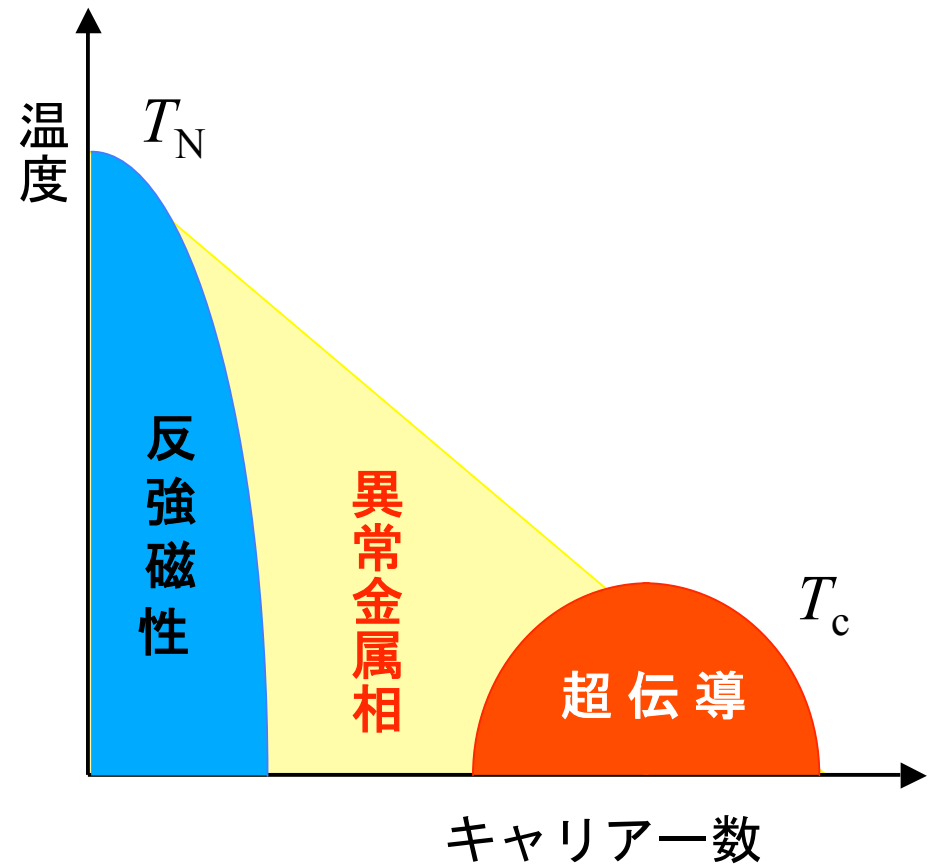
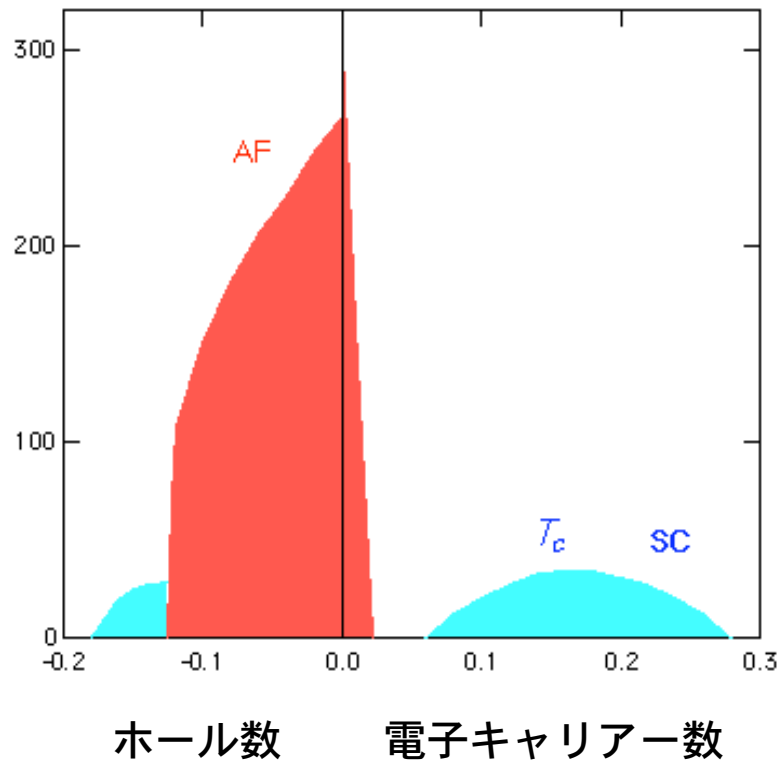
La₂CuO₄



YBa₂Cu₃O₇

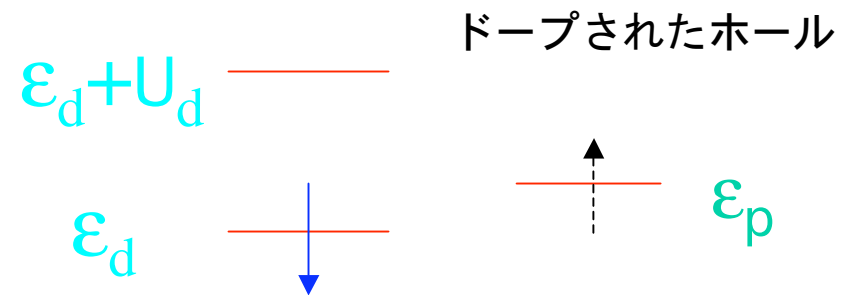
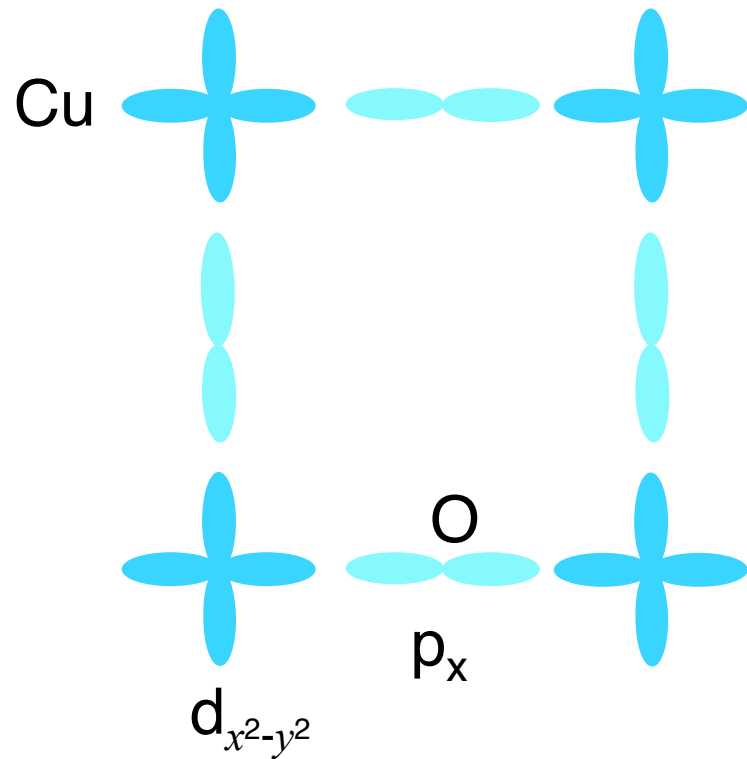
高温超伝導体の相図

高温超伝導体の相図



高温超伝導体のモデル

銅と酸素を含む二次元面



銅酸化物の特徴

- 二次元的である
- スピンが1/2 (小さい)
- 銅, 酸素のhole levelが近い

電気抵抗の温度依存性

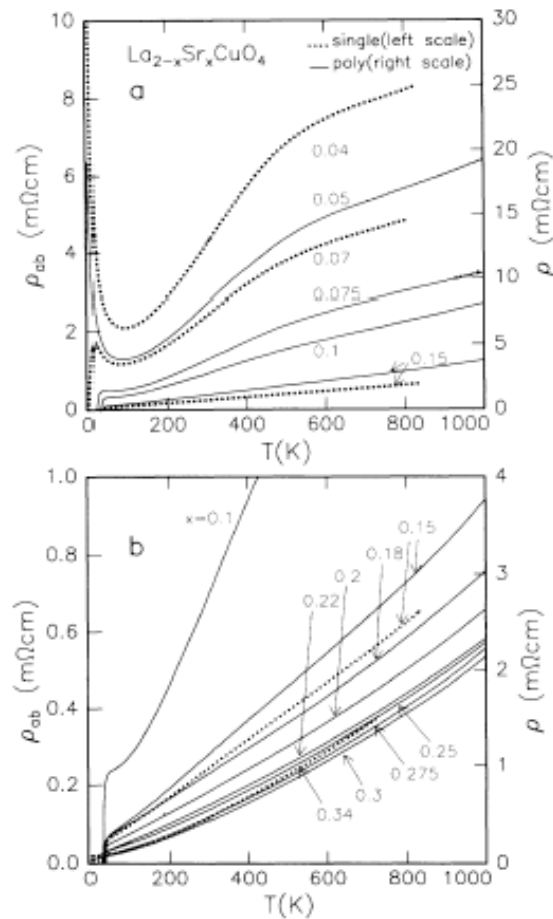


FIG. 1. The temperature dependence of the resistivity for $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. (a) $0 < x \leq 0.15$, (b) $0.1 \leq x < 0.35$. Dotted lines, the in-plane resistivity (ρ_{ab}) of single-crystal films with (001) orientation; solid lines, the resistivity (ρ) of polycrystalline materials. Note, $\rho_M = (h/e^2)d = 1.7 \text{ m}\Omega \text{ cm}$.

H. Takagi et al.

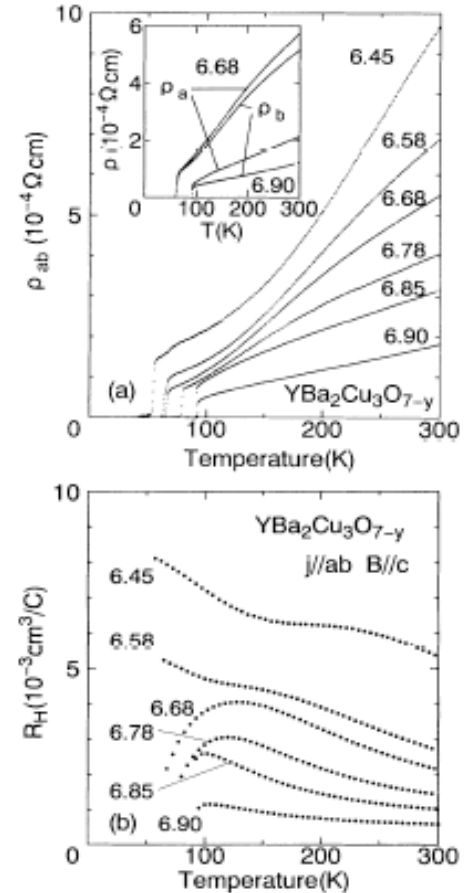


FIG. 1. (a) Temperature dependence of in-plane resistivity of twinned $\text{YBa}_2\text{Cu}_3\text{O}_{7-y}$ crystals with oxygen concentration $7-y \sim 6.90, 6.85, 6.78, 6.68, 6.58, \text{ and } 6.45$. Inset: Temperature dependence of ρ_a and ρ_b for detwinned crystals of $T_c = 90$ and 60 K. (b) Temperature dependence of R_H of twinned crystals measured under $j \parallel ab$ plane and $B \parallel c$ axis at $B = 5 \text{ T}$.

T. Ito et al.

比熱

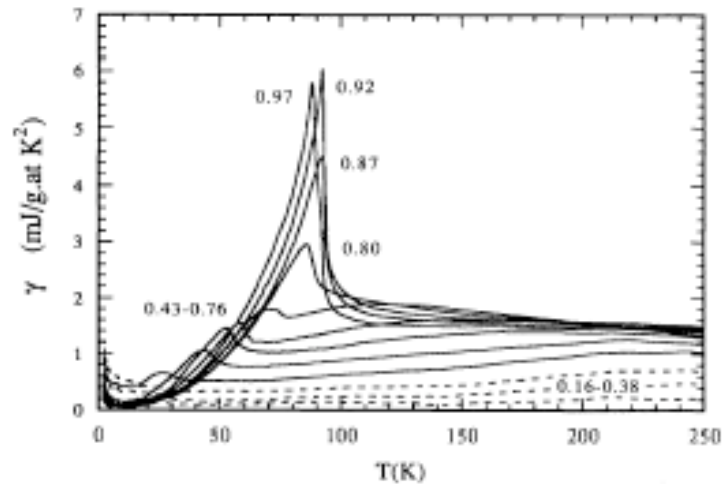


FIG. 4. Electronic specific heat coefficient $\gamma(x, T)$ vs T for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ relative to $\text{YBa}_2\text{Cu}_3\text{O}_6$. Values of x are 0.16, 0.29, 0.38, 0.43, 0.48, 0.57, 0.67, 0.76, 0.80, 0.87, 0.92, and 0.97.

