Calculating noise-to-signal ratios in the horizontal-motion components of circular-array microtremor data

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Formula

An estimate for the signal-to-noise (SN) ratio ε_H in the horizontal-motion components of circular-array microtremor records is obtained by the formula:

$$\varepsilon_H = \frac{\rho_{\text{SPAC−R}}\rho_{\text{SPAC+L}} - (\rho_{\text{SPAC−R}} + \rho_{\text{SPAC+L}})\rho_{\text{Y}} + \rho_{\text{X}}^2}{(\rho_{\text{SPAC−R}} + \rho_{\text{SPAC+L}})\rho_{\text{Y}} - \rho_{\text{X}}^2 + 2/N}, \tag{1}$$

where

$$\rho_{\text{SPAC−R}} = \frac{G_{R1R0}(r,r;\omega)}{G_{R1R0}(0,r;\omega)} \tag{2}$$

is the spectral ratio of the SPAC-R method (equation B2 in Tada *et al*., 2009),

$$\rho_{\text{SPAC+L}} = \frac{G_{R1T0}(r,r;\omega)}{G_{R1T0}(0,r;\omega)} \tag{3}$$

is the spectral ratio of the SPAC+L method (equation 10 in Tada *et al*., 2009),

$$\rho_{\text{X}}^2 = \frac{G_{R1R1}(r,r;\omega)}{G_{R1R1}(0,0;\omega)}, \tag{4}$$

$$\rho_{\text{Y}} = \frac{G_{R1R1}(0,r;\omega)}{G_{R1R1}(0,0;\omega)}, \tag{5}$$

ω denotes the frequency and N is the number of sensors distributed evenly around the circle of radius r.

Notations in the form of $G_{XmYn}(r_1, r_2; \omega)$, where X and Y may stand for either of the subscripts Z (for vertical motion), R (radial component of the horizontal motion) and T (tangential component of the horizontal motion), represent the cross-spectral density between two time series $X_m(t, r_1)$ and $Y_n(t, r_2)$. The time series $X_m(t, r)$ is in turn defined as the mth-order coefficient in the azimuthal Fourier series expansion of the X-component record of microtremors around a circle of radius r (equations 48 and 57 in Cho *et al*., 2006; equations 16 and 17 in Tada *et al*., 2009; equation A.1 and following text in Tada *et al*., 2010).

Derivation

Let us make the following assumptions regarding noise in the circular-array microtremor records:

(i) The *N* seismic sensors are distributed evenly around the circle of radius *r*;

(ii) The seismograms include noise that is uncorrelated with the microtremor signals (i.e. propagating Rayleigh- and Love-wave components);

(iii) The noise is incoherent (uncorrelated) from one sensor to another;

(iv) The noise is incoherent from one component to another in the records of an identical sensor;

(v) The noise has identical intensities for all sensors;

(vi) The noise has identical intensities for the two components of horizontal motion at an identical station.

In this case, it follows from Cho *et al.*'s (2006) theory (see also Tada *et al.*, 2010) that, when directional aliasing and the presence of multiple modes of Rayleigh and Love waves are disregarded, the spectral densities $G_{R1R1}(r,r;\omega)$, $G_{R1R1}(0,r;\omega)$ and $G_{R1R1}(0,0;\omega)$ have the following expressions:

$$G_{R1R1}(r,r;\omega) = \pi^2\left(f_0^R(\omega) + f_0^L(\omega)\right)\left[(J_0 - J_2)^2(rk^R(\omega))\gamma^R(\omega) + (J_0 + J_2)^2(rk^L(\omega))\gamma^L(\omega) + 2\varepsilon_H(\omega)/N\right] \tag{6}$$

$$G_{R1R1}(0,r;\omega) = \pi^2\left(f_0^R(\omega) + f_0^L(\omega)\right)\left[(J_0 - J_2)(rk^R(\omega))\gamma^R(\omega) + (J_0 + J_2)(rk^L(\omega))\gamma^L(\omega)\right] \tag{7}$$

$$G_{R1R1}(0,0;\omega) = \pi^2\left(f_0^R(\omega) + f_0^L(\omega)\right)\left[1 + \varepsilon_H(\omega)\right], \tag{8}$$

where $f_0^R(\omega)$ is the intensity of horizontal motion relative to Rayleigh waves, $f_0^L(\omega)$ its Love-wave counterpart, $J_m(\cdot)$ the *m*th-order Bessel function of the first kind, $k^R(\omega)$ the wavenumber of Rayleigh waves, $k^L(\omega)$ its Love-wave counterpart, $\gamma^R(\omega)$ the power share of Rayleigh waves in horizontal motion ($f_0^R(\omega)$ divided by $f_0^R(\omega) + f_0^L(\omega)$), and $\gamma^L(\omega)$ the power share of Love waves in horizontal motion. The signal-to-noise ratio, $\varepsilon_H(\omega)$, is defined as the total power of noise in the two components of horizontal motion divided by $f_0^R(\omega) + f_0^L(\omega)$, the total power of microtremor signals in the two components of horizontal motion.

Equations (6)-(8) involve six unknown quanties: $f_0^R(\omega) + f_0^L(\omega)$, $(J_0 - J_2)(rk^R(\omega))$, $(J_0 + J_2)(rk^L(\omega))$, $\gamma^R(\omega)$, $\gamma^L(\omega)$ and $\varepsilon_H(\omega)$. The number of equations and the number of unknowns can be matched by taking note of the following relations:

$$\gamma^R(\omega) + \gamma^L(\omega) = 1 \tag{9}$$

$$\rho_{\mathrm{SPAC-R}} = (J_0 - J_2)(rk^R(\omega)) \tag{10}$$

(the basic equation of the SPAC-R method; equation B2 in Tada *et al.*, 2009) and

$$\rho_{\mathrm{SPAC+L}} = (J_0 + J_2)(rk^L(\omega)) \tag{11}$$

(the basic equation of the SPAC+L method; equation 13 in Tada *et al*., 2009). Solving the simultaneous set of equations (6)-(11) for $\varepsilon_H(\omega)$ produces equation (1).

References

Cho, I., T. Tada, and Y. Shinozaki, 2006, A generic formulation for microtremor exploration methods using three-component records from a circular array, *Geophysical Journal International,* **165**, 236-258.

Tada, T., I. Cho, and Y. Shinozaki, 2009, New circular-array microtremor techniques to infer Love-wave phase velocities, *Bulletin of the Seismological Society of. America*, **99**, 2912-2926.

Tada, T., I. Cho, and Y. Shinozaki, 2010, Analysis of Love-wave components in microtremors, Joint Conference Proceedings, 7th International Conference on Urban Earthquake Engineering (7CUEE) & 5th International Conference on Earthquake Engineering (5ICEE), Center for Urban Earthquake Engineering, Tokyo Institute of Technology, 115-124.