A Design Method of Local Communication Area in Multiple Mobile Robot System

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Abstract

When many mobile robots should achieve cooperation, local communication system is considered appropriate from the standpoint of the cost and capacity of communication. This paper presents the optimization of the efficiency of local communication in environments where many mobile robots send out information stochastically. The optimal communication area is derived by minimizing transmission waiting time calculated using the probability of successful information transmission. Computer simulations have been undertaken to verify the analytical results.

1 Introduction

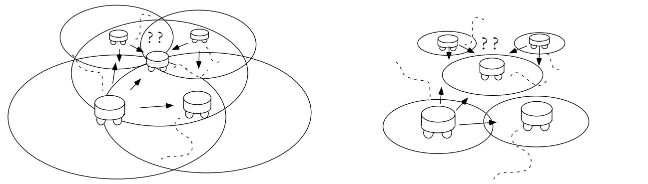
By recent progress in the field of mobile robots, they are now expected to execute complicated and sophisticated tasks by intelligent cooperation. This cooperation needs communication, which can be classified into (a) global communication [1]–[3] and (b) local communication [4]–[8].

Global communication (a) is effective for small number of robots. However, when the robot number increases, this becomes difficult to be realized because of limited communication capacity and increasing amount of information to handle. Thus we adopt local communication system in which each robot transmits information locally. Many examples of this local communication can be seen in nature.

One of the advantages of local communication system is controllable communication area. Let us suppose each robot can adjust the range of information output. If this range is too large, the efficiency of information transmission decreases because all the information from others cannot be treated (Fig. 1(a)). The efficiency is also low if output range is too small (Fig. 1(b)). It is therefore essential to develop a methodology for decision of communication range for efficient information transmission.

Authors have been working on analysis of local communication such as information diffusion and group behavior for efficient information transmission [7][8]. Nevertheless, these studies have not dealt with an environment crowded with robots where the efficiency must be optimized by adjusting communication area.

Although there have been some other researches on local communication [4]∼[6], the design of communication range has hardly been discussed. In communication theory there are studies on this optimization [9], but they are for radio network system and did not take account of information acquisition from multiple robots.



(a) Output range is too large (b) Output range is too small

Fig. 1: Control of Local Communication Range

To make the local communication system efficient, communication delay should be sufficiently small. This means we must minimize the waiting time during which a robot keeps waiting until it obtains necessary information successfully from other robots. We propose in this paper a design method of optimal local communication area which realizes minimal waiting time. We estimate it using "information transmission probability", which represents the probability of successful information transmission.

The information transmission probability can be derived in the form of infinite series, assuming that each robot outputs information stochastically. The analysis is carried out in two cases, the communication with and without transmission interference.

It is desirable that this optimal range should be able to be calculated without heavy computation of infinite series. Thus further analysis is made and the equations of the information transmission probabilitiy are reduced into simpler formulae. These analytical results enable us to obtain the optimal communication area more easily.

We have undertaken the simulation of local information transmission by many robots to verify the effectiveness analytical results.

2 Analysis of Information Transmission Probability

In this chapter, first we show the model of local communication in which each robot sends out information stochastically. Next, we calculate the probability that the information is successfully transmitted to other robots. As mentioned in chapter 1, we call this probability "information transmission probability" and it will be utilized to derive the waiting time in the next chapter.

2.1 Communication Model

We employ a simplified model as briefly described below:

(i) Each robot outputs information with certain probability.

(ii) Each robot moves randomly.

(iii) There is an upper limit in number of robots from which each robot can obtain information.

(iv) Each robot can output and receive information simultaneously.

(v) Each robot gets information at every time unit, which is long enough for information acquisition.

For example, this model corresponds to an environment where robots collect map information by moving randomly and update their internal map data when they detect some change of environment or receive information about it from other robots.

We define the upper limit in (iii) as "information acquisition capacity" c, which is an integer. If a robot finds more than c robots that output information, two cases are possible:

(a) The robot cannot receive information from any robots.