Loram et al., Phys. Rev. Lett. 71, 1740 (1993)

LSCO

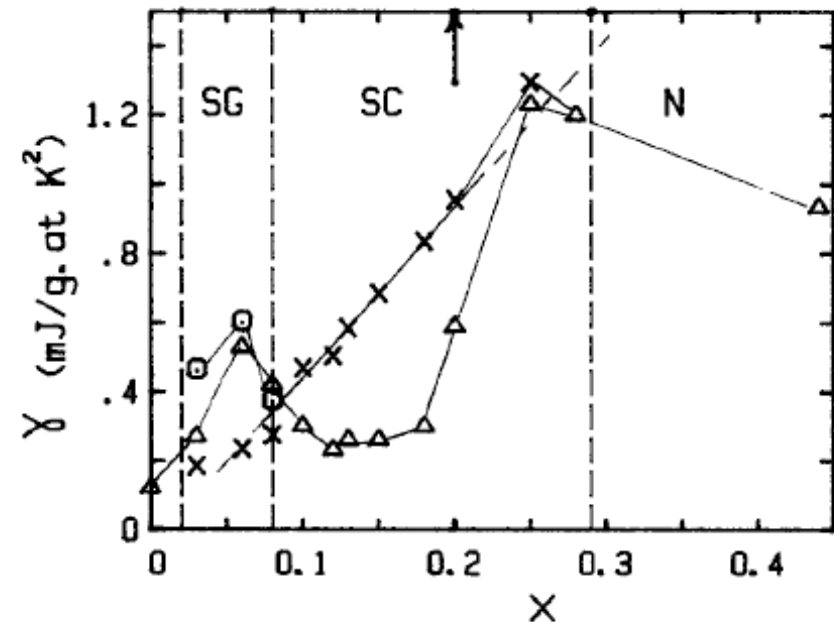


FIGURE 2

γ vs x . for $x \leq 0.08$ $\Delta, \gamma(2\text{K})$; $\circ, \gamma(8\text{K})$; $\times, \gamma(40\text{K})$
for $x \geq 0.1$ $\Delta, \gamma(0)$; \times, γ_n

Loram et al., Physica C162-164, 498 (1989)

磁気緩和率

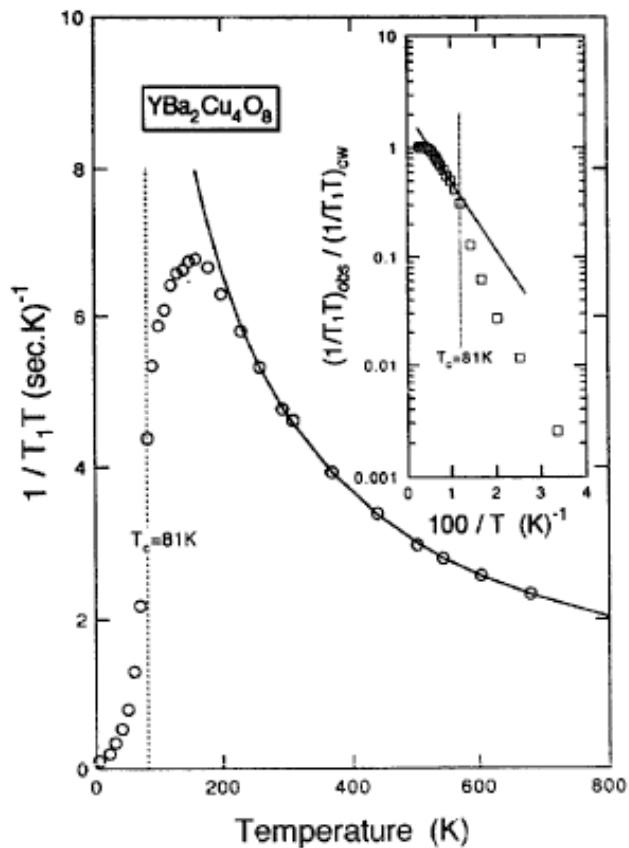


Fig. 1. Temperature dependence of the nuclear spin-lattice relaxation rate $1/T_1T$ for Cu(2) sites of $\text{YBa}_2\text{Cu}_4\text{O}_8$. The solid curve shows the best fit of the data to Eq. (1) for $T > 250$ K. The inset shows the Arrhenius plots for the ratio of the observed $(1/T_1T)_{\text{obs}}$ to the expected $(1/T_1T)_{\text{cw}}$ from Eq. (1), and the best fit of the data to Eq. (2) is shown by the solid line.

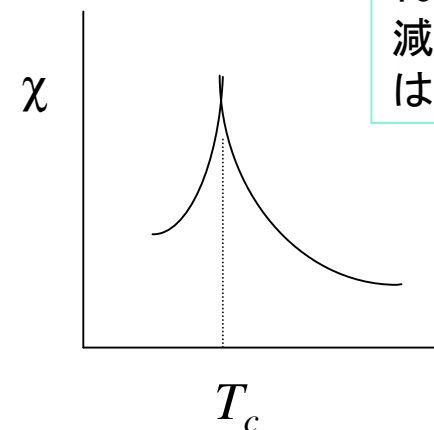
$\text{YBa}_2\text{Cu}_4\text{O}_8$

アンダードープ域 $T_c = 81\text{K}$

T_c より上の温度から帯磁率が下がり始める。

↓
擬ギャップ

スピンゆらぎ理論によると



T_c より上から減少することはない。

量子臨界現象

量子相転移

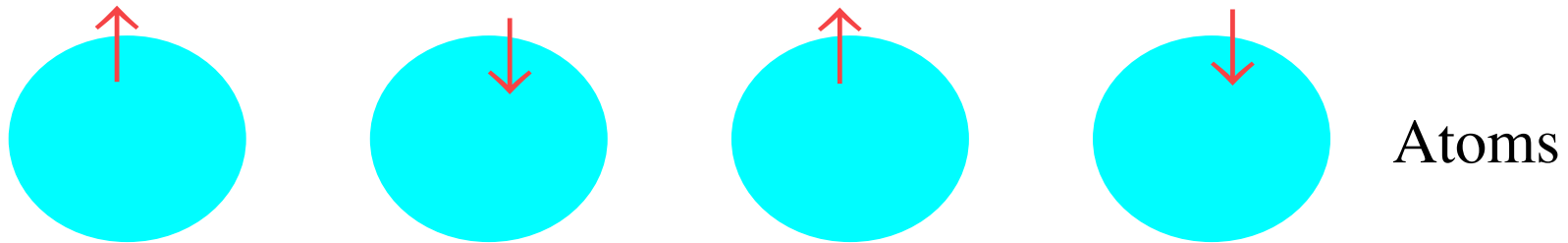
- T=0で起こる相転移
- 量子ゆらぎが重要
 - 量子ゆらぎにより引き起こされる
- 圧力、磁場変化、元素置換等による相転移
- 量子臨界点（相転移点）の近くでは非フェルミ流体の振る舞いが見られることがある。
 - 抵抗、比熱、帯磁率

スピンゆらぎ理論 (Millis, Moriya)

3D	C/T	$\chi(Q)$	ρ	χ_s
Ferro.	$-\ln T$	$T^{-4/3}$	$T^{5/3}$	$T^{-4/3}$
AF	$T^{1/2}$	$T^{-3/2}$	$T^{3/2}$	$T^{-3/4}$
2D	C/T	$\chi(Q)$	ρ	χ_s
Ferro.	$T^{-1/3}$	$(T \ln T)^{-1}$	$T^{4/3}$	$\chi(Q)^{3/2}$
AF	$-\ln T$	T^{-1}	T	T^{-1}

4. Hubbard Model

Itinerant Electrons



Mott insulators

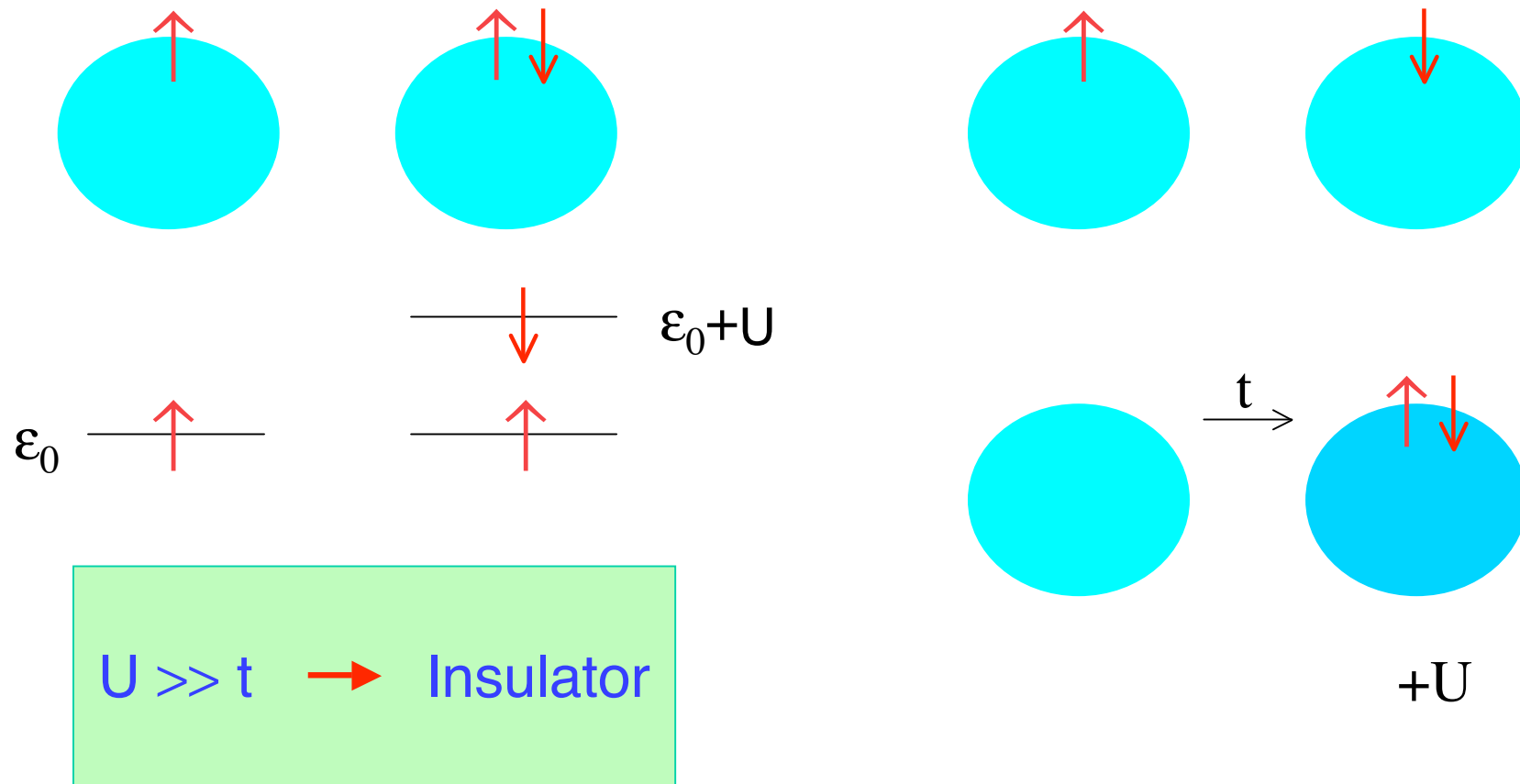
MnO, FeO, CoO, Mn_3O_4 , Fe_3O_4 ,
NiO, CuO

Insulators due to the Coulomb interaction

(Note: Antiferromagnets such as MnO and NiO are not Mott insulators in the strict sense.)

On-site Coulomb Interaction

Coulomb interaction



Gap in the Hubbard Model

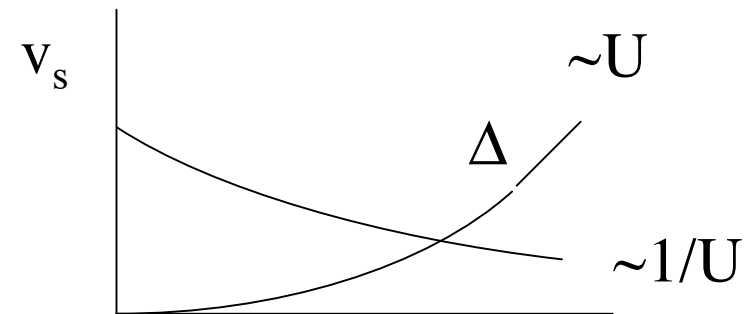
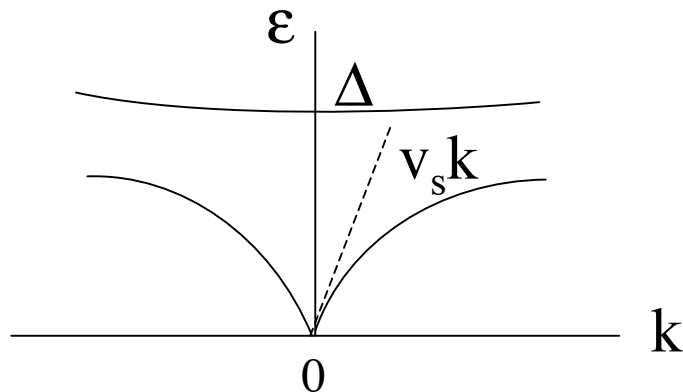
Hartree-Fock theory (Half-filling)

$$\text{AF Gap } \Delta = Um \quad \Delta \sim t e^{-2\pi t/U} \quad d = 1, 3$$

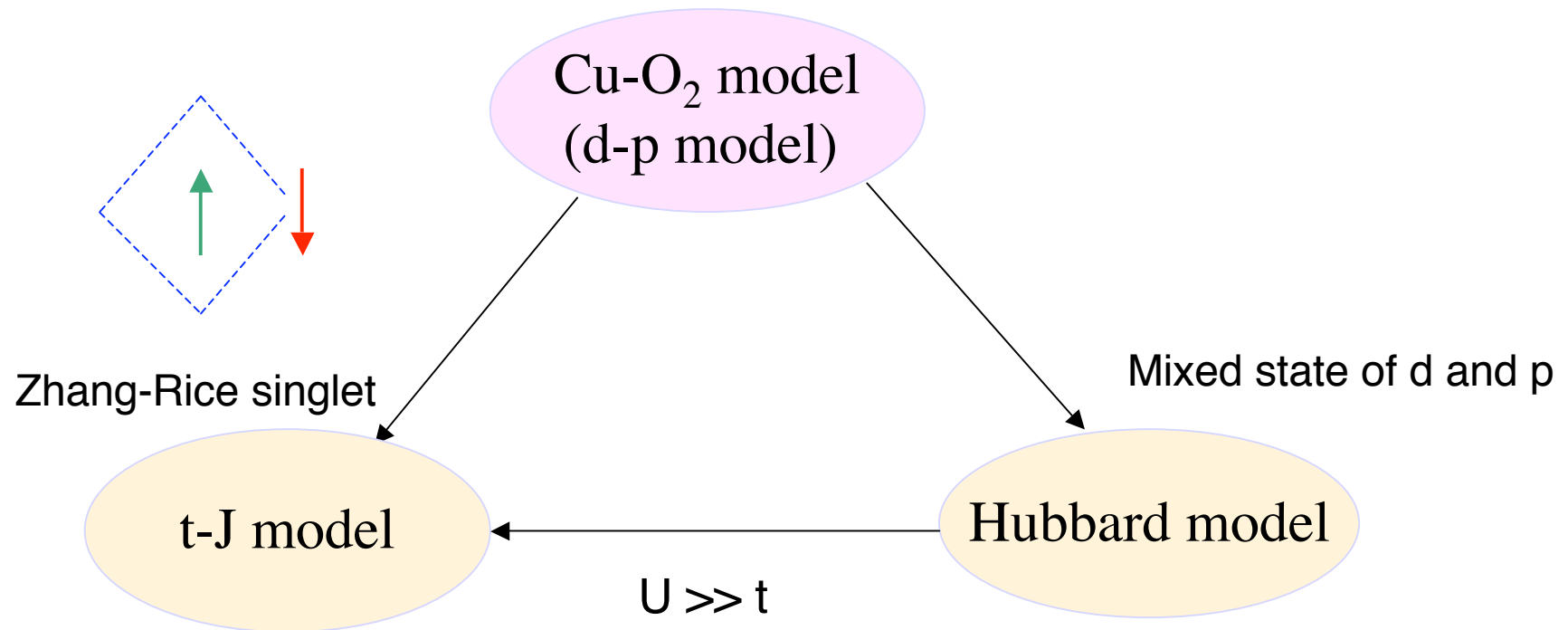
$$\quad \quad \quad \sim t e^{-2\pi(t/U)^{1/2}} \quad d = 2$$

1D Hubbard model

		$U \ll t$	$U \gg t$
Hubbard gap	Δ	$(16/\pi)\sqrt{tU}e^{-\pi/(2U)}$	U
Spin-wave velocity	$2v_s/\pi = J$	$(4t/\pi)(1 - U/4\pi t)$	$4t^2/U$



