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Full paper

An Optimal Control-Based Formulation to Determine Natural Locomotor Paths for Humanoid Robots *

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Abstract

In this paper we explore the underlying principles of natural locomotion path generation of human beings. The knowledge of these principles is useful to implement biologically inspired path planning algorithms on a humanoid robot. By 'locomotion path' we denote the motion of the robot as a whole in the plane. The key to our approach is to formulate the path planning problem as an optimal control problem. We propose a single dynamic model valid for all situations, which includes both non-holonomic and holonomic modes of locomotion, as well as an appropriately designed unified objective function. The choice between holonomic and non-holonomic behavior is not accomplished by a switching model, but it appears in a smooth way, along with the optimal path, as a result of the optimization by efficient numerical techniques. The proposed model and objective function are successfully tested in six different locomotion scenarios. The resulting paths are implemented on the HRP-2 robot in the simulation environment OpenHRP as well as in the experiment on the real robot.

Keywords

Biologically inspired path planning, optimal control, locomotion, humanoid, holonomic and non-holonomic walking

1. Introduction

1.1. Problem Statement: Generation of Natural Locomotion Paths

This paper addresses the question how to best generate the locomotion path of a humanoid robot between two points in a natural and biologically inspired manner. By

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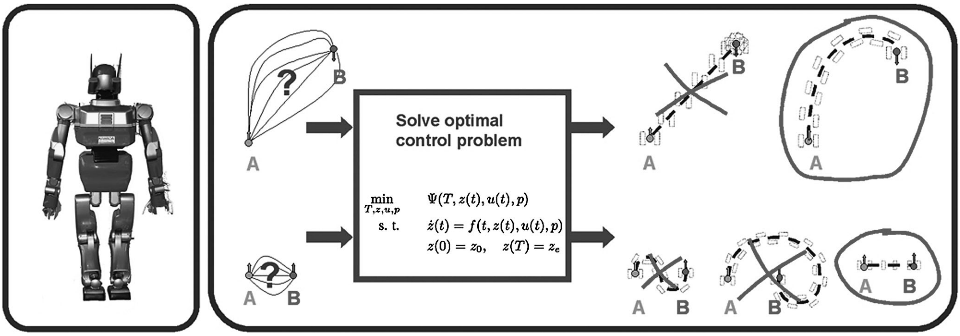

Figure 1. How to choose the most natural path for a humanoid robot? The figure shows desired and undesired behaviors for two example scenarios. The natural choice of the path in this paper is achieved by solving an optimal control problem.

'locomotion path' we denote the motion of the robot as a whole in the plane, i.e., the development of its overall position and orientation, and not the relative joint motions at the robot's internal degrees of freedom.

If a human being is asked to move from a given start point to a given target in an empty room, they will usually choose, out of all possible paths, a very specific one. Obviously this natural path depends on the properties of the individual human being, but there also seem to be some general 'path generation criteria' that always remain valid. Figure 1 gives two example locomotion scenarios and the expected natural as well as undesired behaviors. The aim of this paper is to understand the underlying principles of human path generation and to use them to control the path generation of the humanoid robot HRP-2.

1.2. Holonomic and Non-holonomic Locomotion

It has recently been shown by Arechavaleta *et al.* [1, 2] that human locomotion in many cases is non-holonomic. Non-holonomy is a well-known concept in mobile robotics (see, e.g., Ref. [3]). A system is non-holonomic if not all its degrees of freedom are independently accessible, but instead obey a non-integrable differential coupling, and the direction of motion is identical with the body orientation. One can imagine that the anatomy of the human feet (pointing forward) and the location of strong muscles give preference to walking in the forward direction, i.e., to a purely non-holonomic form of motion. However, in certain situations it seems perfectly natural to include holonomic pieces in human locomotion, e.g., when sideward or oblique steps are performed, not only to avoid obstacles but also to quickly reach close-by targets that do not lie in the current direction of body orientation. Then the translational velocity also has an orthogonal component.

One goal of our research is to provide an efficient way to automatically select, for any given situation, between holonomic and non-holonomic behavior. This is crucial in order to enable an autonomous robot to perform natural paths.

1.3. State of the Art of an Interdisciplinary Research Topic

The question about the natural shape and time history of free locomotion paths of humans between two points is of crucial importance for researchers from many different fields — not only from robotics, but also from neuroscience, biomechanics and computer animation.