(b) The robot can receive information from c robots.

We call the case (a) "communication with interference," and the case (b) "communication without interference."

From (iv), each robot has two standpoints, "receiver" and "transmitter" of information. In the next section, the information transmission probability is calculated from the "receiver" standpoint of each robot, as how much of information emitted from other robots can be received successfully by a "receiver".

The waiting time can be defined as the number of time units described in (v) before a robot receives information output by other robots.

Principle parameters of the model are listed together as follows:

ρ: Density of robot population
p: Probability of information output from a robot
A: Area of output range of information
c: Information acquisition capacity
x: Average number of robots in output range ($=\rho A$)

2.2 Information Transmission Probability by Infinite Series

When robots move randomly as shown in Fig. 2, the number of robots in the output area A is described by Poisson distribution with mean $x = \rho A$. We define $P(n, x)$ as the probability that there are n robots which have the possibility to transmit information to an arbitrary robot. $P(n, x)$ is written as (1).

$$P(n,x) = \frac{x^n}{n!}e^{-x} = \frac{(\rho A)^n}{n!}e^{-\rho A} \quad (1)$$

For an arbitrary robot r, let $Q(n, m, x)$ denote the probability that m robots out of n output information under the condition that robot r is in the output

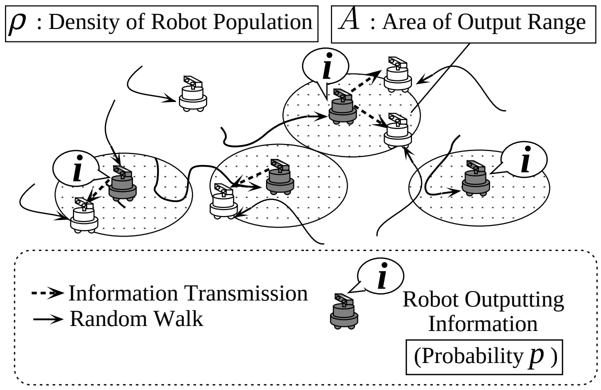


Fig. 2: Local Communication Environment

range of n other robots. $Q(n, m, x)$ is computed multiplying $P(n, x)$ by the probability that m robots out of n output information:

$$\begin{aligned} Q(n,m,x) &= {}_nC_m p^m (1-p)^{n-m} P(n,x) \\ &= {}_nC_m p^m (1-p)^{n-m} \frac{x^n}{n!} e^{-x} \end{aligned} \quad (2)$$

We will compute the information transmission probability for two types of communication, (a) with interference and (b) without it.

From the "receiver" standpoint of each robot, we classify the information transmission into exclusive events E_i ($i = 0 \sim 3$) as follows, and calculate their corresponding probabilities $P_I(E_i)$ and $P_N(E_i)$, for communication with and without interference, respectively. Obviously, information transmission probability is equivalent to $P_I(E_3)$ or $P_N(E_3)$.

E_0: No other robots find the "receiver" robot in their output range.

E_1: Some robots find the "receiver" robot in their output range, but none of them output information.

E_2: There are robots that output information to the "receiver," but the transmitted information is *not* successfully obtained by the "receiver".

E_3: There are robots that output information to the "receiver," and the transmitted information is successfully obtained by the "receiver".

For communication with interference, $P(E_i)$ is computed in the form of infinite series:

$$P_I(E_0) = Q(0,0,x) = e^{-x} \quad (3)$$

$$P_I(E_1) = \sum_{n=1}^{\infty} Q(n,0,x) = e^{-px} - e^{-x} \quad (4)$$

$$P_I(E_2) = \sum_{n=c+1}^{\infty} \sum_{m=c+1}^{n} Q(n,m,x) \quad (5)$$

$$P_I(E_3) = \sum_{n=1}^{c} \sum_{m=1}^{n} Q(n,m,x) + \sum_{n=c+1}^{\infty} \sum_{m=1}^{c} Q(n,m,x) \quad (6)$$