Cu-O₂ Model and Hubbard Model



$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^+ c_{j\sigma} + h.c.) + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^+ c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$\begin{aligned} t_{pd} &\ll U_d - (\epsilon_p - \epsilon_d) \\ t_{pd} &\ll \epsilon_p - \epsilon_d \\ \epsilon_p - \epsilon_d &\ll U_d \end{aligned}$$

$$\epsilon_p - \epsilon_d \sim 0(t_{pd})$$

5. Variational Monte Carlo method

適当な波動関数の期待値をモンテカルロ法により計算する

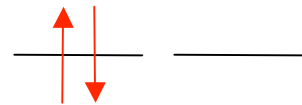
Gutzwiller関数 $\psi_G = P_G \psi_0$

ψ_0 : 試行関数 フェルミ球、反強磁性、超伝導

$P_G = \prod_j (1 - (1 - g)n_{j\uparrow}n_{j\downarrow})$ Gutzwiller演算子

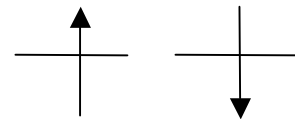
$$0 \leq g \leq 1$$

パラメータgによりオンサイトの相関を制御する



weight g

Coulomb +U



weight 1

変分モンテカルロ法

Normal state ψ_0 Slater 行列式

$$\psi_0 = \sum_{\uparrow} a_{\uparrow} \psi_{\uparrow} \quad \psi_{\uparrow} : \text{実空間での粒子の配置}$$

波数 k_1, k_2, \dots, k_n 座標 j_1, j_2, \dots, j_n を \uparrow 粒子が占めている時

$$\det D_{\uparrow} = \begin{vmatrix} e^{ik_1 j_1} & e^{ik_1 j_2} & \dots & e^{ik_1 j_n} \\ \vdots & \vdots & \ddots & \vdots \\ e^{ik_n j_1} & e^{ik_n j_2} & \dots & e^{ik_n j_n} \end{vmatrix} \quad \text{Slater 行列式}$$

ウェイト $a_{\uparrow} = \det D_{\uparrow} \det D_{\downarrow}$

粒子の配置の総数は大きな数 \rightarrow モンテカルロ法

モンテカルロアルゴリズム

期待値

$$\langle \psi Q \psi \rangle = \sum_{mn} a_m a_n \langle \psi_m Q \psi_n \rangle = \sum_m \frac{a_m^2}{\sum_l a_l^2} \sum_n \frac{a_n}{a_m} \langle \psi_m Q \psi_n \rangle$$

ψ_m の出現確率が $P_m = \frac{a_m^2}{\sum_l a_l^2}$ に比例するようにサンプルを生成すると

$$\langle \psi Q \psi \rangle = \frac{1}{M} \sum_m \left(\sum_n \frac{a_n}{a_m} \langle \psi_m Q \psi_n \rangle \right) \quad m = 1, L, M$$

Metropolis法

ψ_j の次に ψ_n を生成した時 (例えば、どれかの電子を動かす)

$$R = \begin{cases} |a_n|^2 / |a_j|^2 \geq \xi & \text{なら } \psi_n \text{ を採用} \\ < \xi & \psi_j \text{ のまま} \end{cases} \quad \xi: \text{一様乱数} \quad 0 \leq \xi < 1$$

$\langle \psi_m Q \psi_n \rangle$ の計算には余因子展開を使うとcpu時間を稼げる

超伝導状態の波動関数

$$\Psi_s = P_N P_G \Psi_{BCS}$$

$$\Psi_{BCS} = \prod_k (u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+) |0\rangle$$

P_N : 電子数をN個に固定

$$\begin{aligned} \Psi_s &= P_G P_N \exp\left(\sum_k \frac{v_k}{u_k} c_{k\uparrow}^+ c_{-k\downarrow}^+\right) |0\rangle \\ &= P_G \left(\sum_k \frac{v_k}{u_k} c_{k\uparrow}^+ c_{-k\downarrow}^+\right)^{N/2} |0\rangle \\ &= P_G \left(\sum_k a_{ij} c_{i\uparrow}^+ c_{j\downarrow}^+\right)^{N/2} |0\rangle \\ a_{ij} &= \frac{1}{V} \sum_k \frac{v_k}{u_k} e^{ik \cdot (R_i - R_j)} \end{aligned}$$

$N_\uparrow = N_\downarrow$ の時

↑電子が $i_1, i_2, \dots, i_{N/2}$; ↓電子が $j_1, j_2, \dots, j_{N/2}$ にいる時、ウェイトは行列式

$$\begin{vmatrix} a_{i_1 j_1} & a_{i_1 j_{N/2}} \\ \vdots & \vdots \\ a_{i_{N/2} j_1} & a_{i_{N/2} j_{N/2}} \end{vmatrix}$$

で与えられ、normal stateと同様に計算できる。

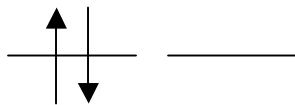
▲ 一般のペアー状態は、近藤さんのレクチャーにあるように、一つのSlater行列式では書けないが、BCS状態は一体近似をしているので、行列式で書くことができる。

Superconducting state

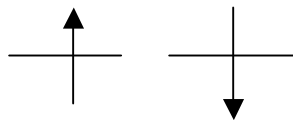
$$\psi_{cdS} = P_G \prod_k (u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+) |0\rangle$$

Gutzwiller Projection P_G

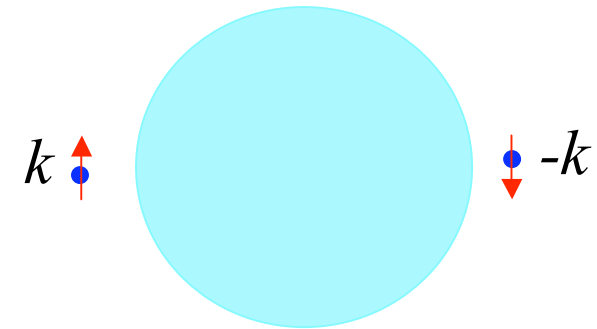
To control the on-site strong correlation



Weight g
Coulomb $+U$



Weight 1
Parameter $0 < g < 1$



Equivalent
to
RVB state (Anderson)

Superconducting condensation energy

SC Condensation energy

$$\Delta E_{SC} = \Omega_n - \Omega_s = \int_0^{T_c} (S_n - S_s) dT$$

$$= \int_0^{T_c} (C_s - C_n) dT$$

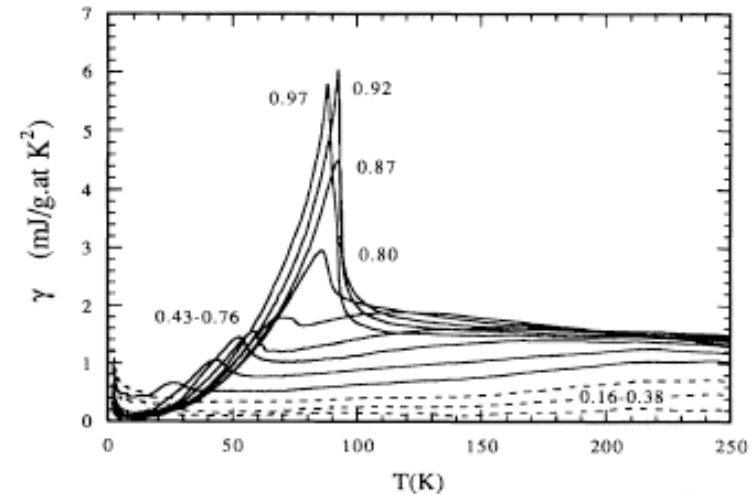
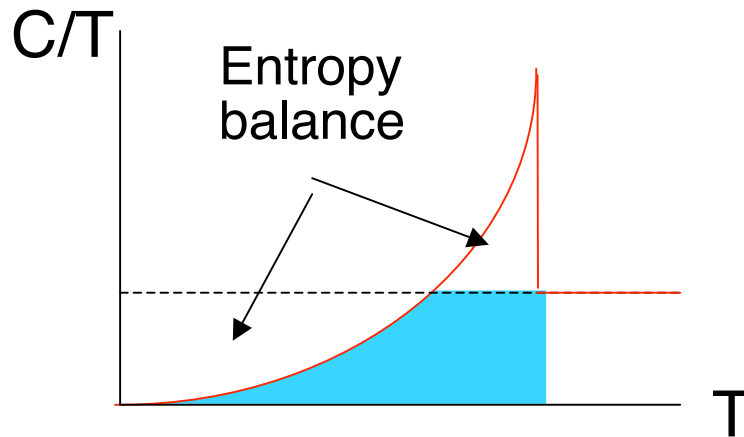
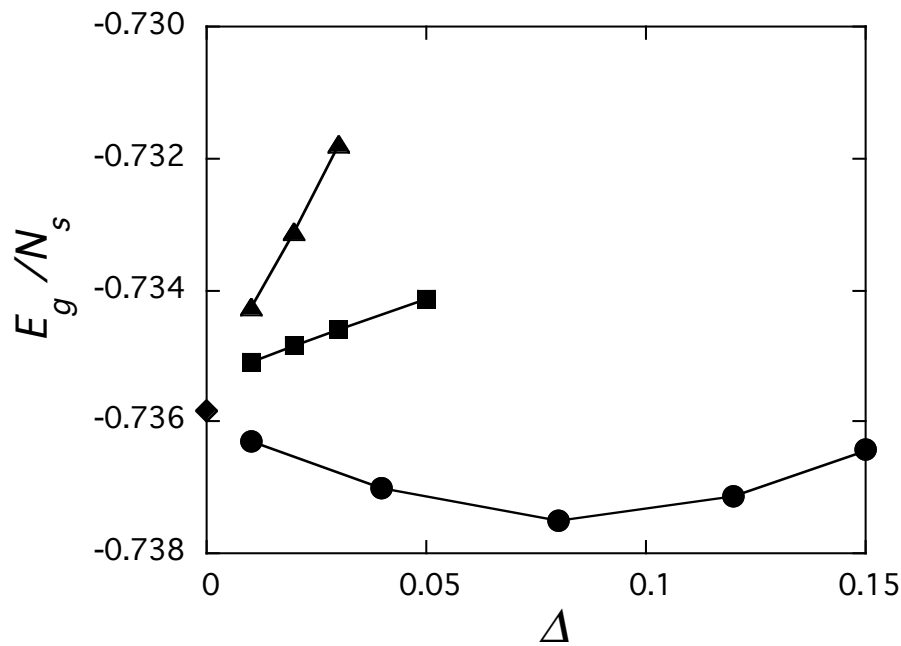


FIG. 4. Electronic specific heat coefficient $\gamma(x, T)$ vs T for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ relative to $\text{YBa}_2\text{Cu}_3\text{O}_6$. Values of x are 0.16, 0.29, 0.38, 0.43, 0.48, 0.57, 0.67, 0.76, 0.80, 0.87, 0.92, and 0.97.

Loram et al. PRL 71, 1740 ('93)
optimally doped YBCO

SC Condensation energy
 $\sim 0.2 \text{ meV}$

Evaluations in the superconducting state



K. Yamaji et al., Physica C304, 225 (1998)

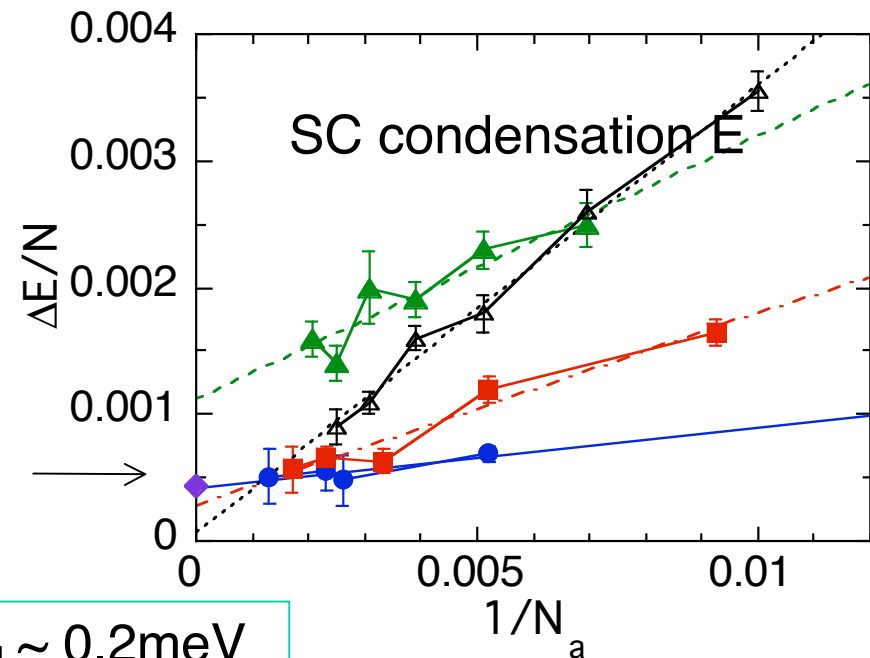
T. Yanagisawa et al.,
Phys. Rev. B67, 132408 (2003)

YBCO →

$E_{\text{cond}} \sim 0.2\text{meV}$

Variational Monte Carlo method
10x10 Hubbard model $U=8$

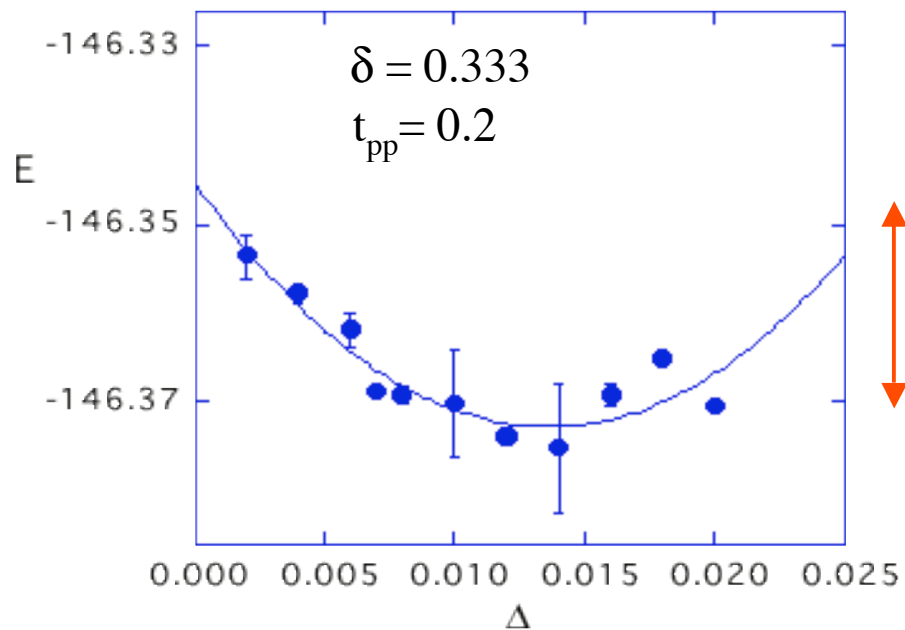
T. Nakanishi et al. JPSJ 66, 294 (1997)