In robotics, for the general problem of motion planning, many algorithms have been proposed (see, e.g., Refs [3–5]) and applied to mobile robots. However, the focus here was more on the search for a feasible path and the handling of complex geometries with obstacles, etc., than on the determination of a biologically inspired natural path. In the context of humanoid robots, real-time path planning and adaption based on sensor information of the robot has been investigated by several researchers (e.g., Refs [6–10]). Offline path generation based on biological selection criteria has not yet been performed for humanoid robots. However, such natural paths might well serve as a basis for whole-body motion planning, such as in Ref. [11].

In biomechanics, the goal is to fully understand the mechanics and control mechanisms of normal and pathological gait, e.g., of joint kinematics, required forces and torques in the locomotor system, processes related to force generation in the muscles, and control signals to produce the motion. Most of the investigations focused on straight-ahead walking in standardized conditions such as treadmill walking (e.g., Refs [12–14]), but this setting is far away from everyday walking conditions. It was pointed out [15] recently that the biomechanics of changing direction while walking have been largely neglected despite the obvious relevance to functional mobility.

From a neuroscience perspective, it is interesting to determine which perceptions are used to steer the path generation process and how this task is achieved using a generally highly redundant control space (see, e.g., Refs [16–18]). For human walking along a curved path, segment orientation as well as the influence of vision have been investigated in Refs [19, 20].

One major objective of the computer animation domain is to virtually reproduce human appearance and behavior in a believable manner, with particular interest in navigation and locomotion tasks. Motions on the joint level are mostly produced based on motion capture techniques. Recorded data can be adapted to a desired context, e.g., by smoothly deforming the motion [21], mixing several data sources [22] or snapping together short motion sequences to compose new motions [23]. Yamane and Nakamura [24] present an inverse kinematics-based method that can create natural motions from scratch, but can also be used to edit or re-target captured motion. In computer graphics, autonomy for virtual actors also on the level of locomotion path selection is achieved by mixing motion synthesis techniques with path planning [25, 26] and reactive navigation techniques [27]. Multon *et al.* [28] propose a framework that allows reactive virtual reality animations of dozens of characters in real-time. In all these works, the task of planning and reactivity to static and dynamic obstacles focuses on the generation of a collision-free trajectory. Realism and natural smoothness of the followed path and speed profile mainly depend on the artistic skills of the animators.

Several recent research projects on locomotion benefit from a synergetic multidisciplinary view (e.g., Refs [19, 29–31]), making the frontiers between disciplines more and more fuzzy. The research presented in this paper contributes to this tendency.

1.4. Contribution of this Paper: Optimal Control-Based Formulation for Natural Path Planning

The contribution of this paper is the novel way of formulating the natural locomotion path planning problem as an optimal control problem. A natural path performed by a human can be interpreted as the solution of an optimization problem with some *a priori* unknown objective function. The question we are addressing in this paper is the following: is there a general description of the locomotion dynamics and a unified objective function that is valid for all walking scenarios and paths — no matter if short or long, fast or slow, curved or straight, forwards, backwards or sidewards? Or is it necessary to choose different dynamic models (e.g., for holonomic and non-holonomic motions) and different objective functions that are each only valid for a certain class of motions? The contribution of this paper is to show how the model of walking can be formulated in a unified way by considering a single control system. The transfer between holonomic and non-holonomic parts of locomotion is not made based on a switching model, but it appears in a continuous form. We essentially complete an acceleration level form of the classical non-holonomic model with two inputs (namely the forward and rotational acceleration) by a third input modeling the sidewards motion (the orthogonal acceleration). The key then lies in the formulation as an optimal control problem with an appropriately designed cost function that lets the choice between holonomic and non-holonomic behaviors appear as a result of the optimization. We evaluate the proposed objective function by demonstrating that it performs well, generating optimal walking in six very different test cases (see Fig. 2), i.e., for six combinations of start and end point and orientation that are expected to produce different types of locomotion. The optimization runs are performed applying characteristic velocity and acceleration bounds of the HRP-2 robot [32], and the resulting motions are visualized using the tool OpenHRP [33] and tested on the real HRP-2 robot. The application of the approach presented in this paper is, however, not limited to humanoid robots, but it can also be used to produce natural motions in computer graphics. In addition, the developed model gives useful insights for researchers in biomechanics.

