時系列信号の学習を行うニューラルハードウェアの記憶容量評価
Storage Capacities of Temporal-Coding Neural Hardware

藤田 大地 (PY), トバー ジェシカ マリア, 浅井 哲也, 雨宮 好仁
Daichi Fujita(PY), Gessyca Maria Tovar, Tetuya Asai and Yoshihito amemiya
北海道大学 大学院情報科学研究科
fujita@sapiens-ei.eng.hokudai.ac.jp

Abstract— On the basis of a model for the storage of temporal sequences [1], we proposed a neural model that is suitable for implementation with analog metal-oxide-semiconductor (MOS) circuits, and demonstrated the circuit operations [2]. Through extensive numerical and circuit simulations, we here evaluate the storage capacity by introducing complexity of input patterns and pattern overlaps between the input and output sequences.

Keywords— **Neural hardware, neural networks, temporal coding, storage capacity**

1 Target Model and Demonstrations

Fukai proposed a model for the storage of temporal sequences in [1]. The main function of this model is learning and recalling the temporal input stimuli. Based on this model, we proposed a modified model for learning and recalling temporal sequences that is suitable for implementation with analog MOS circuits [2]. The modified model is shown in Fig. 1. As in the original model, the primary function is to learn (record) temporal input sequence $I(t)$ of length T and to recall it as recorded sequence $u(t)$. The model consists of N neural oscillators whose outputs $Q_i(t)$ $(i = 1, ..., N)$ are time-varying periodic square waves with different fundamental frequencies. Each of the oscillators is connected to an output cell through synaptic connections whose weights are denoted by w_i $(i = 1, ..., N)$. The output cell calculates the weighted sum of the oscillator's outputs as

$$u(t) = \sum_{i=1}^{N} w_i Q_i(t). \quad (1)$$

Through cyclic learning processes, w_is in Eq. (1) are updated at every cycle to achieve $u(t) \to I(t)$. Notice that this expression, *i.e.*, a weighted sum of square-wave functions with various fundamental frequencies, corresponds to a form of the Walsh series expansion [3] which is a mathematical method to approximate a certain class of functions, like the Fourier series expansion.

Now, given a periodic input signal $(I(t))$ with period T and the output $(u(t))$, we define the mean square error (E) between them as:

$$E = \frac{1}{2T}\int_{jT}^{(j+1)T} [I(t) - u(t)]^2\, dt \quad (j = 0, 1, 2, \cdots), \quad (2)$$

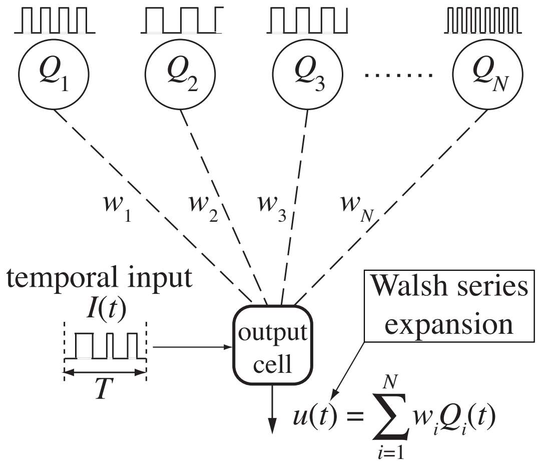

Fig. 1: Proposed temporal coding model.

where j represents the learning cycle. To learn the input signal $(I(t))$ correctly, we need to minimize this error. This is achieved by modifying the weights (w_i) between the oscillators and the output cell according to the gradient descent rule:

$$\delta w_i = -\eta \frac{\partial E}{\partial w_i}, \quad (3)$$

where η represents a small positive constant indicating the learning rate. Substituting E in Eq. (2) into Eq. (3), we obtain

$$\delta w_i = \frac{\eta}{T}\int_{jT}^{(j+1)T} [I(t) - u(t)] Q_i(t)\, dt. \quad (4)$$

The weights are updated at the end of each learning cycle $(t = (j + 1)T)$ as

$$w_i^{\text{new}} = w_i^{\text{old}} + \delta w_i. \quad (5)$$

The procedures above, *i.e.*, numerical calculations of Eqs. (1), (4) and (5), are repeated $(j = 0, 1, \cdots)$ until the error between the input and the output becomes small enough.