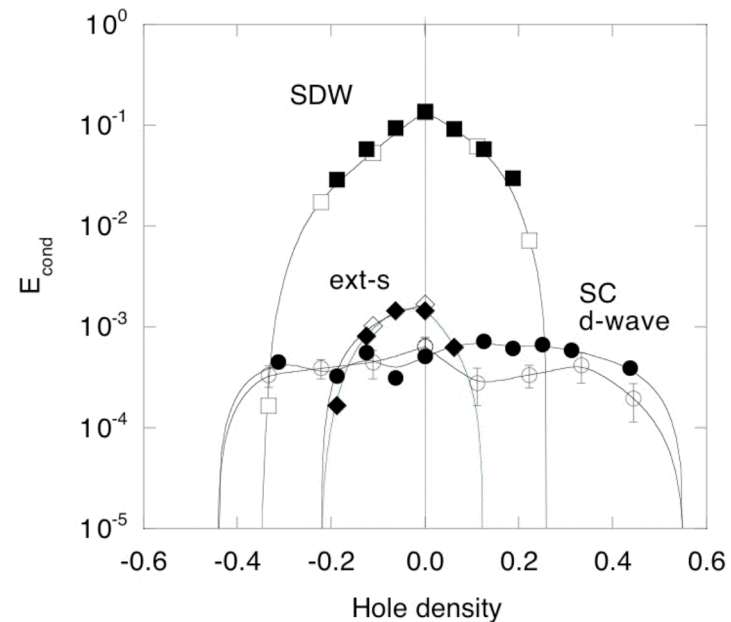
Condensation Energy for d-p model

Condensation energy

$$E_{\text{cond}} \sim 0.00038 t_{\text{dp}} \\ = 0.56 \text{ meV/site}$$



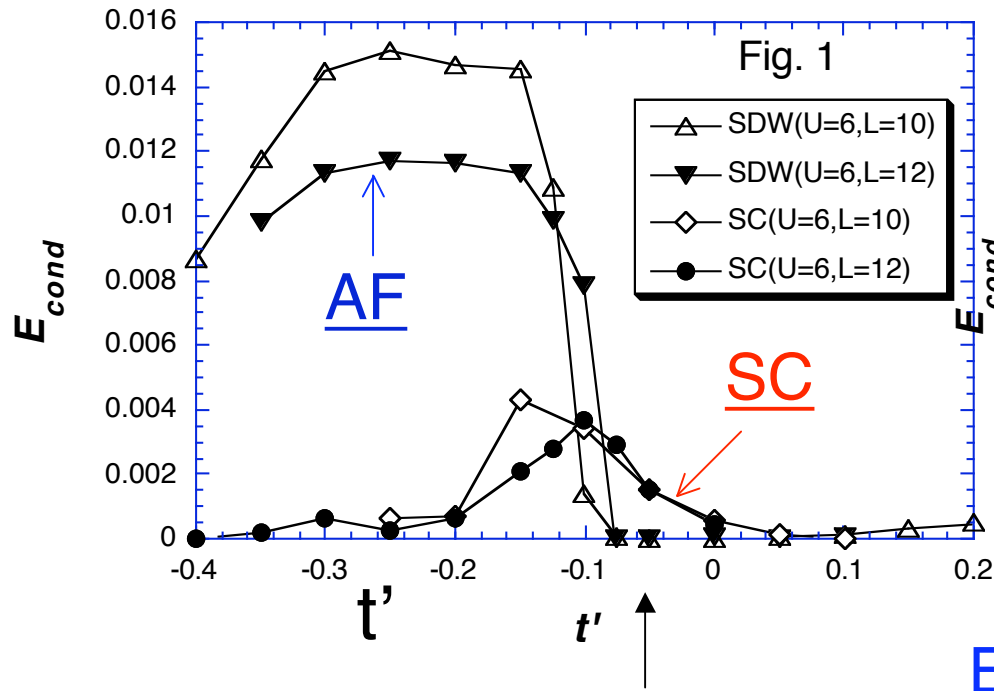
2D d-p model 6x6 and 8x8



T.Yanagisawa et al., PRB64, 184509 ('01)

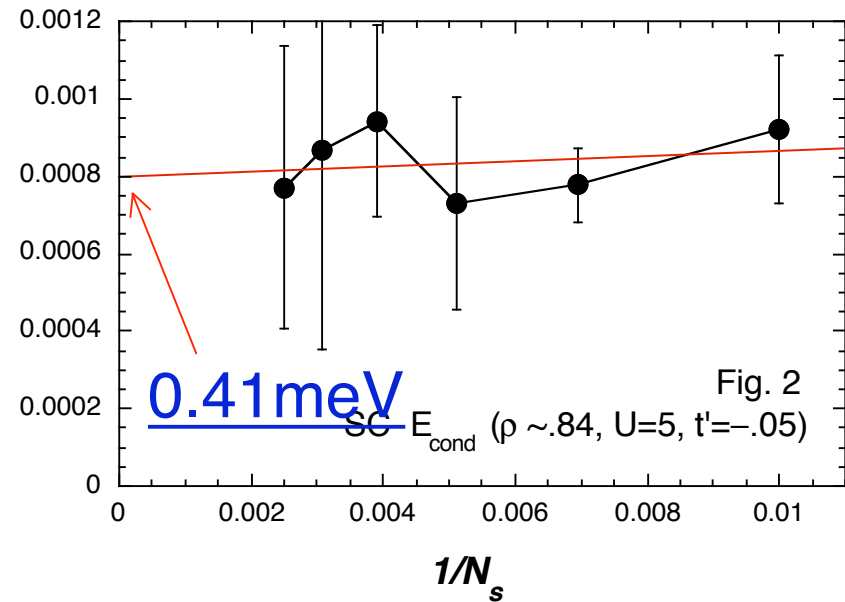
Superconductivity and Antiferromagnetism

Competition



Pure d-wave SC

Size dependence of SC condensation energy



Experiments

0.26 meV/site

(critical field H_c)

0.17~0.26

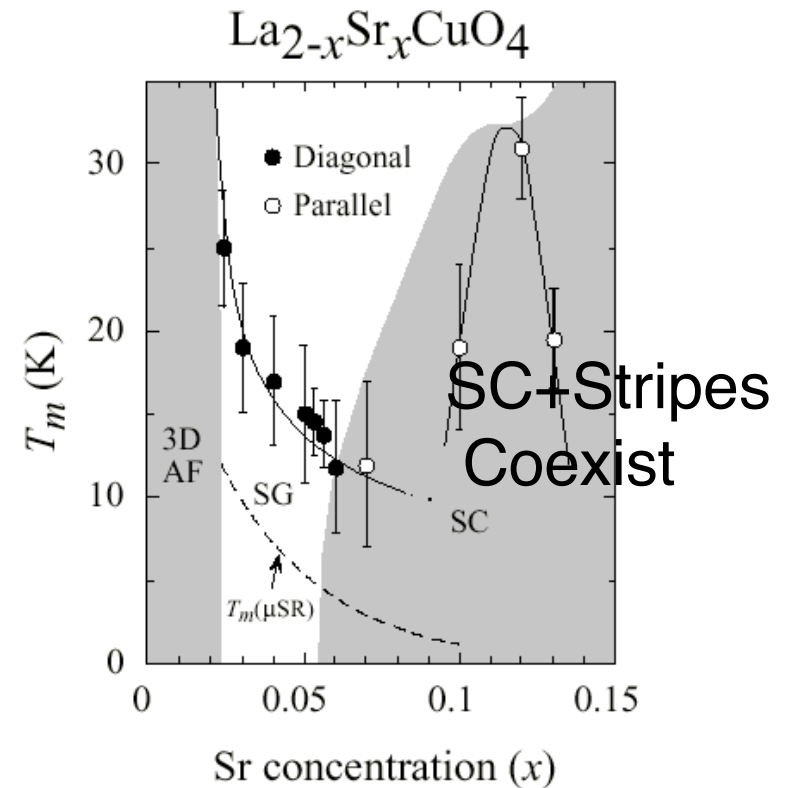
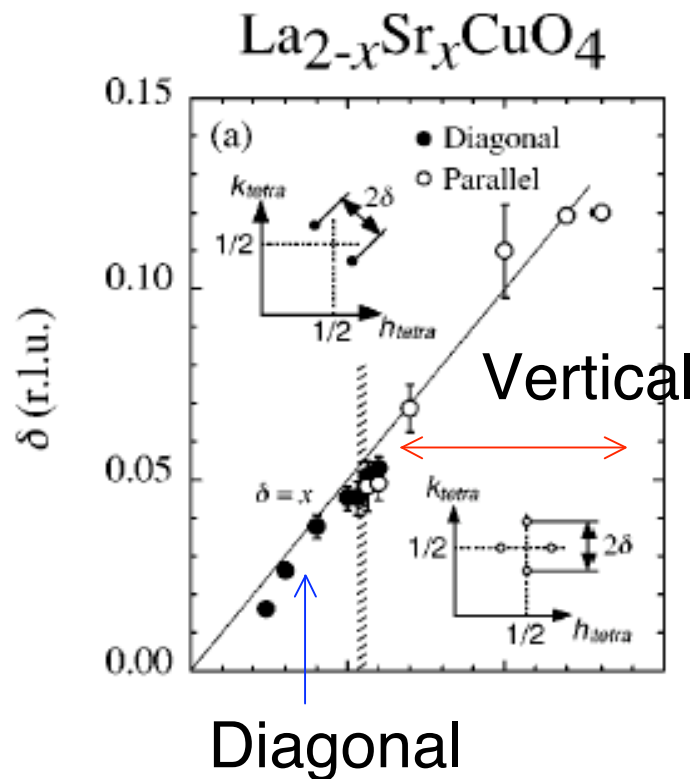
(C/T)

6. Stripes in high-Tc cuprates

- Vertical stripes for $x > 0.05$
- Diagonal stripes for $x < 0.05$

AF coexists with SC?

Neutron scattering

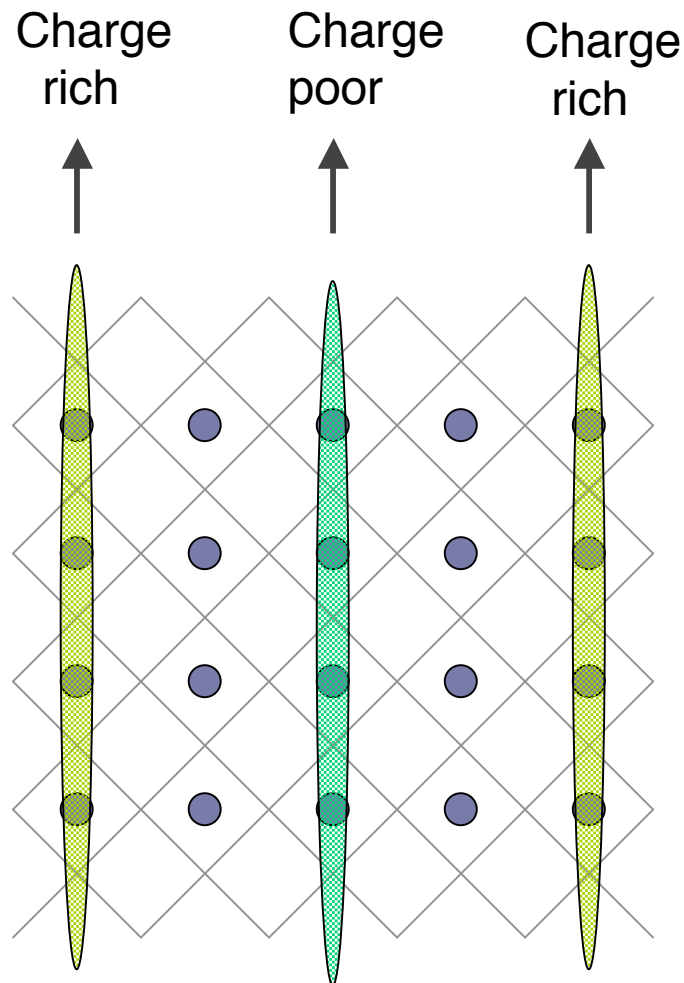


M.Fujita et al. Phys. Rev.B65,064505('02)

S.Wakimoto et al. PRB61, 3699('00)

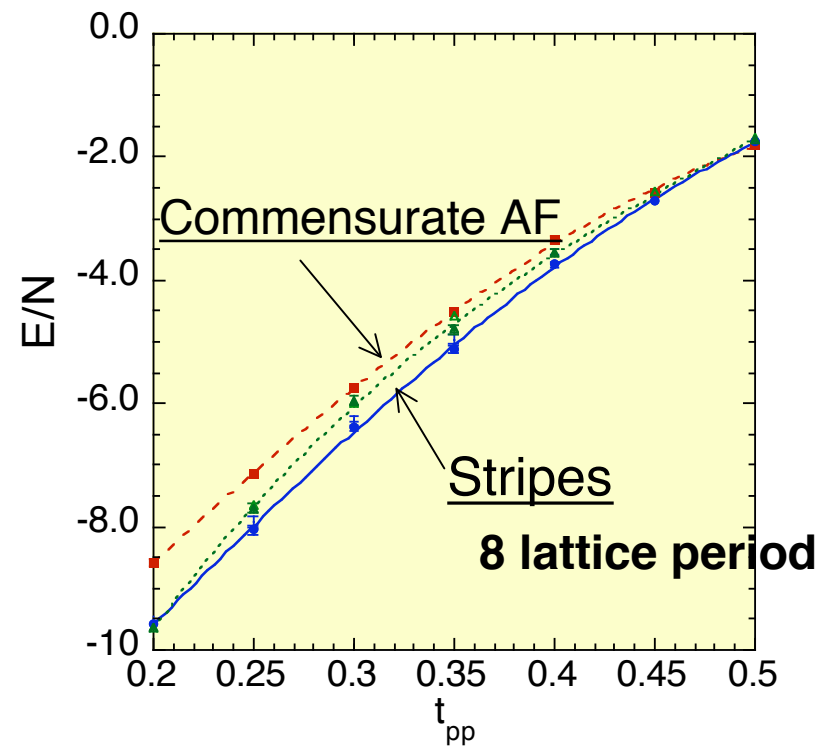
Vertical Stripes in the under-doped region

Vertical stripes: 8 lattice periodicity (Tranquada)



VMC

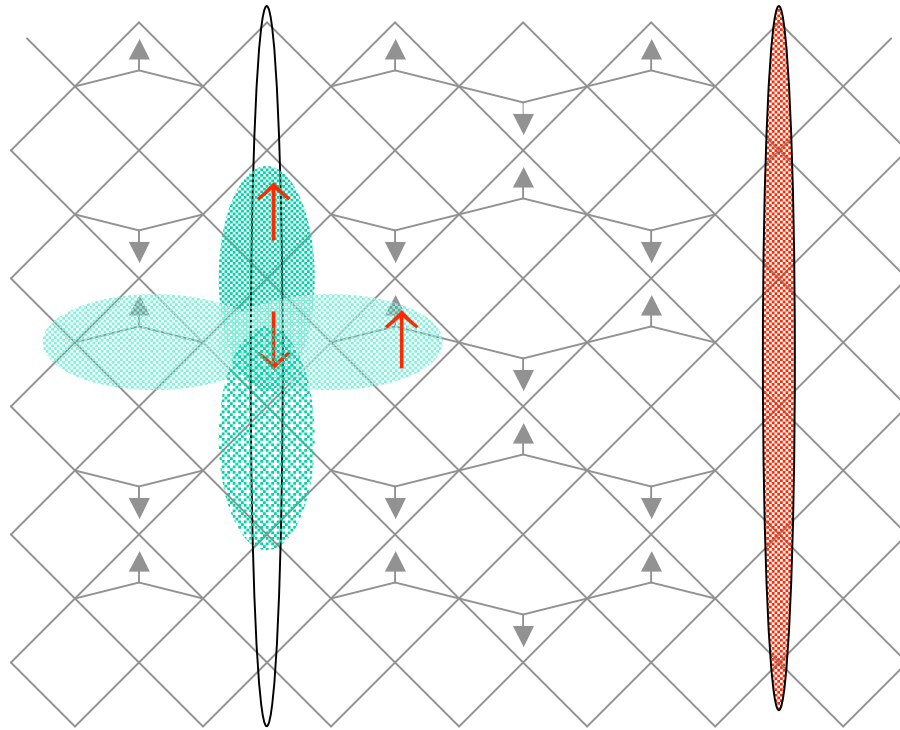
$x = 1/8$ 3-band Hubbard



T.Y. et al., J.Phys.C14,21('02)