For communication without interference, $P_N(E_i)$ can be given through similar consideration. $P_N(E_0)$, $P_N(E_1)$ are the same in (3), (4).

$$P_N(E_2) = \sum_{n=c+1}^{\infty}\sum_{m=c+1}^{n}\frac{m-c}{m}Q(n,m,x) \quad (7)$$

$$P_N(E_3) = \sum_{n=1}^{c}\sum_{m=1}^{n}Q(n,m,x) + \sum_{n=c+1}^{\infty}\sum_{m=1}^{c}Q(n,m,x) + \sum_{n=c+1}^{\infty}\sum_{m=c+1}^{n}\frac{c}{m}Q(n,m,x) \quad (8)$$

In the case of communication without interference, if a "receiver" finds m ($m > c$) "transmitter" robots that output information toward it, we calculated $P_N(E_2)$ and $P_N(E_3)$ on the assumption that the part c/m of $1 - \{P_N(E_0) + P_N(E_1)\}$ contributes to $P(E_3)$ and the rest $1 - c/m$ contributes to $P_N(E_2)$ in (7) and (8) respectively.

On the other hand, for communication with interference, if a "receiver" finds more than c robots outputting information, no information is received by the "receiver". So all the possibility of finding more than c "transmitter" robots contributes to $P_I(E_2)$ in (6).

2.3 Simplification of Information Transmission Probability

So far the information transmission probability in local communication is represented in the form of infinite series. But it is not preferable that robots should carry out heavy numerical computation to obtain the optimal output range when they adjust their communication area in real time.

To reduce computation load, the equations of information transmission probability (6) and (8) will be rewritten from infinite series into more simplified formulae.

2.3.1 Communication with Interference

Through a series of transformation of equation utilizing the exponential series and binomial distribution, $P_I(E_3)$ in (6) is reduced into the form of (9).

$$P_I(E_3) = e^{-px}\left(\sum_{n=0}^{c}\frac{(px)^n}{n!} - 1\right) \quad (9)$$

$P_I(E_3)$ is derived by computing $1-\{P_I(E_0)+P_I(E_1)+P_I(E_2)\}$ from (3) $\sim$ (5).

$P_I(E_3)$ in (9) changes according to information acquisition capatcy c. In particular:

$$P_I(E_3) = \begin{cases} pxe^{-px} & (c=1) \\ 1-e^{-px} & (c \to \infty) \end{cases} \quad (10)$$

The values of $P_I(E_i)$ are plotted in terms of x in Fig. 3 for parameters $p = 1$, $c = 1$. These parameters can be thought to represent a fundamental case in which robots output information continuously and can receive information from one robot. Note that $P_I(E_1)$ always equals to zero because robots send out information with probability $p = 1$. We can also see the information transmission probability $P_I(E_3)$ has one maximal value.

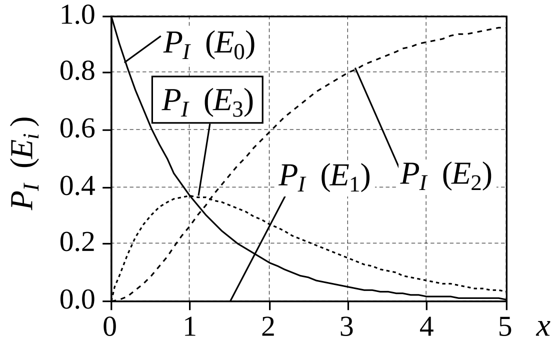


Fig. 3: $P(E_i)$ [$i = 0 \sim 3$], (p=1, c=1)