For the solution of the optimal control problem we use efficient numerical optimal control techniques based on the direct multiple shooting approach [34, 35]. These same techniques have since long been used and extended by the first author to solve other classes of optimal control problems in robotics (optimal walking and running based on complex models on the multibody system level, e.g., Refs [36–38]) and have proven to be very powerful.

This paper is organized as follows. In Section 2, we introduce the optimal control problem formulation, proposing a continuous holonomic–non-holonomic model and a unified objective function. In Section 3, we briefly present the numerical techniques that are used for the optimal control problem solution. Section 4 describes the six walking scenarios and presents optimization results for all cases. In Section 5, we show snapshots visualizing the resulting optimal paths on OpenHRP.

2. General Optimization Model of Locomotion

A natural walking path produced by a human can be seen as the solution of an optimization problem minimizing some cost function while at the same time satisfying dynamic constraints. These constraints in the form of differential equations give a physically correct representation of the evolution in time of the locomotor path. This optimization problem with dynamic constraints, also called an (offline) optimal control problem, can be stated in the following form:

$$\min_{T,z,u,p} \Psi(T, z(t), u(t), p) \tag{1}$$

$$\text{s.t. } \dot{z}(t) = f(t, z(t), u(t), p) \tag{2}$$

$$z(0) = z_0, \qquad z(T) = z_\mathrm{e}, \tag{3}$$

with state variables $z(t)$, control or input variables $u(t)$, model parameters p and duration T.

In the remainder of this section we will propose a general model of the locomotion dynamics as well as a parameterized objective function, the combination of which will produce a human-like behavior. With such a unique problem formulation it would be possible to solve the corresponding optimal control problem on a real robot given the task to go from A to B, to let it autonomously decide on the path. We focus here on unperturbed locomotion on flat ground without any obstacles.

2.1. Dynamic Model Equations of Holonomic and Non-holonomic Motion

This section deals with the proper choice of (2) in the optimal control problem formulation. We would like to emphasize that in this context we are not interested in the multi-body dynamics of the locomotor system with all its internal joint degrees of freedom, but only on the movement of the whole system in this plane, described by its two-dimensional position coordinates x, y and its orientation ϕ.

However, as outlined above, locomotion in many cases is non-holonomic i.e., its tangential velocity is coupled to the body orientation that can be formulated by the following very simple model:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} v_\mathrm{f} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega, \tag{4}$$

with linear velocity in the forward direction of the body v_f (here equivalent to the tangential velocity) and rotational velocity ω. Pure rotations on the spot are included in this non-holonomic model with $v_\mathrm{f} = 0$.

If in certain situations the motion becomes holonomic due to the appearance of orthogonal velocity components, v_{orth}, the model becomes:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} v_{\text{f}} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \omega + \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} v_{\text{orth}}, \tag{5}$$

v_{f} remains the velocity component linked to the forward orientation of the body, but it is not tangential any more. This model is of course equivalent to the very general 3-d.o.f. model of a rigid body in a plane, but the parameterization along forward and orthogonal directions is crucial in the following. For our investigations in this paper, we use an extension of this model, going down to the acceleration level for all three velocity components v_{f}, v_{orth} and ω:

$$\begin{aligned} \dot{x} &= \cos\phi v_{\text{f}} - \sin\phi v_{\text{orth}} \\ \dot{y} &= \sin\phi v_{\text{f}} + \cos\phi v_{\text{orth}} \\ \dot{\phi} &= \omega \\ \dot{v}_{\text{f}} &= u_1 \\ \dot{\omega} &= u_2 \\ \dot{v}_{\text{orth}} &= u_3, \end{aligned} \tag{6}$$

which will replace (2) with $z^{\text{T}} = (x, y, \phi, v_{\text{f}}, \omega, v_{\text{orth}})$. The variables u_1, u_2 and u_3 denote the respective accelerations in the forward, angular and orthogonal direction, and are used as control (i.e., input) variables in the optimal control problem. Obviously the non-holonomic case is still included in model (6) for $u_3 \equiv 0$ and $v_{\text{orth}}(0) = 0$ ($\rightarrow v_{\text{orth}} \equiv 0$). The key property of this model formulation is that it allows transitions from holonomic to non-holonomic behavior and *vice versa* to happen in a smooth and continuous way, not requiring any discrete switching events.