Stripes and Superconductivity

Compete and Collaborate



Nano-scale SC

SC coexists with stripes (AF)

Bogoliubov-de Gennes eq.

$$\begin{pmatrix} H_{ij\uparrow} + F_{ij} \\ F_{ji}^* - H_{ji\downarrow} \end{pmatrix} \begin{pmatrix} u_j^\lambda \\ v_j^\lambda \end{pmatrix} = E^\lambda \begin{pmatrix} u_i^\lambda \\ v_i^\lambda \end{pmatrix}$$

$$\alpha_\lambda = u_i^\lambda a_{i\uparrow} + v_i^\lambda a_{i\downarrow}^+$$

$$\bar{\alpha}_\lambda = \bar{u}_i^\lambda a_{i\uparrow} + \bar{v}_i^\lambda a_{i\downarrow}^+$$

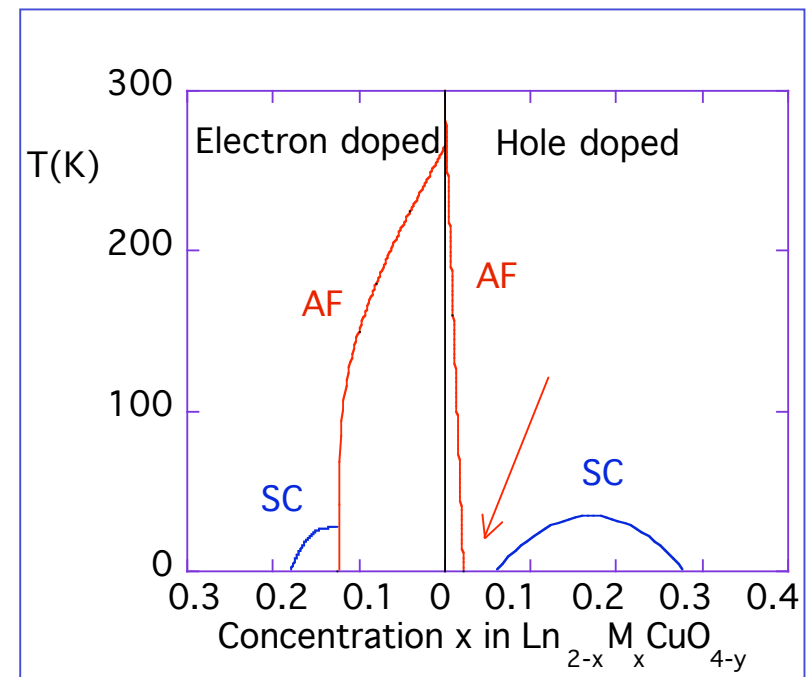
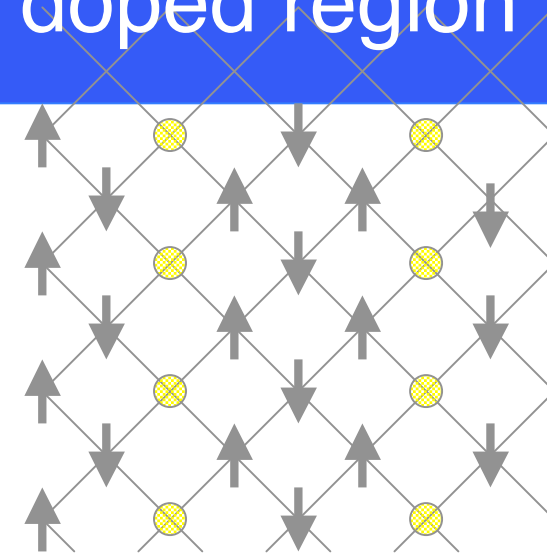
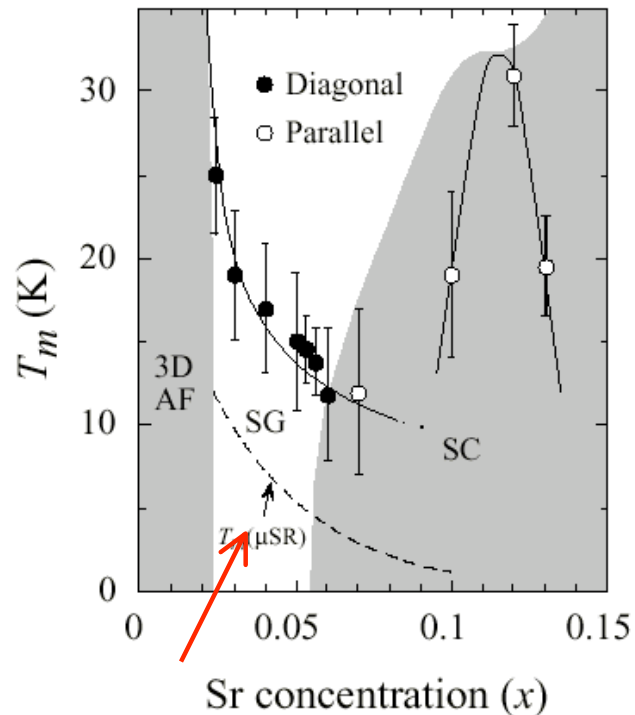
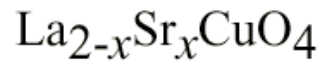
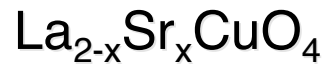
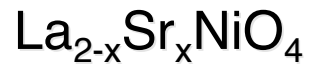
Wave function

$$V_{\lambda j} = v_j^\lambda \quad (\bar{U})_{\lambda j} = \bar{u}_j^\lambda$$

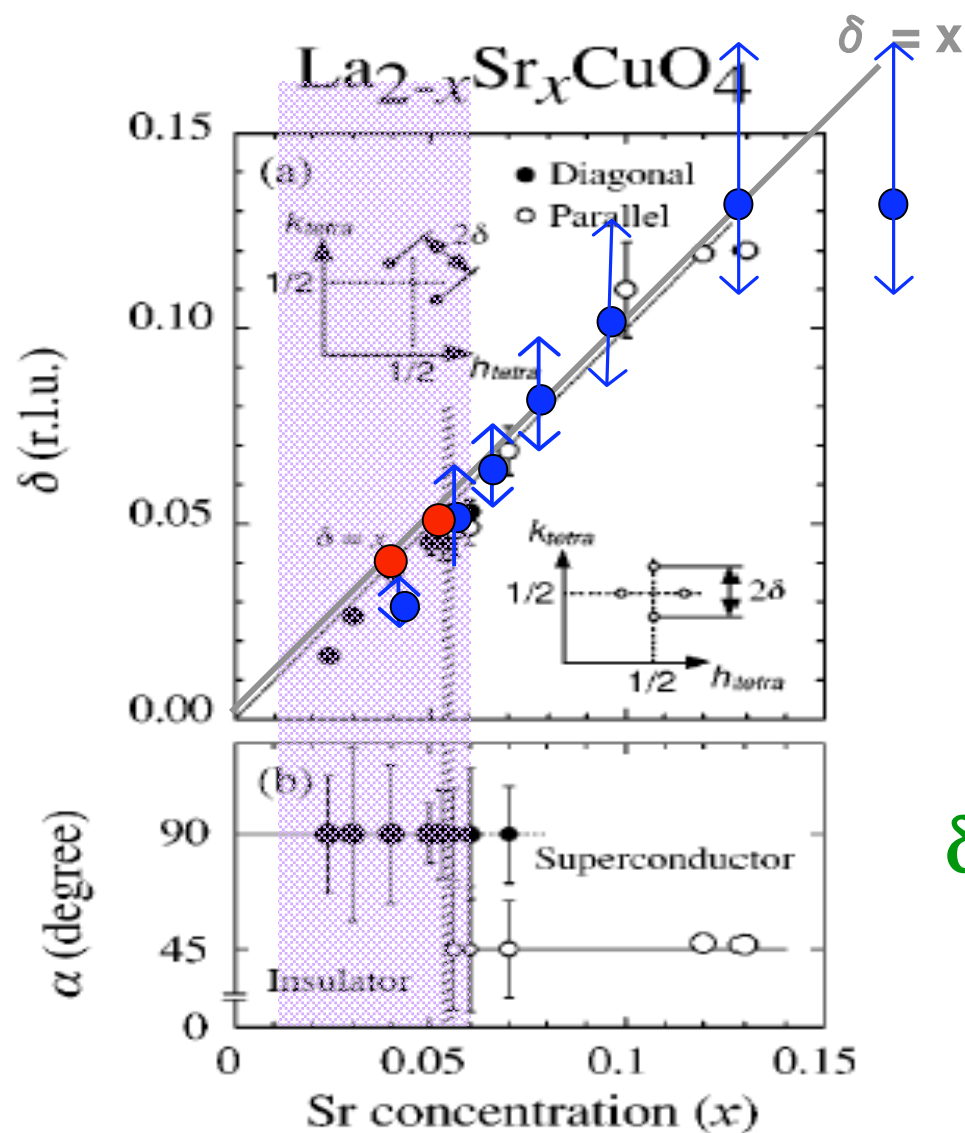
$$\psi_{SC} = P_G P_{N_e} \prod_\lambda \alpha_\lambda \bar{\alpha}_\lambda^+ |0\rangle \propto P_G \left(\sum_{ij} (U^{-1}V)_{ij} a_{i\uparrow}^+ a_{j\downarrow}^+ \right)^{N_e/2} |0\rangle$$

Diagonal stripes in lightly doped region

Diagonal stripes are observed for



Incommensurability: Comparison with Experiments



● Vertical stripes

● Diagonal stripes

$U=8.0$ $t'=-0.2$

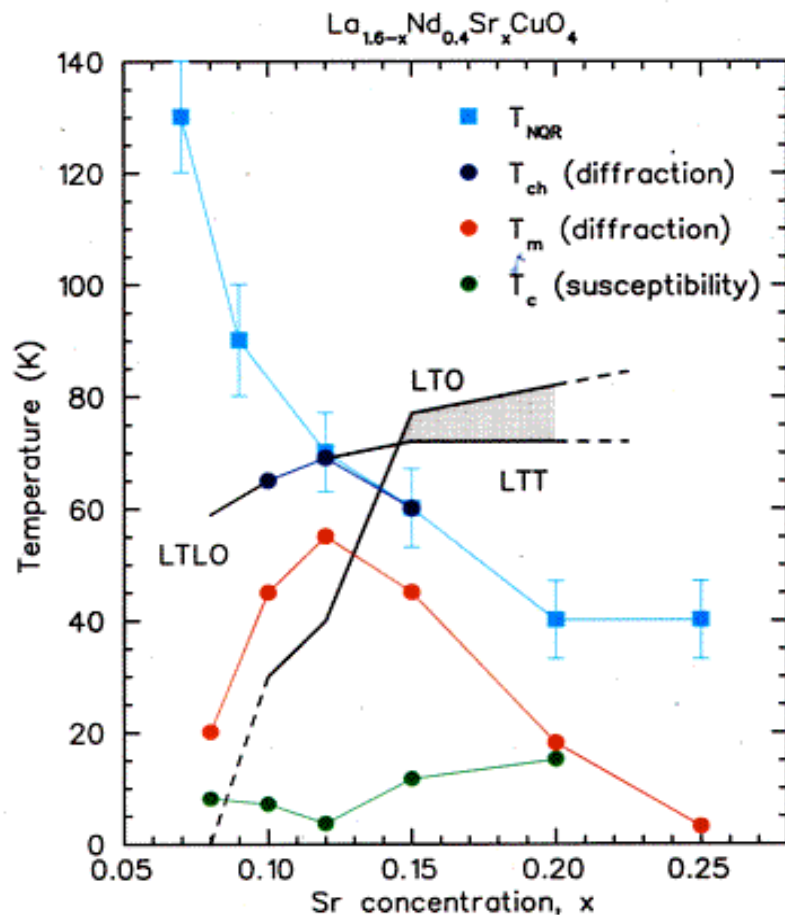
δ can be explained by 2D Hubbard model.

Stripes and Structural transition

Structural transitions: Lattice distortions

LTT, LTO, LTLO, HTT

Stripes: suggested by Incommensurability



N. Ichikawa et al.
PRL 85, 1738 ('00)

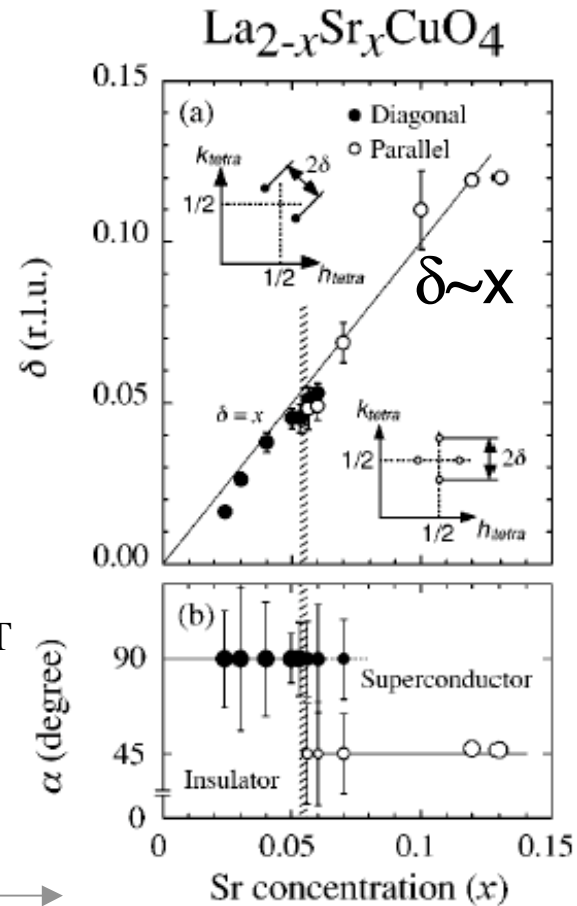
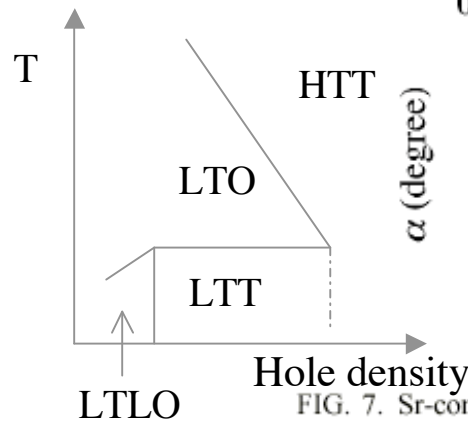


FIG. 7. Sr-concentration dependence of (a) the incommensurability δ and (b) the angle α defined in Fig. 3. Previous results for $x=0.024$ (Ref. 11), 0.04 (Ref. 10), 0.05 (Ref. 10), 0.12 (Ref. 5), 0.1 (Ref. 15), and 0.13 (Ref. 15) are included. In both figures, the solid and open symbols represent the results for the diagonal and parallel components, respectively.