2.3.2 Communication Without Interference

We are going to simplify the equation of information transmission probability $P_N(E_3)$ in (8) to clarify its characteristics. Using a similar method as in 2.3.1, namely calculating $1-\{P_N(E_0)+P_N(E_1)+P_N(E_2)\}$, we obtain a simplified form of $P_N(E_3)$:

$$P_N(E_3) = e^{-px}\left(\sum_{k=0}^{c}\frac{p^kx^k}{k!} + c\sum_{k=c+1}^{\infty}\frac{1}{k}\frac{p^kx^k}{k!} - 1\right) = P_I(E_3) + ce^{-px}\sum_{k=c+1}^{\infty}\frac{1}{k}\frac{p^kx^k}{k!} \quad (11)$$

We can see $P_N(E_3)$ is obtained by adding a term of summation to $P_I(E_3)$ in (9). This is a part of an infinite series known as exponential integral $E_i(px)$[1].

3 Derivation of Optimal Communication Range

In order to make local communication system efficient, the waiting time required for information transmission should be made as short as possible, as mentioned in chapter 1. This improves the efficiendy of the system so that it can follow dynamic environmental changes.

Here we define W as how many time units an arbitrary robot must wait until it receives information from other robots. This waiting time W will be computed using probabilities $P_I(E_3)$ and $P_N(E_3)$ in (11).

Let x_{opt} be the average number x ($= \rho A$) minimizing W. Then optimal output range A_{opt} is calculated as:

$$A_{opt} = \frac{x_{opt}}{\rho} \quad (12)$$

Our goal is now to obtain x_{opt} because actual optimal output range A_{opt} can be calculated easily using (12). We call x_{opt} also the "optimal output range" hereafter.

In this chapter, first we will show that the waiting time W is minimized by calculating the maximum of the information transmission probability. And next, the optimal communication range will be obtained.

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[1] $E_i(px) = \int_{\infty}^{px}\frac{e^t}{t}dt = C + \log px + \sum_{k=1}^{-\infty}\frac{1}{k}\frac{p^kx^k}{k!}$

3.1 Calculation of Waiting Time

The waiting time W denotes the time required so that a "receiver" robot receives information from others. "Transmitter" robots keep outputting the information until successful transmission to certain number of robots, and the reception is supposed to be detected using acknowledgment scheme.

We define W_I and W_N as the waiting time for communication with and without interference respectively. These values will be computed using infomation transmission probability $P_I(E_3)$ and $P_N(E_3)$.

In the case of communication with interference, a robot must wait for W_I because it cannot get information during that time due to interference. Unlike this, especially for large x, W_N represents the interval at which a robot gets information from a particular robot in its communication range, and the robot can obtain information from other robots during this interval W_N. Thus when x is large, W_N is related to the efficiency of information acquisition from its neighborhood, while W_I is computed based on the problem of whether a robot can get information or not.

We give an assumption that p is the probability of information output including retransmitted information [9].

Here $P(E_3)$ corresponds to $P_I(E_3)$ or $P_N(E_3)$. The probability of waiting only 1 time unit is just $P(E_3)$, the information transmission probability. The probability of waiting 2 time units is $P(E_3)(1-P(E_3))$; thus the probability of waiting i time units is $P(E_3)(1-P(E_3))^{i-1}$.

The mean value of waiting time W can be computed using geometric distribution as:

$$W = \sum_{i=0}^{\infty} i \cdot P(E_3)(1-P(E_3))^{i-1} = \frac{1}{P(E_3)} \quad (13)$$

From (13), the problem of obtaining the optimal output range x_{opt} minimizing W_I and W_N is equivalent to that of obtaining x_{opt} maximizing $P_I(E_3)$ and $P_N(E_3)$ for the communication with interference.

Fig. 4 shows the waiting time W_I and W_N plotted versus x, for the same paramaters (p=1, c=1) as in Fig. 3. W_I takes the minimum at the same x where $P_I(E_3)$ takes the maximum. This is the optimal output range x_{opt} for communication with interference. As to W_N, we can see x_{opt} is greater than that of W_N.