2.2. Choice of Objective Function

In this section we present the adequate choice of an objective function Ψ. As discussed above, the goal here is to find a unique objective function that performs well in all possible walking scenarios and not to adapt the function to any new situation. The function proposed in this section has been established based on qualitative empirical observation and it is shown to work qualitatively well in the results section.

A function describing the optimality criterion of human locomotion may be expected to get very complex if all details are considered, but the purpose here is to evaluate if the behavior can be approximated by a combination of just a few simple base functions.

We base our choice of the objective function on a few general guidelines — deduced from observations of human locomotion — that seem to be adequate for the description of natural human walking paths:

- The time of the path is not *a priori* fixed.

- Quicker paths should be preferred over slower ones.
- In most cases, smooth paths are desired.
- Large variation of velocities should be avoided.
- In general, the motion should be (close to) non-holonomic, except for near targets and similar initial and final orientations.

These ideas lead us to the following parameterized formulation of the objective function:

$$\begin{aligned}
&\Psi(T, x(t), u(t), p)\\
&\quad = \alpha_0 \cdot T + \alpha_1 \int_0^T u_1^2 \,\mathrm{d}t + \alpha_2 \int_0^T u_2^2 \,\mathrm{d}t + \tilde{\alpha}_3(\Delta\phi, d) \int_0^T u_3^2 \,\mathrm{d}t\\
&\quad = \int_0^T \left[\alpha_0 + \alpha_1 u_1^2 + \alpha_2 u_2^2 + \tilde{\alpha}_3(\Delta\phi, d) u_3^2\right] \mathrm{d}t,
\end{aligned} \tag{7}$$

i.e., a combined weighted minimization of total time and the integrated squares of the three acceleration components. The last term in the objective function serves to penalize the orthogonal motion, i.e., the related control $u_3(t)$ (the existence of which turns the non-holonomic motion into a holonomic motion and we, therefore, call it the 'holonomic term'). α_0, α_1 and α_2 are constants, but the factor $\tilde{\alpha}_3$ of the holonomic term is a function of the distance between the start point and end point d and of the change in orientation $\Delta\phi$:

$$\begin{aligned}
&\tilde{\alpha}_3(\Delta\phi, d) = \alpha_3 \left(1 + \frac{\Delta\phi}{\xi_1}\right)\left(1 + \frac{d^2}{\xi_2}\right)\\
&\quad \text{with } \alpha_3 = \text{cst.}, \quad \Delta\phi = \phi_\mathrm{e} - \phi_\mathrm{s}, \quad d = \sqrt{(x_\mathrm{e} - x_\mathrm{s})^2 + (y_\mathrm{e} - y_\mathrm{s})^2}.
\end{aligned} \tag{8}$$

This results in large penalization factors $\tilde{\alpha}_3$ for the holonomic part, i.e., a stronger emphasis on minimizing u_3, in the case of larger distances or larger relative orientation changes and to small factors $\tilde{\alpha}_3$ for close goals with little orientation change, such that holonomy here is more likely to occur. The constants ξ_1 and ξ_2 in (8) are scaling factors that allow us to adjust the model, e.g., to unities used, characteristic size of the locomotor system, etc. (here we have used the numbers $\xi_1 = 10$ for ϕ in degrees, i.e., $\pi/18$ for ϕ in rad and $\xi_2 = 0.5$ for d in m, which have been heuristically determined).

Overall, we now have an objective function with four parameters $\alpha_0, \alpha_1, \alpha_2$ and α_3. In the following, we will show that using the same set of constants α_i, objective function (7)/(8) in combination with model (6) leads to a qualitatively natural behavior for very different locomotion tasks, i.e., different sets of boundary conditions (3). In this paper, we focus on motions with zero initial and final velocity. In the more general case, in particular with $v_\mathrm{orth}(0) \neq 0$, the acceleration-based term $\tilde{\alpha}_3 u_3^2$ would not be sufficient and a term based on v_orth^2 would have to be added.

3. Numerical Optimization Methods

For the solution of the optimal control problems described in the previous section, efficient numerical techniques are required. An optimal control problem is fundamentally different from a 'standard' constrained nonlinear optimization problem due to the fact that its free variables $z(t)$ and $u(t)$ are not vectors in $\mathbb{R}^n$ but vector functions in time, and, therefore, the solution does not consist in a point in n-dimensional (i.e., finite-dimensional) space, but in trajectories $z^*(t)$ and $u^*(t)$ which can be seen as 'infinite-dimensional' optimization variables.