M. Fujita et al. Phys. Rev. B 65, 064505 ('02)

What happens under lattice distortions?

1. Anisotropy of the transfer integrals

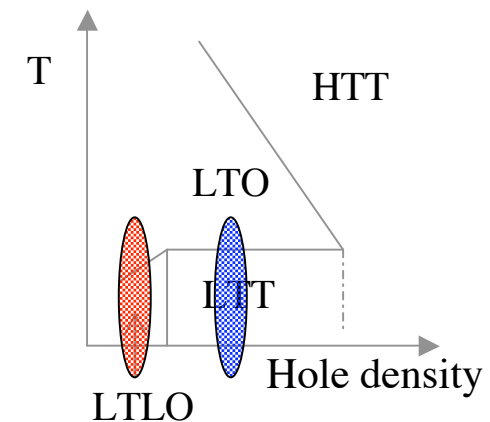
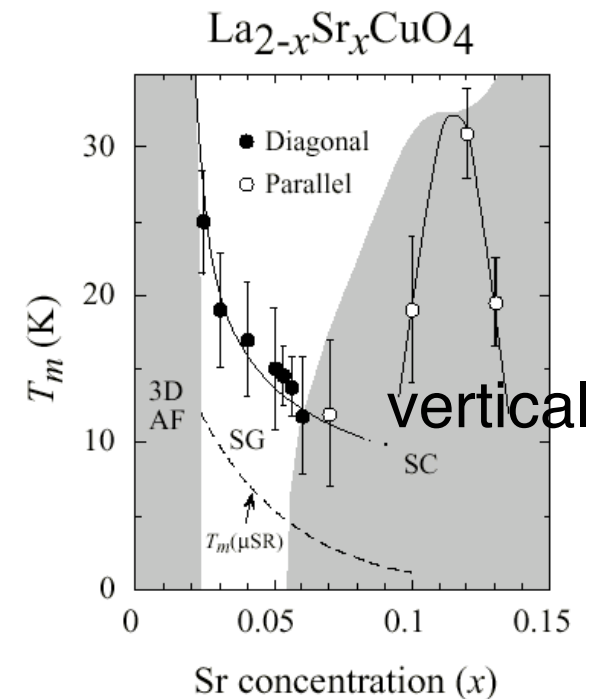
Anisotropic electronic state

vertical stripes

Diagonal stripes $x < 0.05$

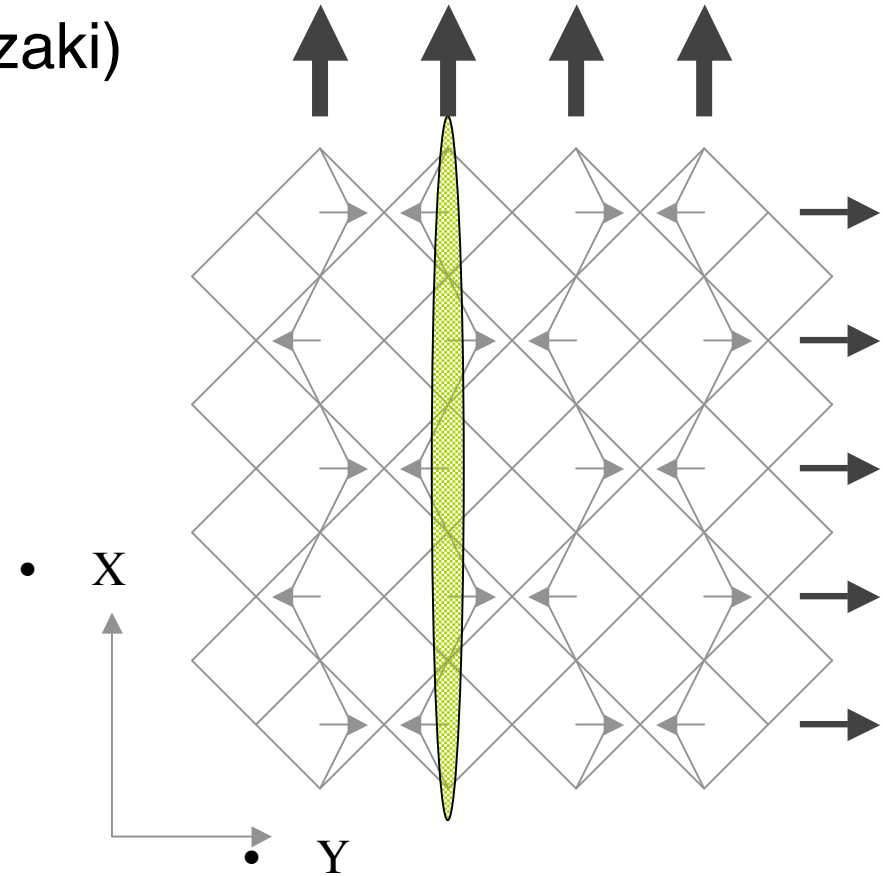
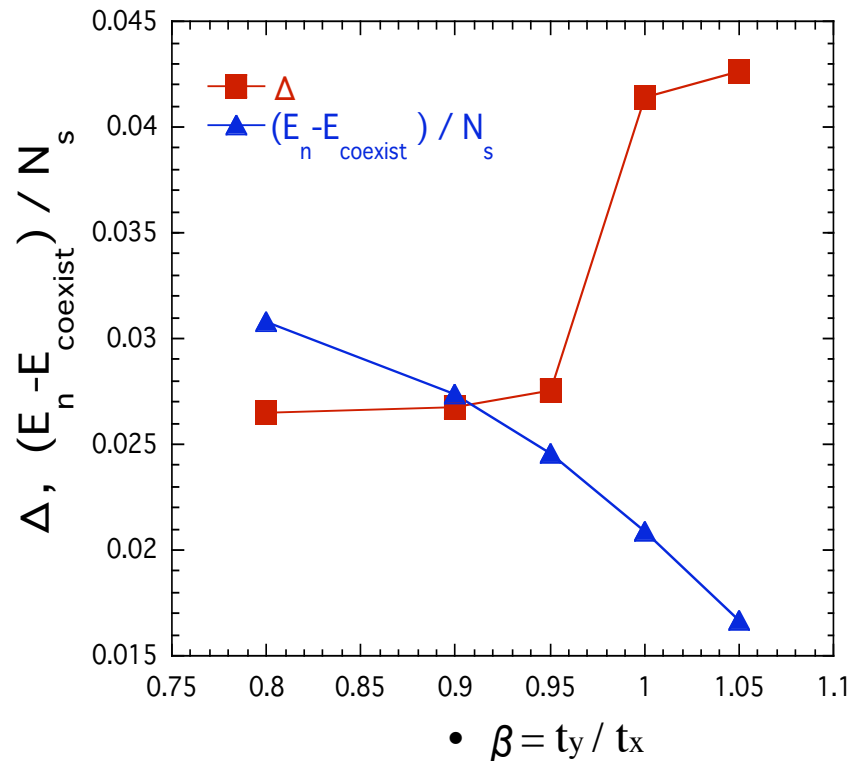
2. Spin-Orbit Coupling induced from lattice distortions

3. Electron-phonon interaction



Anisotropy of the transfer integrals in LTT phase

One-band Hubbard model (Miyazaki)



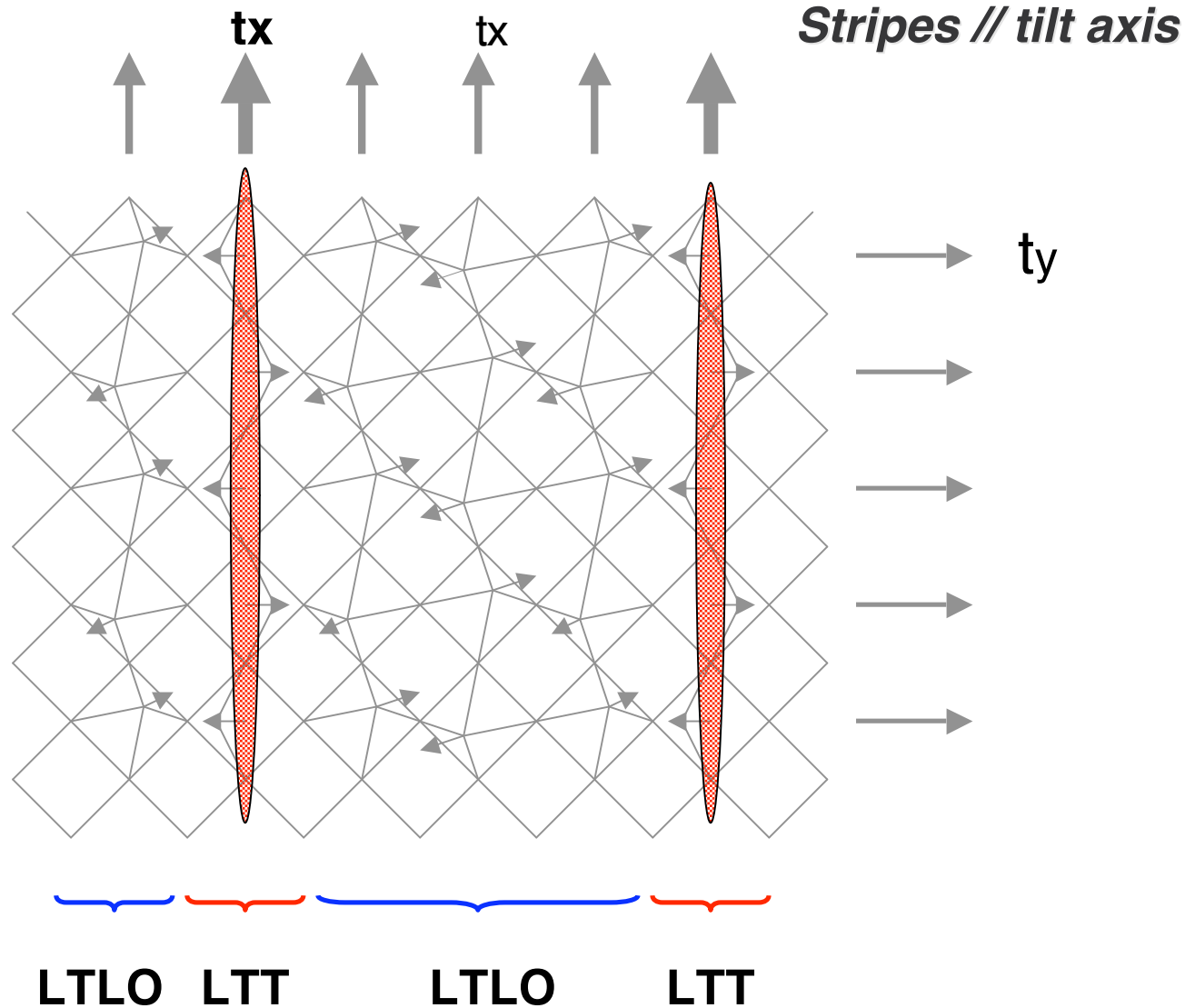
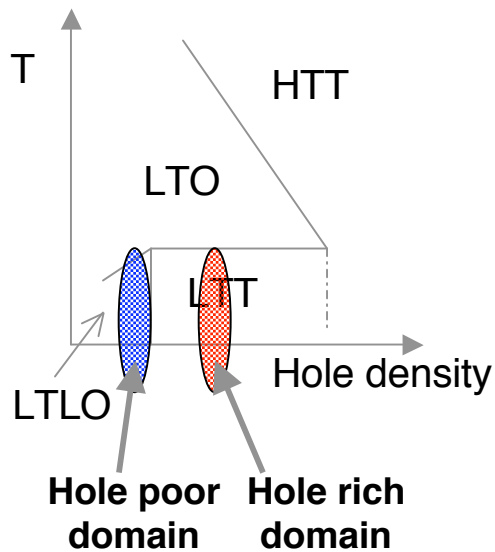
LTT structural transitions stabilize stripes.

