DEVELOPMENT OF OBLIQUE METAL SPINNING WITH FORCE CONTROL

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ABSTRACT

The aim of this study is to use a novel metal spinning approach to create a product that contains a flange that is inclined in relation to its original plane. Metal spinning is a plastic forming process in which a metal sheet is formed on a rotating mandrel using the force of a roller. This forming process is usually performed while the flange of the workpiece is kept flat and in a position perpendicular to the rotational axis. In this paper, the thrust force along the flange is maintained at a constant value using an impedance control method, while the position of the roller is constrained on the inclined flange plane. Thus, neither the three-dimensional data for a die nor tool trajectory data are needed for flexible manufacturing. The effectiveness of the proposed method was experimentally verified by forming of an aluminum sheet of 1.0-mm thickness.

INTRODUCTION

Shear spinning Metal spinning is a collective term for plastic forming processes in which sheet or tube-shaped metals are fixed onto a rotating mandrel through the force exerted by a roller or paddle. In contrast to press working, metal spinning usually requires some minutes of production time. On the other hand, the die costs are minimal, since the product is formed using only a male die of the desired shape. Therefore, this technique is suitable for the manufacture of a wide variety of familiar products, such as car wheels, gas canisters, kitchenware, tableware, lighting fixtures, parabola antennas, pulleys, nozzles, and brass instruments.

A spinning process in which a roller tool is forced down onto a mandrel so as to shear the blank is called shear spinning (Fig. 1). The flange of the workpiece is kept flat and



Figure 1: Shear spinning

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perpendicular to the axis of rotation throughout the process. In addition, the wall thickness *t* of the product is determined by wall angle α and original thickness t_0 of the blank according to the sine law:

$$t = t_0 \sin \alpha . \tag{1}$$

Therefore, the only degree of freedom to adjusting wall thickness *t* is the original thickness *t*₀, since α is used in shaping the product.

Products formed by metal spinning have traditionally been inherently limited to straight-axis and circular-sectional shapes. However, in recent years, spinning methods have been developed to allow the formation of curved/inclined shapes or non-axisymmetric-sectional shapes through structural improvements to the machinery and intelligent technologies. There is a growing need for these methods, particularly in the manufacture of car parts and instruments for physics and chemistry. A tube spinning method, in which roller tools revolved around a fixed blank, has been described by Shindo et al. [1]. With this technique, it is possible to shape inclined or decentered ends by necking the original pipe, so it is used to produce space-saving exhaust ducts in cars. In our recent study [2], we proposed a die-less sheet-spinning method for curved-axis and non-axisymmetric-sectional shell shaping that involves inclining the flange of the workpiece. In this process, the motions of the tool and workpiece are synchronized in the axial and radial directions with the rotation of the spindle axis.



Figure 2: Scheme of oblique metal spinning using force control

A numerically controlled spinning machine moves the spherical head tool along a trajectory, which is calculated from the intended shape and takes into consideration contact with the tool. This system has the advantage of low-cost rapid prototyping, since no forming die is needed when using a pre-hemmed blank disc.

In the present paper, we propose a system of oblique shear spinning that uses a force-control method with coordinate conversion for shaping products that have inclined processing sites, and evaluate the basic features of this process by forming an obliquely truncated cone shell. A fundamental strategy of this method is discussed in terms of organizing the specifications of current shear spinning methods.

METHODOLOGY

In the currently used process of shear spinning, a numerically controlled machine directs a roller tool along the profile of a rotating mandrel, while maintaining clearance for the tool trajectory equivalent to the wall thickness, as calculated from the sine law in the tool-positioning command. Τo streamline this process, Arai [3] developed a force-controlled spinning method for the manufacture shear of non-axisymmetric products. In this process, the axial location of the tool is positionally controlled, and the normal force applied to the mandrel is controlled through a position/force hybrid control system. Neither the three-dimensional data for a die nor tool trajectory data are needed, since forming is performed by a roller tool that is moved along the contour of The flange plane of the workpiece is the mandrel. perpendicular to the rotational axis in this process.