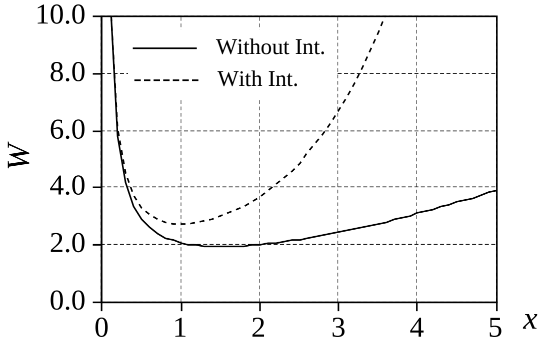

Fig. 4: Waiting time W (p=1, c=1)

3.2 Calculation of Optimal Output Range

We will derive the optimal output range x_{opt} in this section. While x_{opt} can be derived analytically for communication with interference, it is not the case for communication without interference because of the existence of infinite series in (11). Thus we will show a method that makes it possible to reduce computation cost using a lookup table.

3.2.1 Communication With Interference

Let us derive the optimal output range x_{opt} from (9). Differentiating $P_I(E_3)$ in terms of x, we obtain:

$$\frac{d}{dx}P_I(E_3) = \frac{d}{dx}e^{-px}\sum_{n=1}^{c}\frac{(px)^n}{n!} = pe^{-px}\{1-\frac{(px)^c}{c!}\} \quad (14)$$

By solving (14) =0, we can derive x_{opt} that maximizes $P_I(E_3)$ in a simple formula:

$$x_{opt} = \sqrt[c]{\frac{c!}{p^c}} = \frac{\sqrt[c]{c!}}{p} \quad (15)$$

In (15), x_{opt} is in inverse proportion to p, which is the probability of information output of each robots. As to the relationship between px_{opt} and information acquisition capacity c, we show a graph of px_{opt} versus c in Fig. 5.

In paticular, $px_{opt} = 1$ when $c = 1$, and if $c \to \infty$, then $x_{opt} \to \infty$. We can see that x_{opt} has approximately linear relationship with c in Fig. 5. However, we must pay attention to the fact that it is not simply proportional to c such as $x_{opt} = c/p$. Actual optimal output range A_{opt} can be calculated using (12).

We can also know the maximum value of information transmission probability $P_{Imax}(E_3)$ by substituting x_{opt} in (15) for x (9) as follows:

$$P_{Imax}(E_3) = e^{-\sqrt[c]{c!}}(\sum_{n=0}^{c}\frac{(\sqrt[c]{c!})^n}{n!} - 1) \quad (16)$$

It is very interesting that $P_{Imax}(E_3)$ depends only on the information acquisition capacity c. Thus for

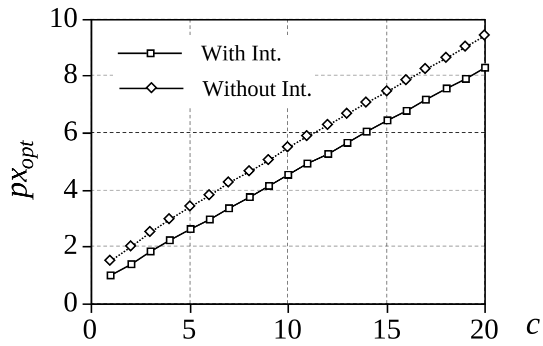

Fig. 5: The Value px_{opt} veusus c (With and Without Interference)

a fixed value of c, $P_{Imax}(E_3)$ takes always the same value regardless of probability of information output p. For example, if $c = 1$, $P_{Imax}(E_3) = 1/e$ for any p. The minimum value of W_{Imin} is easily obtained as $1/P_{Imax}(E_3)$.

Looking at Fig. 3 and 4, it is observed that the optimal output range x_{opt} equals to 1, as calculated from (15).

3.2.2 Communication Without Interference

The value of x_{opt} maximizing $P_N(E_3)$ cannot be derived analytically because of infinite series $E_i(px)$. But on the analogy of communication with interference, x_{opt} is likely to be inversely proportional to p, the probability of information output from each robot. After verifying this conjecture, we will propose a method for computing x_{opt} using a lookup table.