Possible Stripe Structure 1

Mixed phase of
LTT and LTLO

Stabilize stripes

M. K. Crawford et. al.

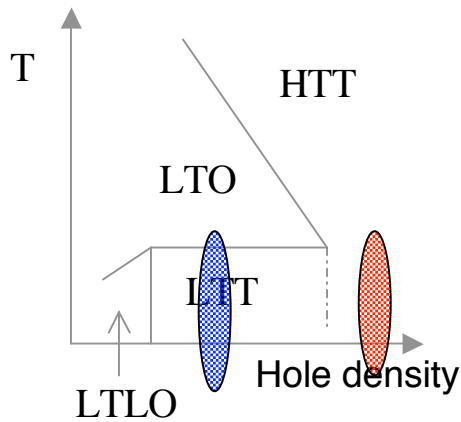


Possible Stripe Structure 2

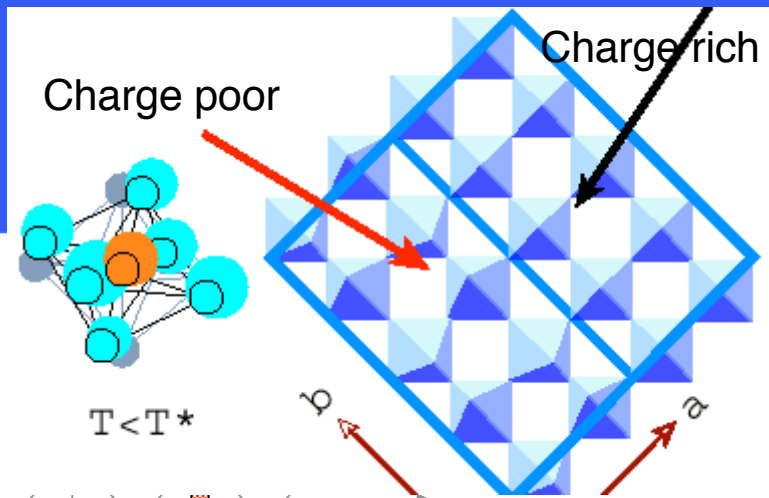
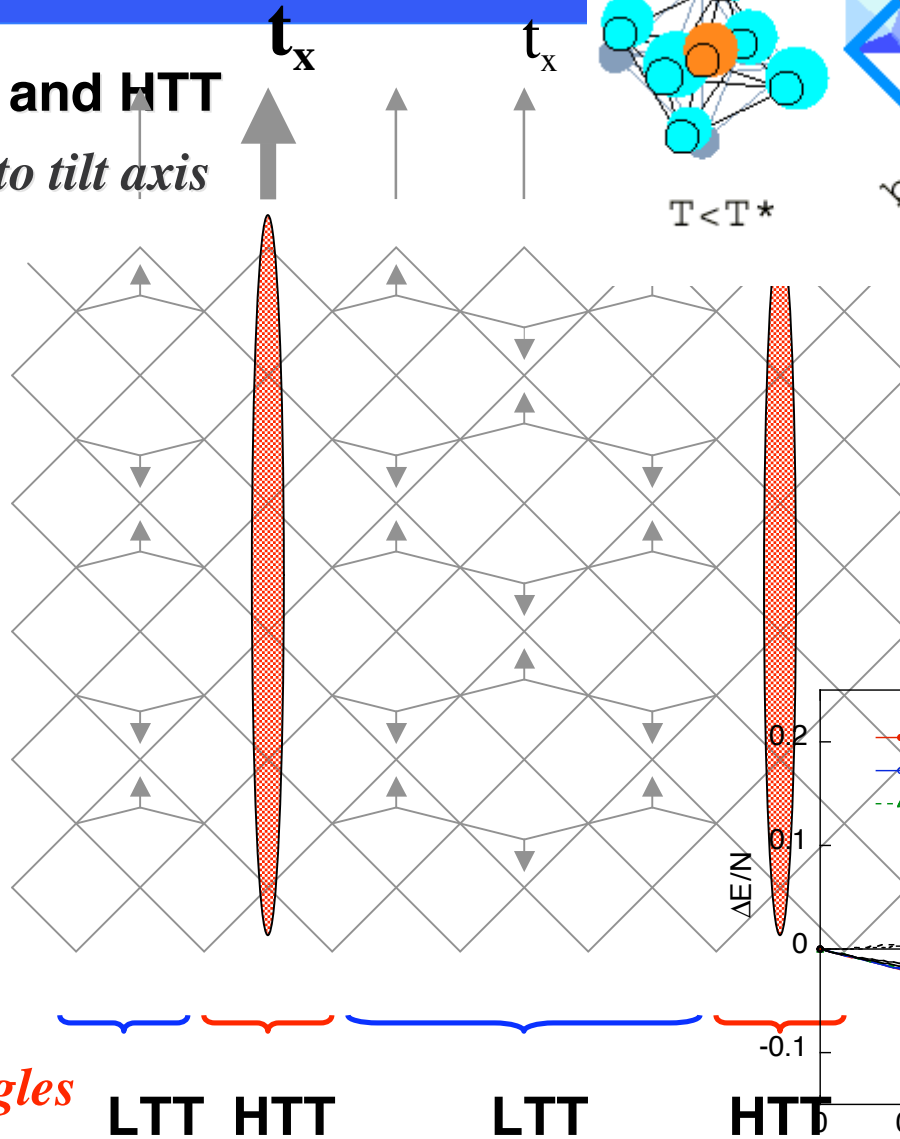
Mixed phase of LTT and HTT
Stripes perpendicular to tilt axis

Stable

M. K. Crawford et. al.

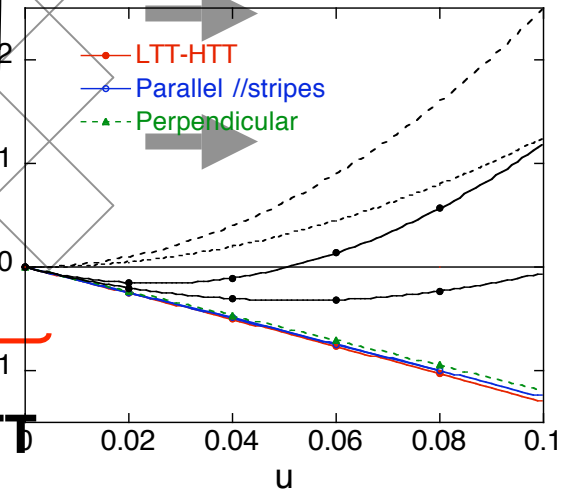


Oscillation of tilt angles



H. Oyanagi
 A. Bianconi

t_y



7. Spin-orbit coupling and Lattice distortion

Spin-Orbit Coupling induced by the Lattice distortion

Tilting

Friedel et al., J.Phys.Chem.Solids 25, 781 (1964)

$$\langle p_x(x - a/2, y) \uparrow | H_{dp} | d_{xz}(r) \uparrow \rangle = -t_{xz} e^{-ik_x/2 \cdot a}$$

$$\langle p_y(x, y - a/2) \uparrow | H_{dp} | d_{yz}(r) \uparrow \rangle = -t_{yz} e^{-iky/2 \cdot a}$$

$$H_{SO} = \xi(r) L \cdot S$$

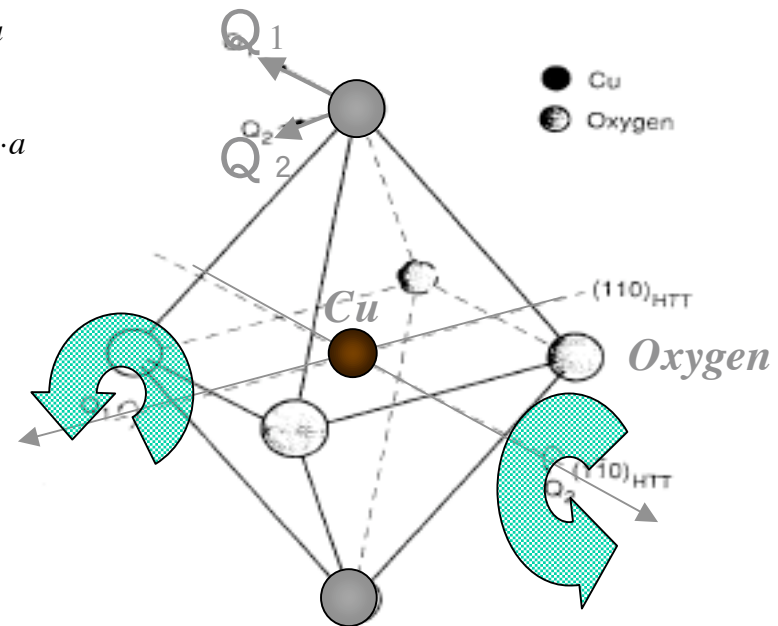
$$\langle d_{xz}(r) \uparrow | H_{SO} | d_{yz}(r) \uparrow \rangle = -\frac{i}{2} \xi$$

$$\langle d_{yz}(r) \uparrow | H_{SO} | d_{xz}(r) \uparrow \rangle = \frac{i}{2} \xi$$

$$\langle d_{x^2-y^2}(r) \uparrow | H_{SO} | d_{yz}(r) \downarrow \rangle = \frac{i}{2} \xi$$

$$\langle d_{x^2-y^2}(r) \uparrow | H_{SO} | d_{xz}(r) \downarrow \rangle = \frac{1}{2} \xi$$

Effective $i\xi$ term for p-p transfer



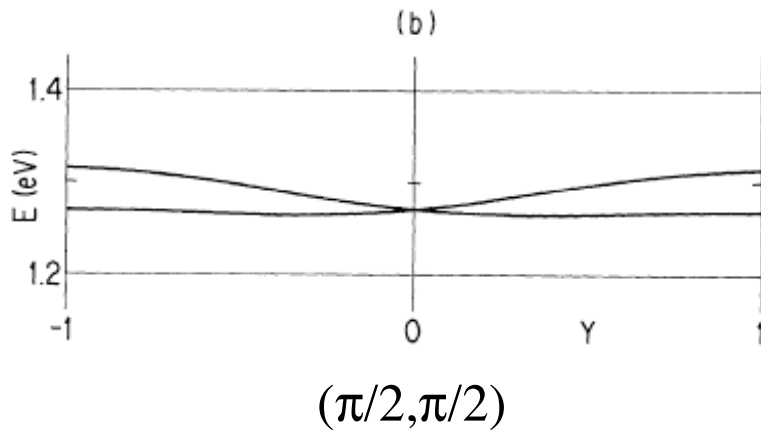
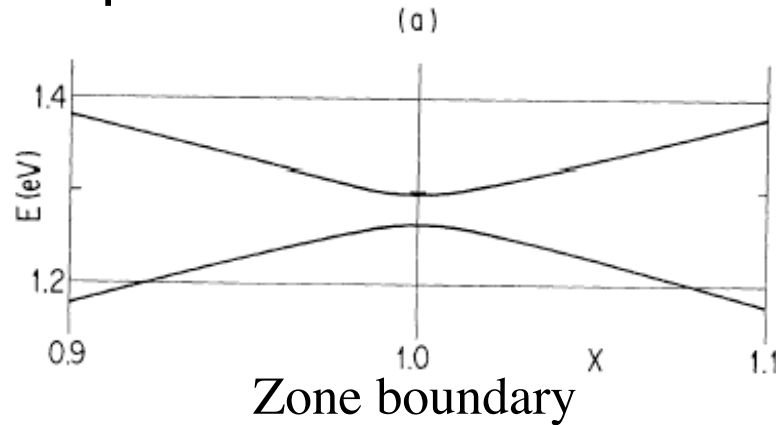
$$t_{xz}, t_{yz} \neq 0 \sim \text{tilt angle}$$

Five orbitals \times ($\uparrow\downarrow$):

$$(d_{x^2-y^2}, d_{xz}, d_{yz}, p_x, p_y)$$

Dispersion in the presence of spin-orbit coupling

d-p model

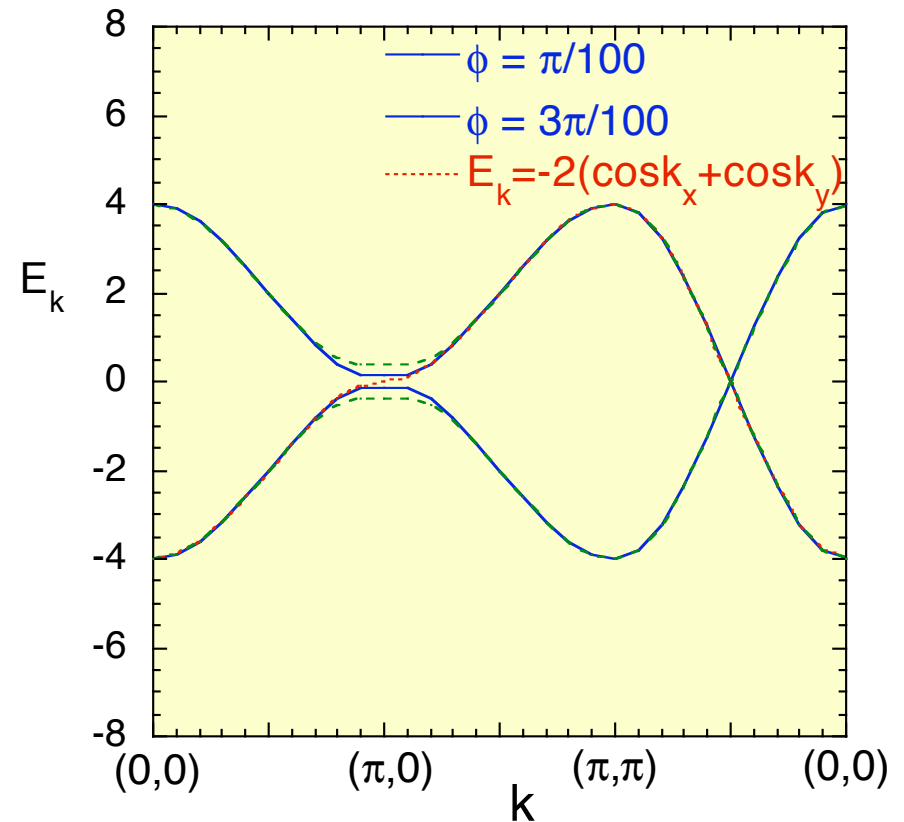


$\xi = 0.4$ (K. Yamaji, JPSJ(1988))

One-band effective model

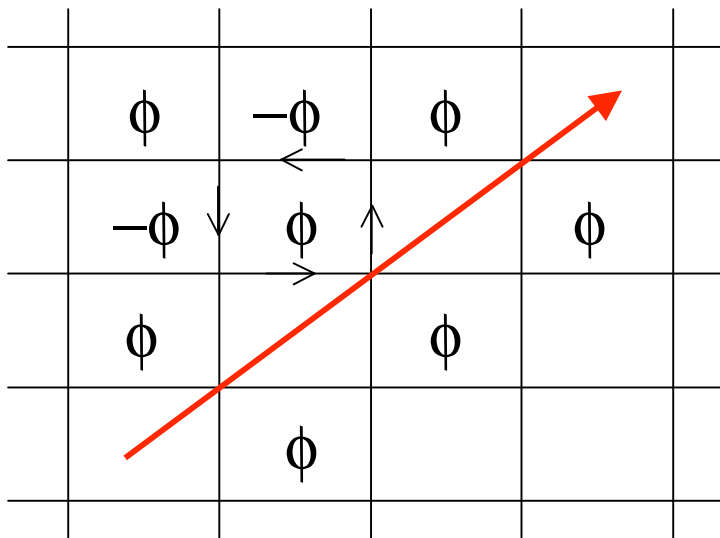
$$H_{kin} = -\sum_{ij\sigma} (t_{ij} + ic \sigma \theta_{ij}) d_{i\sigma}^+ d_{j\sigma}$$

(Bonesteal et al., PRL68,2684('92))



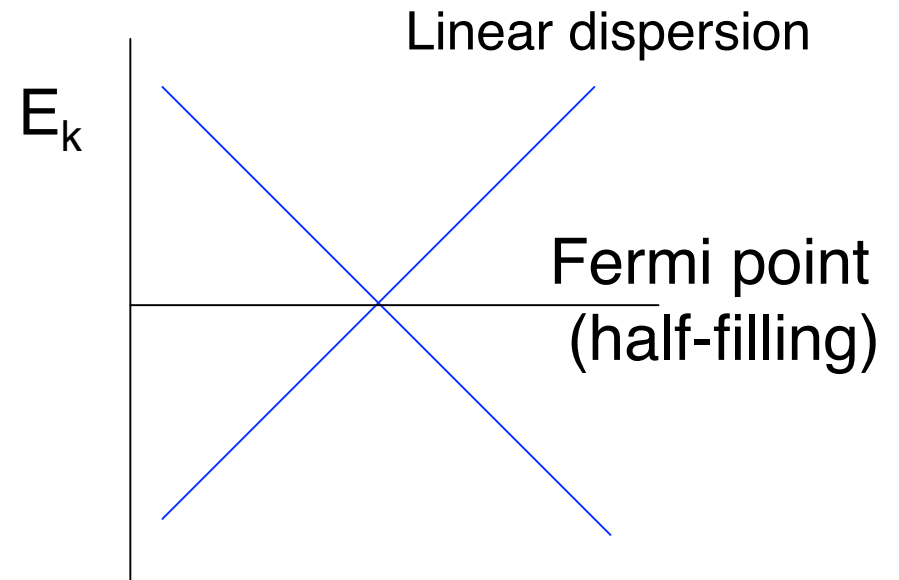
Flux state

$$E(k_x, k_y) = \pm \left| e^{i\phi/4} e^{ik_x} + e^{-i\phi/4} e^{ik_y} + e^{i\phi/4} e^{-ik_x} + e^{-i\phi/4} e^{-ik_y} \right|$$