The aim of the present study is to create a product that contains a flange that is inclined relative to its original plane. In the operational coordinate, the thrust force along the mandrel is maintained at a constant value using an impedance control method, while the tip-position of the roller is constrained on the inclined flange plane. As in the traditional system, the feeding direction the X of roller is fixed perpendicular to the plane, while the forcing direction Y is fixed horizontal to the plane and crossing the spindle axis. However, as illustrated in Figure 2, the feeding (constraining) direction and the forcing direction should be tilted dynamically depending on the angle of the spindle, since the inclined flange of the workpiece is rotated with a mandrel. In this present study, an operational coordinate system (X, Y) is used to intersect the inclined plane of the flange and the rotational axis (Fig. 3). This is achieved using a variable coordinate transformation between the operational coordinate (X, Y) and the mechanical coordinate, together with adequate control methods. The position and direction of the operational coordinate are determined from the origin displacement on the spindle, x_c , and the tilting angle of the operational coordinate, γ , which are expressed as variables of the inclination angle ϕ of the flange and the rotational angle

θ of the spindle.

The production from sheet blanks of parts with an inclined flange using NC metal spinning has already been achieved [2].

However, in this method, the precise shaping of the part to a desired profile and the estimation of errors are not easily achieved, since the method is executed without the support of a forming die. In contrast, the process described in the present study requires the use of a dedicated die. Overall, the costs associated with this new technique are offset by the improved profile accuracy achieved by forcing the metal onto a mandrel, as well as the opportunities it provides to improvise with various shapes of mandrels.

ROLLER & SPINDLE CONTROL

Coordinate Transformation



Figure 3: Notation of force-controlled oblique metal spinning

In the present method, metal spinning is performed with a roller tool on a constrained plane, so as to generate a product with an inclined flange while maintaining flatness. Thus, the tip of the roller is set on a constraint line, which corresponds to the intersection of the constraint plane and the movable plane of the roller. During the forming process, the roller follows a constraint line path that is tilted by the angles of the spindle (Fig. 3). The inclination angle γ of the line is defined by the following equation:

$$\gamma = \tan^{-1}(\tan\phi\cos\theta),\tag{2}$$

where ϕ is the obliquity angle of the constraint plane from the original normal-plane, and θ is the angle of the spindle. This equation is differentiable with regard to time in the realistic range of $-\pi/2 < \phi < \pi/2$. Therefore, γ and γ can be calculated from ϕ , θ , and their time derivatives according to the following expressions:



Figure 4: Relation between a constraint plane and a constraint line

$$\dot{\gamma} = \frac{\phi \cos \theta - \dot{\theta} \sin \phi \cos \phi \sin \theta}{\sin^2 \phi \cos^2 \theta + \cos^2 \phi},$$
(3)

$$\ddot{\gamma} = [\ddot{\phi} \cos \phi \cos \theta (\sin^2 \phi \cos^2 \theta + \cos^2 \phi) - \ddot{\theta} \sin \phi \cos^2 \theta \sin \theta (\sin^2 \phi \cos^2 \theta + \cos^2 \phi) + 2\dot{\phi}^2 \sin \phi \cos \theta \{\cos^2 \phi - (1 - \sin^2 \phi) \cos^2 \theta\} - \dot{\theta}^2 \sin \phi \cos^2 \phi \cos \theta \{(1 - \cos^2 \theta) \sin^2 \phi + 1\} + 2\dot{\phi}\dot{\theta} \cos \phi \sin \theta (\sin^2 \phi \cos^2 \theta - \cos^2 \phi)]/\{\cos \phi (\sin^2 \phi \cos^2 \theta + \cos^2 \phi)^2\}.$$
(4)

In order to perform necessary coordinate transformations, the above equation are used for computing the variables γ , $\dot{\gamma}$, and $\ddot{\gamma}$ during the process online.