In the same way as 3.2.1, the derivative of $P_N(E_3)$ in terms of x is:

$$\begin{aligned} \frac{d}{dx}P_N(E_3) &= \frac{d}{dx}P_I(E_3) + \frac{d}{dx}\sum_{k=c+1}^{\infty}\frac{1}{k}\frac{p^k x^k}{k!} \\ &= pe^{-px}(1 - \frac{(px)^c}{(c+1)!} + c\sum_{k=c+1}^{\infty}\frac{1}{k}\frac{(px)^k}{k!} \\ &\quad + c\sum_{k=c+1}^{\infty}\frac{(px)^k}{(k+1)!}) \end{aligned} \quad (17)$$

Although it is not possible to get the analytical solution of (17)=0, we can easily verify that the solution $x_{opt} \propto 1/p$ by substituting y/p for x.

Then all we have to know is the relationship between px_{opt} and c. We can obtain it from numerical computation, as shown in Fig. 5. We can see px_{opt} has approximately linear relationship with c also in the case of communication without interference, and is always greather than px_{opt} for communication with interference.

Since the information aquisition capacity c is an integer, the relationship between px_{opt} and c as in Fig. 5 can be stored in each robot as a lookup table without consuming large memory. When robots need to calculate their information output range, they have only to refer to this table and get px_{opt}, then multiply this value by p.

Please note that the characteristics of x_{opt} is different from the case of communication with interference. When robots can communicate without interference, the waiting time for information acquisition from an arbitrary robot keeps decreasing as output range x increases. But robots from which a robot obtains information become diverse, which makes it difficult to collect information in a robot's neighborhood efficiently. Therefore it follows that the usage of x_{opt} has an advantage of making local information collection efficient.

4 Simulation for Analysis Verification

In order to verify the optimal output range derived in previous sections, several simulations have been executed. We implemented an environment where many

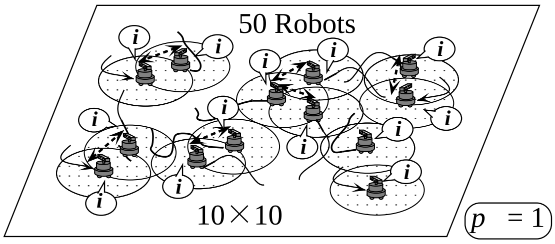

Fig. 6: Simulation Environment

robots moves randomly and communicate locally as shown in Fig. 6. In the environment 10×10, we implemented 50 robots moving randomly with velocity 0.2 at each time unit. Information output probability p is set to 1, which implements an environment where each robot outputs continuously such information as dynamically changing map.

We will first verify the analysis of information transmission probability $P_I(E_i)$ and $P_N(E_i)(i = 1 \sim 3)$, and the waiting time W_I and W_N next.

4.1 Information Transmission Probability

We can obtain probability $P(E_i)$ by counting how many times each event E_i occurs. Simulation results of $P(E_i)$ are acquired by executing the simulation for 300 time units. These simulation results are compared to the theoretical values calculated using the analysis in previous chapters.

Figs. 7 and 8 show the simulation results (indicated by "Simulation") compared to theoretical values (indicated by "Theory") for parameters p=1,c=1.

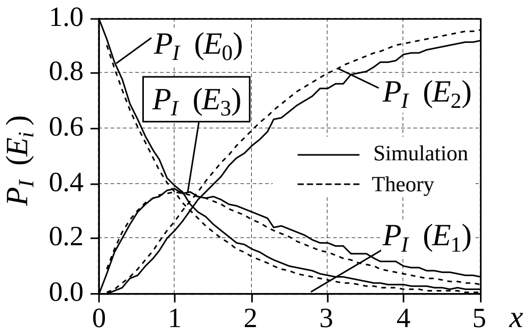

Fig. 7: $P_I(E_i)$ $[i = 0 \sim 3]$ (With Interference)