Inhomogeneous d-density wave

Excitation: Dirac fermion



Small Fermi surface

Pseudo-gap in the density of states

Flux state

→ Pseudo-gap

An origin of pseudo-gap

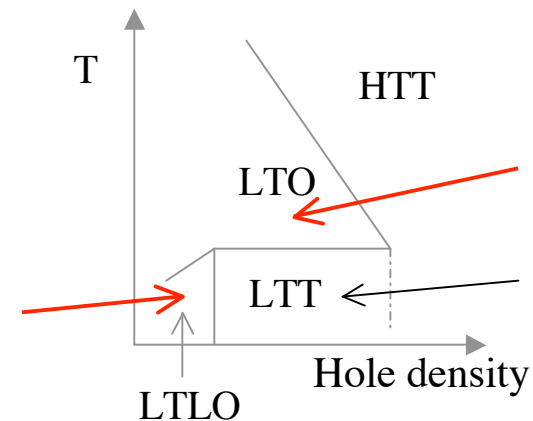
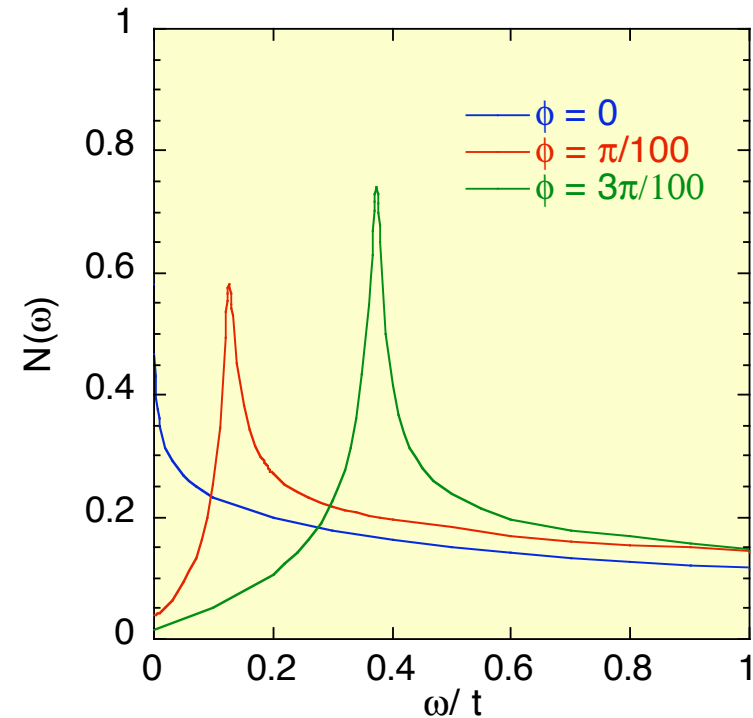
Density of states

$$N_{\sigma}(k, \varepsilon) = -\frac{1}{\pi} \text{Im} G_{\sigma}(k, \varepsilon + i\delta)$$

Eigenfunction $\varphi_{\sigma m}(r)$

$$H\varphi_{\sigma m}(r) = E_{\sigma m} \varphi_{\sigma m}(r)$$

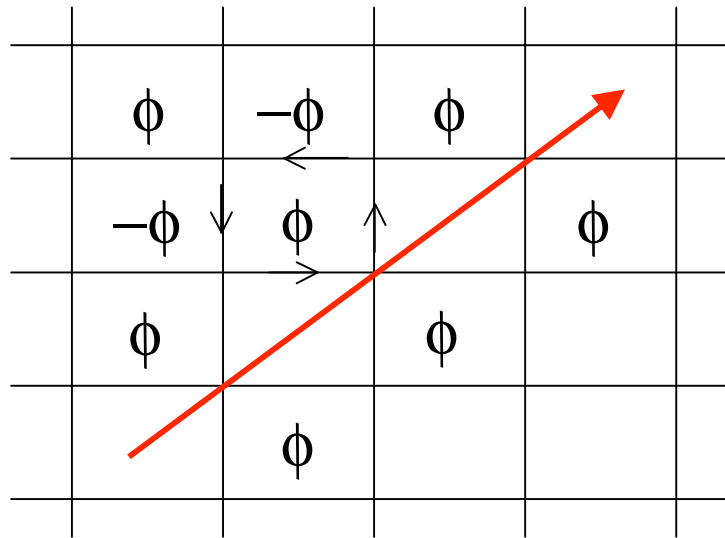
$$G_{\sigma}(r, r', i\omega) = \sum_m \frac{\varphi_{\sigma m}(r)\varphi_{\sigma m}^*(r')}{i\omega - E_{\sigma m}}$$



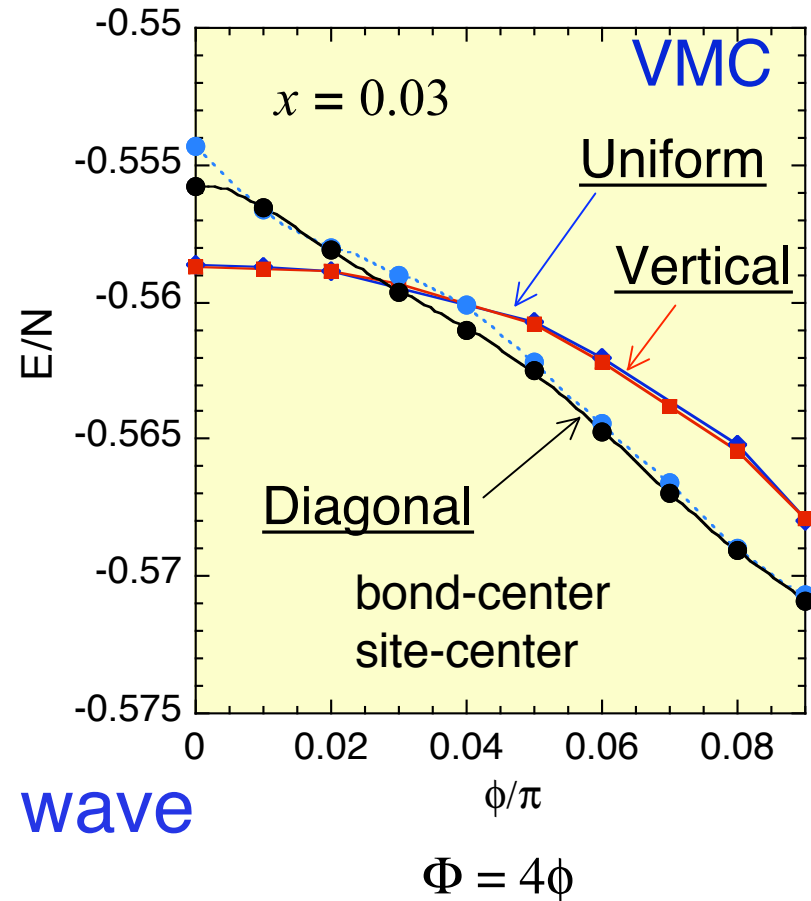
Diagonal stripes with Spin-orbit coupling

Spin-orbit coupling induces flux.

Spin-orbit coupling stabilizes the diagonal stripes.



Diagonal Stripe & d-density wave



d-density wave

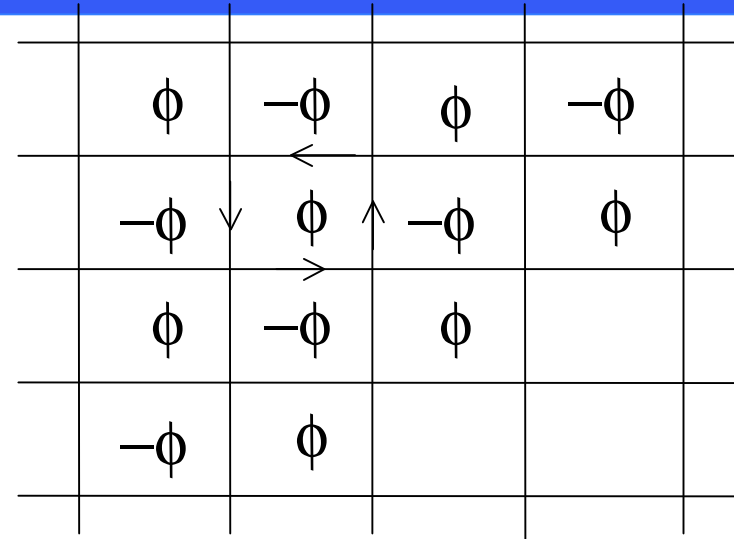
d-density wave

$$i\Delta_Q Y(k) = \langle c_{k+Q\sigma}^+ c_{k\sigma} \rangle \quad Q = (\pi, \pi)$$

$$Y(k) = \cos(k_x) - \cos(k_y)$$

Nayak, Phys. Rev. B62, 4880 ('00)

Chakravarty et al., PRB63, 094503 ('01)



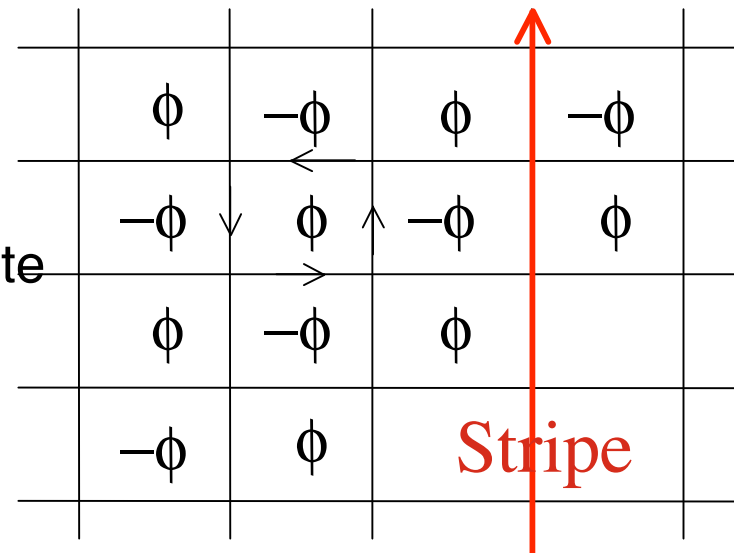
Inhomogeneous density wave

$$i\Delta_Q Y(k) = \langle c_{k+Q\sigma}^+ c_{k\sigma} \rangle \quad \text{d-symmetry}$$

$$\Delta_{lQ_s\sigma} = \sum_k \langle c_{k+lQ_s\sigma}^+ c_{k\sigma} \rangle \quad \text{incommensurate}$$

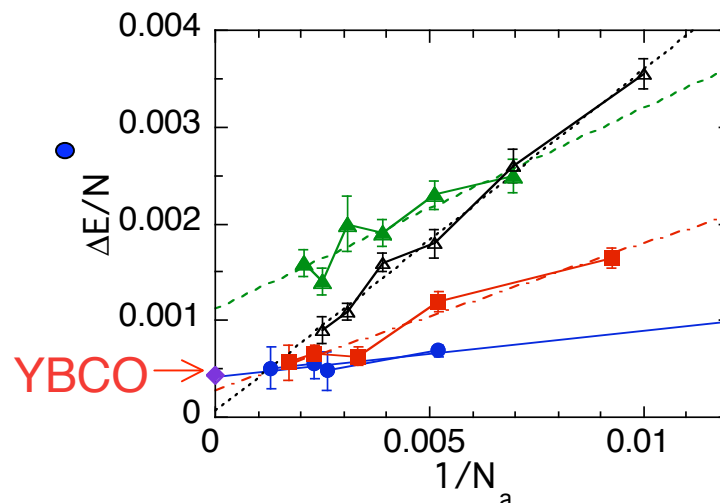
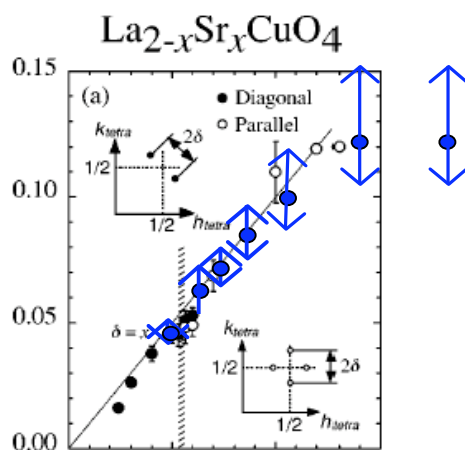
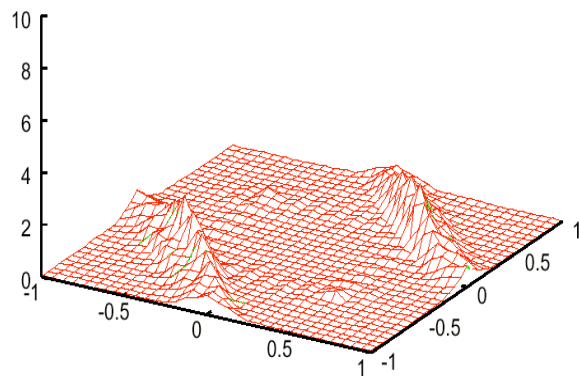
$$Q_s = (\pi + 2\pi\delta, \pi) \quad \text{vertical}$$

$$Q_s = (\pi + 2\pi\delta, \pi + 2\pi\delta) \quad \text{diagonal}$$

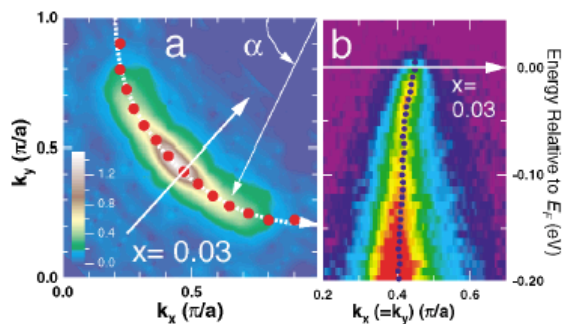


8. Summary

$$\psi_{CDs} = P_G \prod_k (u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+) |0\rangle$$

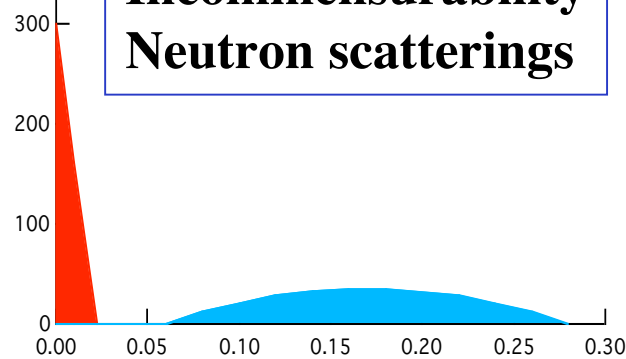


Explanation of spectra



ARPES Measurements

Incommensurability Neutron scatterings



underdope

overdope

Theoretical estimate of SC condensation energy

Agreement with Exp.

$$E_{\text{cond}} \sim 0.2 \text{meV}$$