In addition, to facilitate transformations between the roller-tool coordinate and the spindle axis, a mechanical coordinate system (u, v) on the XY table and an absolute coordinate system (x, y) on the spindle axis (i.e., top center of a mandrel) are defined as shown in Fig. 3. The transformation from the mechanical coordinate (u, v) to absolute coordinate (x, y) can simply be expressed as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \begin{pmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} u_c \\ v_c \end{bmatrix} = J_1(\boldsymbol{u} - \boldsymbol{u}_c), \quad (5)$$

where β is the mounting angle of the spindle, and u_c is the distance between each origin. In addition, we established an operational coordinate system (X, Y), which designates the translation x_c and inclination γ of the moving constrained plane from the absolute coordinate system (x, y). Thus, position $\mathbf{X} = [X, Y]^{\mathrm{T}}$ can be defined as a function of x_c and γ in the following formula:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{bmatrix} \begin{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_c \\ 0 \end{bmatrix} = J_2(\mathbf{x} - \mathbf{x}_c) .$$
(6)

Using the definition of $J_1J_2 = J$, and performing the necessary derivatives we obtain the following relations:

$$\dot{J} = j\dot{\gamma}, \qquad \ddot{J} = -J\dot{\gamma}^2 + j\ddot{\gamma}, \dot{J}_2 = j_2\dot{\gamma}, \qquad \ddot{J}_2 = -J_2\dot{\gamma}^2 + j_2\ddot{\gamma}.$$

$$(7)$$

Consequently, the transformations between the mechanical coordinate (u, v) and the operational coordinate (X, Y) are expressed in the following equations:

$$\boldsymbol{X} = J(\boldsymbol{u} + \boldsymbol{u}_c) - J_2 \boldsymbol{x}_c \,, \tag{8}$$

$$\dot{\boldsymbol{X}} = J\dot{\boldsymbol{u}} + j\dot{\gamma}(\boldsymbol{u} + \boldsymbol{u}_c) - J_2 \dot{\boldsymbol{x}}_c - j_2 \dot{\gamma} \boldsymbol{x}_c , \qquad (9)$$

$$\mathbf{X} = J\{\ddot{\mathbf{u}} - \dot{\gamma}^2 (\mathbf{u} + \mathbf{u}_c)\} + j\{2\dot{\gamma}\dot{\mathbf{u}} + \ddot{\gamma}(\mathbf{u} + \mathbf{u}_c)\} - J_2(\ddot{\mathbf{x}}_c - \dot{\gamma}^2 \mathbf{x}_c) - j_2(2\dot{\gamma}\dot{\mathbf{x}}_c + \ddot{\gamma}\mathbf{x}_c)\}$$
(10)

Control method

Rotation of the spindle θ is controlled by the simple PD

positional control method. The torque output τ_{θ} is calculated

from

$$\tau_{\theta} = I_{\theta} \{ \ddot{\theta}_d + k_{v\theta} (\dot{\theta}_d - \dot{\theta}) + k_{p\theta} (\theta_d - \theta) \}, \qquad (11)$$

where I_{θ} is the total inertia moment of the main spindle, and $\dot{\theta}_d$, $\dot{\theta}_d$, and θ_d are the desired values for the rotation. In our experiments, the feedback gains $k_{\nu\theta} = 320 \text{ s}^{-1}$ and $k_{\rho\theta} = 25600 \text{ s}^{-2}$ were found to be appropriate for an accurate spinning process.

As proposed earlier, along the direction of the *X* axis, the tip of the roller should be set on the constraint line during the process. As a result, the acceleration value of feedback, \ddot{X}_f , is determined with $X_d = 0$ according to the following equation for PD feedback:

$$\ddot{X}_{f} = 0 + k_{vX}(0 - \dot{X}) + k_{pX}(0 - X) \}.$$
(12)

Since the operational coordinates (X, Y) are a

function of $\gamma(\theta, \phi)$ in Eq. (2) and (8), the desired

values of *X* cannot be determined independently of θ

or ϕ . Therefore, the direct method which involves changing the desired value of the X-axis presents a problem. Therefore, the roller that feeds perpendicular to the constraint plane is specified in an indirect way, i.e., by assigning *X* to zero and translating the origin of the operational coordinate by x_c on the spindle axis. In Equation (12), the values of the feedback gains used are: $k_{vX} = 500 \text{ s}^{-1}$ and $k_{nX} = 62500 \text{ s}^{-2}$.