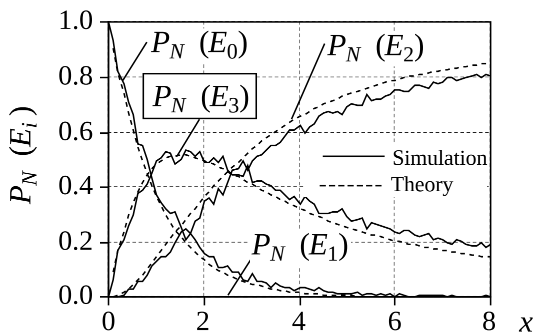

Fig. 8: $P_N(E_i)$ $[i = 0 \sim 3]$ (Without Interference)

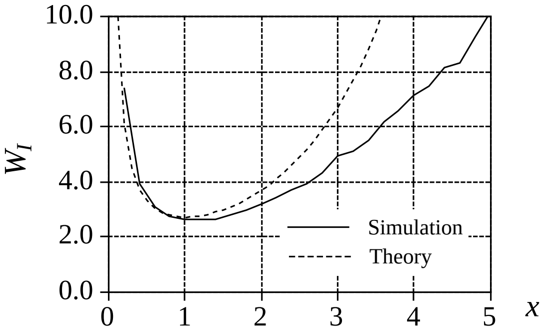

Fig. 9: Waiting Time W_I (With Interference)

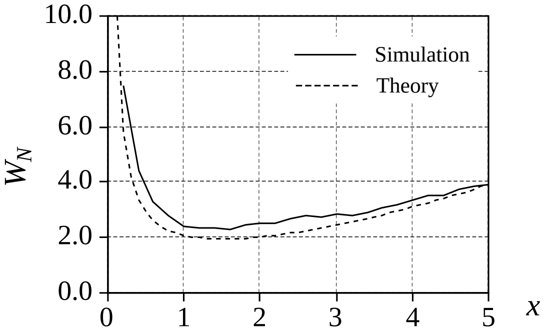

Fig. 10: Waiting Time W_N (Without Interference)

Although some modeling errors can be observed, theoretical values of $P_I(E_i)$ and $P_N(E_i)$ describe simulation results closely. Thus the validity of analysis on information transmission probability is verified.

4.2 Waiting Time and Optimal Output Range

Figs. 9 and 10 show the waiting time W_I and W_N for same parameters as in 4.1.

The theoretical value of x_{opt} is 1 and 1.5 for each case. In these graphs, W_I and W_N take the minimum values at almost the same x as theoretically predicted x_{opt}, it is therefore shown that the communication is optimized using the optimal output range.

For W_I, the difference between theory and simulation becomes large as x increases. This is because in very crowded situations with very large x, robots keeps information output continuously but hardly received, while the theory estimates certain value of waiting time. However this effect does not have much significance because the model is precise enough near around the x_{opt} derived from the analysis, where the simulation result takes the minimum.

From these simulation results, the effectiveness of the optimal output range is demonstrated.

5 Conclusion

We have analyzed the efficiency of information transmission by local communication. The efficiency is expressed by The efficiency is expressed by waiting time, namely the number of time units required before a robot receives output information from other robots. It was shown that this waiting time can be represented using the information transmission probability, described using infinite series.

After, we proved that the optimal output range can be derived analytically for communication with interference. This allows robots to optimize the output range of information in real time. This optimal value is not obtained in such a simple formula for communication without interference, but this is given quite easily using a lookup table method.

The effectiveness of these analyses was verified by computer simulation of many robots. The derived design method of communication area can help to construct an efficient local communication system for many robots.

We intend as future work to extend the analysis to obtain the optimal output range for information diffusion to multiple robots. For the application of the analyses in this research to real system, we are now implementing local communication system for mobile robots combining the relative position/orientation measurement system we have developed [10] and infrared communication device.

Acknowledgments

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