Numerical simulations were conducted to confirm the operation of the model. In the simulation, output of the oscillatory units $Q_i(t)$ was defined by:

$$Q_i(t) = H[\sin(2\pi f_i t)] \quad (6)$$

where f_i represents the random frequency distributed

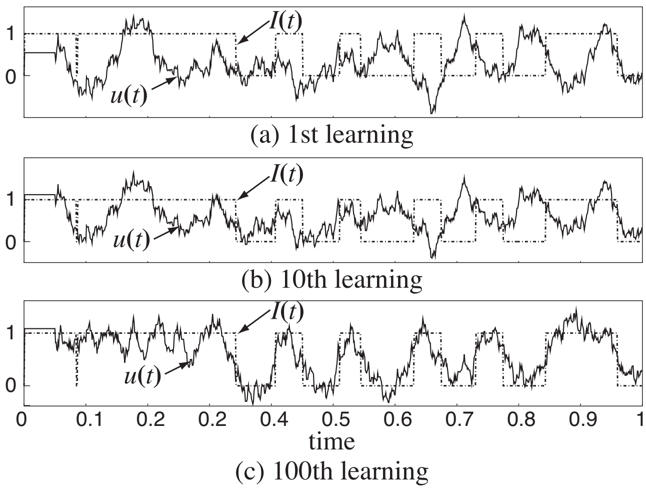

Fig. 2: Input ($I(t)$) and output sequences ($u(t)$) of proposed network with 200 oscillatory units after first (a), 10th (b) and 100th learning (c).

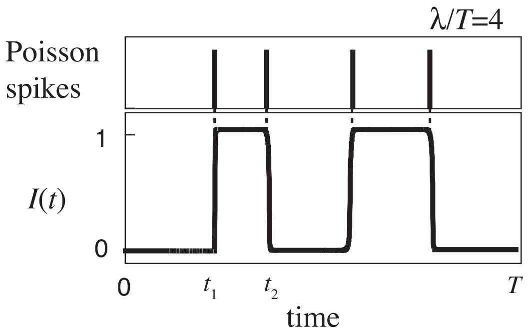

Fig. 3: Input sequence ($I(t)$ generated by Poisson spikes with $\lambda = 4$.

between 1 and 10, and $H(x)$ is the step function defined as:

$$H(x) = \begin{cases} 1 & (x > 0) \\ 0 & (x < 0) \end{cases}. \tag{7}$$

The results are shown in Fig. 2 ($N = 200$, $T = 1$ and $\eta = 0.01$). After the first learning (Fig. 2(a)), the input ($I(t)$) and the output sequences ($u(t)$) were completely different, however, $u(t)$ approached to $I(t)$ as repeating the learning (Figs. 2(b) for 10th and (c) for 100th learning).

2 Numerical Estimation of Storage Capacity

We evaluate the storage capacity of the proposed network by defining pattern overlaps between the input and output sequences, as a function of N and complexity of input sequences. To define the complexity ($\equiv \lambda$), we use Poisson spikes whose mean firing rate is represented by λ. Let us assume binary input sequence $I(t)$ with period T and $I(0) =$ "0". The expected number of spikes within period T is thus λ/T. The value of the input sequence is flipped and kept

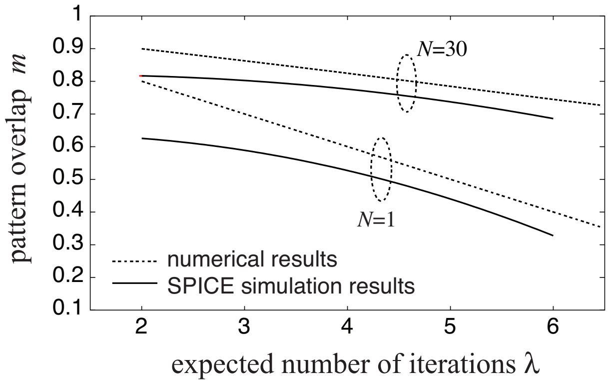

Fig. 4: Numerical and SPICE results showing pattern overlaps between input and output sequences for different Ns and complexity of input sequence λ.

when a spike is generated, *i.e.*, $I(t)$ ($t > 0$) remains "0" if no spikes were generated, whereas $I(t)$ ($t > t_1$) is flipped to "1'' when a spike is generated at $t = t_1$. When the subsequent spike is generated at $t = t_2$, $I(t)$ ($t > t_2$) is flipped to "0". Figure 3 shows the examples with $\lambda/T = 4$. This process is repeated while $t \leq T$

The pattern overlap between the input ($I(t)$) and the output sequences ($u(t)$) is defined by

$$m \equiv \frac{4}{T} \int_0^T \left(I(t) - \frac{1}{2} \right) \times \left[H\left(u(t) - \frac{1}{2} \right) - \frac{1}{2} \right] dt, \tag{8}$$

where $I(t)$ is expanded to ± 1, and Boolean values of threshold evaluation ($u(t) > 0.5$) is also expanded to ± 1. We calculated the pattern overlaps of the previously proposed MOS circuits (see [2] for the circuit details) for different sets of input sequences (λ). The calculations were carried out for 1 and 30 neuron networks. Figure 4 shows the averaged pattern overlap between 10 different sets of the input sequences and their outputs. For the comparison, numerical results of the network model with the same number of neurons were superimposed in the figure. This result shows that the circuit network of $N = 30$ can retrieve input sequence of $\lambda/T = 6/(0.7\ \mu\text{s}) \approx 8.6 \times 10^6$ (s^{-1}) with the accuracy of 72% ($m \approx 0.72$), which indicates that the circuit can learn and recall temporal sequence of 4.3 MHz under our device setups.

References

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