Along the direction of the Y-axis, the thrust force F_Y parallel to the constraint line is regulated around a constant desired value by an impedance control method, which is based on virtual internal model, proposed by Kosuge et al. [4]. In this control method, the desired value of the Y-axis is calculated online through the following formulas by simulating a virtual mass-damper model, as defined in

$$\begin{cases} \ddot{Y}_d = (F_Y - F_{Yd} - B_V \dot{Y}_d) / M_V \\ \dot{Y}_d = \int \ddot{Y}_d dt \end{cases}$$
(13)

These virtual parameters were adjusted to a mass value of $M_V = 50$ kg and a viscous resistance value of $B_V = 150$ Ns/m for the efficient performance of the spinning process in our experiments. The acceleration values of the feedback on the Y-axis were determined with a gain of $k_{vY} = 400$ s⁻¹ using

$$\ddot{Y}_{f} = \ddot{Y}_{d} + k_{vY}(\dot{Y}_{d} - \dot{Y})$$
 (14)

Finally, the Y-position coordinate, which indicates the distance from the spindle axis, is dependent upon the profile of the mandrel. Therefore, this method has the advantage of allowing improvisations with various shapes of mandrels. In Equation (10), the acceleration values on the mechanical coordinates \ddot{u}_f , \ddot{v}_f can simply be calculated from \ddot{X}_f and \ddot{Y}_f for the feedback values. Accordingly, the force outputs, f_u and f_u , for a mechanism that moves a roller tool can be obtained by multiplying the real mass of the mechanism, M_{uv} , by \ddot{u}_f and \ddot{v}_f , and subsequently adding the desired thrust force F_{Yd} with transformation to the mechanical coordinate as follows:

$$\begin{bmatrix} f_u \\ f_v \end{bmatrix} = M_{uv} \begin{bmatrix} \ddot{u}_f \\ \ddot{v}_f \end{bmatrix} + J^T \begin{bmatrix} 0 \\ F_{Yd} \end{bmatrix}.$$
 (15)



Figure 5: Experimental setup

Table 1: Specification of the machine

Axis	u,v-axis	a-axis
Rated force/torque	800 N	7.01 Nm
Rated speed	0.25 m/s	273 min ⁻¹
Active stroke	0.15 m	-
Resolution	0.61 µm	0.004 deg.

Table 2: Dynamics parameters of each axis

Axis	u-axis	v-axis	a-axis
Coulomb friction	138 N	184 N	0.454 Nm
Viscous friction	701 Ns/m	555 Ns/m	0.0317 Nms/rad
Inertia	34.6 kg	51.5 kg	0.0103 kgm ²



Figure 6: Obliquely cut mandrel

A TRIAL FORMING EXPERIMENT

Spinning Machine and Roller

The experimental setup used in the present study is depicted in Fig. 5, and the specification of the machine is indicated in Table 1. The roller tool is moved by the XY table driven by AC servo motors of 200 W with a ball screw of 5-mm pitch. The main spindle is driven by the same servo motor with low-backlash gears and a reduction ratio of 11, and is



Figure 7: Three steps in forming



Figure 8: Forming experiment

mounted at an angle of $\beta = 60^{\circ}$ from the mechanical axis of *u*.

The position of the roller and the spindle angle are measured by rotary encoders at the motors. A 6-axis force/torque sensor is fixed at the base of the roller tool. The signals from the rotary encoders and force sensor are transmitted to a personal computer (CPU, Pentium II; clock frequency, 450 MHz) via external interface boards, and the values of torque outputs are sent to the motor drivers as analog voltages via a D/A board. The sampling period for the control is 1 ms. The maximum thrust forces are limited by the rated torques of the servo motors. The quenched roller tool, which is made of alloy tool steel SKD11, is supported with bearings for free rotation. Its diameter and nose radius are D = 70 mm and $\rho_R = 9.5$ mm, respectively.

System Identification

Before the experiment, the parameters of coulomb friction, viscous friction, and inertia at each axis were identified from the relationship of motor speed and torque output by moving them at a constant velocities and constant accelerations. The results are indicated in Table 2. The values of the inertia are used to calculate the torque outputs from the acceleration values for each axis.

Blank Sheet and Mandrel

Commercially available pure aluminum discs (A1100P-H24) of nominal thickness $t_0 = 1.0$ mm and outer



(a) Straight, $\phi_I = 0^{\circ}$ (b) Inclined, $\phi_I = 10^{\circ}$

Figure 9: Products of different inclination angle of the flange in forming

diameter 150 mm were used as blank sheets. Each disc has a hole of 20-mm diameter at the center, so as to allow fixing to the mandrel, as shown in Fig. 6. The original half-angle of the

cone was 30°; and it was obliquely cut at an angle of 10° using wire-electrical discharge machine. Therefore, the wall angle of the mandrel to the spindle axis is 20° on the steepest side and

40° on the opposite side. The mandrel was composed of S45C

carbon steel. General purpose lubricating oil (CRC5-56) was sprayed onto the blank disc before every forming.

Forming Procedure

To ensure proper forming while maintaining the flatness of the flange, the desired motion of the tool is important in terms

of the angle of the spindle, $\theta_d(t)$, the inclination angle of the

constraint plane, $\phi(t)$, the translation length of the operational coordinate, $x_c(t)$, and the time derivatives thereof. In this process, a blank disc is shaped onto the mandrel while inclining its flange plane via three forming steps, as shown in Fig. 7.

In the first step, a shallow shape that is looser than shoulder profile of the mandrel (Fig. 7(a)), was formed by controlling the tip-position of the roller on the absolute coordinate system (u, v), since a roller position with force control is not supported radially on the flat blank. In this experiment, a roller feed per revolution of 0.4 mm, feed length of 5 mm, which includes the initial thickness of the blank, and rotary speed for the spindle of $\omega = 120$ rpm were configured as a working condition.

From the second step, the proposed force control method was used to incline the flange plane around a flexural center (Fig. 7(b)). Thus, the constraint plane was relatively inclined

and translated to the mandrel while keeping constant the thrust force parallel to the plane, F_{γ} . The center is placed outside the desired shape at a radial distance of $R_{\phi} = 50$ mm to the steepest side. In this step, an inclination angle per revolution of $p_{\phi} =$

0.3°, rotary speed for the spindle of $\omega = 30$ rpm, and thrust

force $F_Y = 500$ N were used. The total inclination angle

employed for the flange, ϕ_I , was 10° in the inclined test, and 0°

in the normal case used for comparison. For the origin displacement on the spindle, x_c and its time derivatives, an axial translation with inclination of the flange plane around the center was calculated according to

$$R_{\phi} | \tan \phi | -\rho_R \left(\frac{1}{\cos \phi} - 1 \right), \tag{16}$$

and its derivatives were added to each other.

In the final step, the constraint plane was translated a feed depth L_S in the normal direction (Fig. 7(c)), while maintaining its oblique angle. The depth was relatively configured as $L_S = (35 - R_{\phi} \phi_l)$ mm, to conform to the forming lengths for different inclination angles. The roller feed rate was the same as in the first step, i.e., $p_L = 0.4$ mm. The desired thrust force F_Y and the spindle speed ω were as in the previous step.

Forming Results



(b), the wall angle was 30° at around, and the top was inclined

10° to its flange, since the direction of inclination was matched

to the inclined axis of the mandrel. The outer surfaces of the products were covered with the glossy feed-marks of the roller trajectory, while inner surfaces were smooth by due to the pressure of the mandrel. It is known that the surface roughness R_{surf} to the feeding direction in a straight feeding area (in this case, the first step or third step) is macroscopically obtained from the feed rate p and nose radius ρ_R by



Figure 11: Position and thrust forces of roller at a

round of
$$x_c = 30 \text{ mm} (\phi = 0^\circ)$$

$$R_{surf} = \rho_R - \sqrt{\rho_R^2 - (p/2)^2} = 2.1 \,\mu\text{m} \,. \tag{17}$$

In the second step, the roughness was not constant to

the round, since the roller tracks on the inner surface of the inclination marked structurally finer than those on the outer surface.

The outer profiles of each product and the mandrel,

measured every 45° of the half-round using a laser range

sensor, are indicated in Fig. 10. The graph shows that in both cases the entire material of the wall was attached closely to the

mandrel. In the non-oblique forming ($\phi_I = 0^\circ$), the flange part

was maintained at the original diameter of 150 mm and was

slanted slightly (by about 4°) in the reverse direction to feeding.

In addition, in the oblique forming ($\phi_I = 10^\circ$), the measured

difference in the flange angle between the inner and outer

profiles was 20.5°, which is corresponded to the value of $2\phi_l$.

The wall thicknesses t at an axial position from the top of each product on the steepest/gentlest sides

(wall angle $\alpha = 20^{\circ}$, 40°) were measured as Table 3.

For the product in which there was no inclination of the flange, the thicknesses show a trend to charge in accordance with the sine law in Eq. (1), while this was not true for the inclined flange product.

Figures 11 and 12 show the roller position to the constraint plane and thrust forces for a round of the (X, Y)-origin displacement on the spindle, at $x_c = 30$ mm for both inclination angles of the flange ϕ_l . In the oblique process, the roller moves backward to the axial direction during the former half-round, and then pushes forward in remaining half-round. The ideal value of X is closer to zero with low-magnitude variation, since it indicates a positional error in relation to the desired plane. The roller position on the tilting X-axis floats at an average distance of 50 µm from the constraint plane in these graphs. The error is potentially minimized by replacing the simple positional PD feedback in Eq. (12) with an alternative control system. Nevertheless, the amplitude of X in Figure 12, 100 µm, appeared to be sufficiently low to retain the flatness of the flange plane during the shear spinning process. The thrust force in the radial direction on the plane, F_{Y_2} was maintained close to the desired value of 500 N, and the force normal to the plane, F_{X_2} was 700 N on average. In this machine, a 1.5-mm-thick pure aluminum blank cannot be successfully formed due to thrust saturation of the actuator of v-axis.

CONCLUSION

A trial forming experiment was conducted successfully using an oblique metal-spinning process that involved force control and an obliquely cut mandrel. The results of this experiment and our conclusions can be summarized as follows:

- To conduct force-controlled oblique spinning, a control system that includes coordinate transformation is proposed. This system can be adapted to other mechanisms, e.g., an intelligent spinning lathe with a serial link manipulator
 [5].
- A product of non-traditional shape (having an inclined flange) in terms of sheet spinning, was experimentally formed from a pure aluminum blank of 1.0-mm thickness. In contrast to the previously used process [2], the method described herein has advantages of higher profile accuracy in shaping the product and allowing improvisation of the forming process with various shapes of mandrels.
- 3. For the oblique flange generated in this experiment, the measured wall thicknesses did not match those observed for the non-oblique flange, whereby the latter were in accordance with the sine law.



Figure 12: Position and thrust forces of roller at a

round of $x_c = 30 \text{ mm} (\phi = 10^\circ)$

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Table 3: Wall thickness of products

Wall angle α [°]	20		40	
Sine law $t = t_0 \sin \alpha$	0.342		0.643	
Incline angle ϕ [°]	0	10	0	10
Thickness t [mm]				
at $z = 5 \text{ mm}$	0.692	0.696	0.682	0.681
15 mm	0.333	0.372	0.640	0.454
25 mm	0.272	0.519	0.660	0.470
35 mm		0.540		