We solve this problem using the optimization techniques implemented in the optimal control code MUSCOD developed at IWR in Heidelberg [34, 35].

The MUSCOD core algorithm is based on:

- A direct method for the solution of the optimal control problem (also termed a 'first discretize then optimize method'): instead of using arbitrary (i.e., infinite dimensional) control functions $u(t)$, we restrict the controls to a discretized space described by a finite set of parameters. For numerical efficiency, functions with local support are used, in this case piecewise constant functions on a grid with m intervals.

- A multiple shooting state parameterization: the basic idea of this technique is to split the long integration interval $[0, T]$ into many smaller ones and to introduce the values of the state variables x at all those grid points as new variables. The original boundary value problem is, thus, transformed into a set of initial value problems with corresponding continuity conditions between the integration intervals. For numerical reasons the multiple shooting grid is chosen to be identical to the control grid described above. The multiple shooting approach is very favorable for a number of reasons. The rough knowledge that one usually has about the trajectory can be exploited in the generation of starting data for the multiple shooting points (i.e., one just defines where to start the search, but one does not fix anything). Since the integration intervals are much shorter than in the original problem, the chances for finding a solution of the initial value problem obtaining sufficiently accurate derivatives increase significantly, even if all values are still far from the final solution.

Using these discretization techniques, the original infinite-dimensional optimal control problem is transformed into a finite-dimensional nonlinear programming problem which is of large dimension but exhibits clear structural matrices and can, therefore, be solved efficiently by a tailored SQP algorithm exploiting the structure of the problem (for details, see Ref. [35]).

It is important to note that the treatment of the dynamical model equations is not part of the discretized optimal control problem; this task must, however, be handled in parallel in order to provide the required information for the evaluation of objective functions, continuity constraints and the derivatives thereof. For this purpose, fast and reliable integrators are used that also include a computation of sensitivities based on the techniques of internal numerical differentiation [39].

For the problems treated in this paper we have chosen a discretization grid of 50 control and multiple shooting intervals. Note that this is not the number of points with which the dynamical model is discretized. Due to the use of integrators on each of the 50 multiple shooting intervals with respective subgrids, the number of discretization points for the dynamics is much higher.

The grid of 50 control/multiple shooting intervals was actually finer than necessary, but we did not care about this fact since the computations here were not time critical and the required computation times of 0.5–0.9 s were low enough. If lower computation times were desired it would easily be possible to reduce the number of intervals to 20 for the problems treated here, to obtain solutions in approximately 0.2–0.3 s.

4. Generating Optimal Locomotion Paths

In order to evaluate the optimal control model presented in Section 2, we have chosen six test cases that are expected to result in fundamentally different types of locomotion. These test cases will be described in Section 4.1. For each of these test cases we have performed a variety of different optimization trials each leading to different locomotion paths and speeds. In order to demonstrate how much the choice of objective function parameters influences the resulting path, we give a comparison of two example cases in Section 4.2. Looking at all these optimization results, we were in fact able to identify a set of parameters that leads to qualitatively good behavior of our model in all cases investigated. The optimization results are presented in Section 4.3. The computations have been performed for the model of the HRP-2 robot, using its velocity constraints $0 \leqslant v_{\mathrm{f}} \leqslant 0.4$, $|v_{\mathrm{orth}}| \leqslant 0.4$ and $|\omega| \leqslant 0.5$.

4.1. Six Different Locomotion Test Cases for the Proposed Optimal Control Model

The six selected test cases are shown in Fig. 2 and the expected natural behavior in each case will be described in this section. In all cases, translational as well as rotational velocity is imposed to be zero at the origin and goal positions of the path.

In the first test case, the goal is at 3 m to the front and 3 m to the right from the origin, and the orientation changes by $-\pi$. In this case, one expects a smooth non-holonomic movement performing forward motion and rotation simultaneously (i.e., no rotations on the spot). In the second case, the goal is positioned much closer to the point of departure, at 1 m to the right. The change of orientation is $-\pi/2$. In this case, we would expect the human to perform at first a (more or less) pure rotation and then walk forward (in a roughly non-holonomic way) until the goal is reached. For both the third and the fourth test case, there is no relative change in orientation between the origin and goal point. In the third test case the goal is very close, at 1 m to the right, and we expect a sequence of sidewards steps with no rotation and no forward motion, i.e., a clearly holonomic form of movement. In contrast, for test

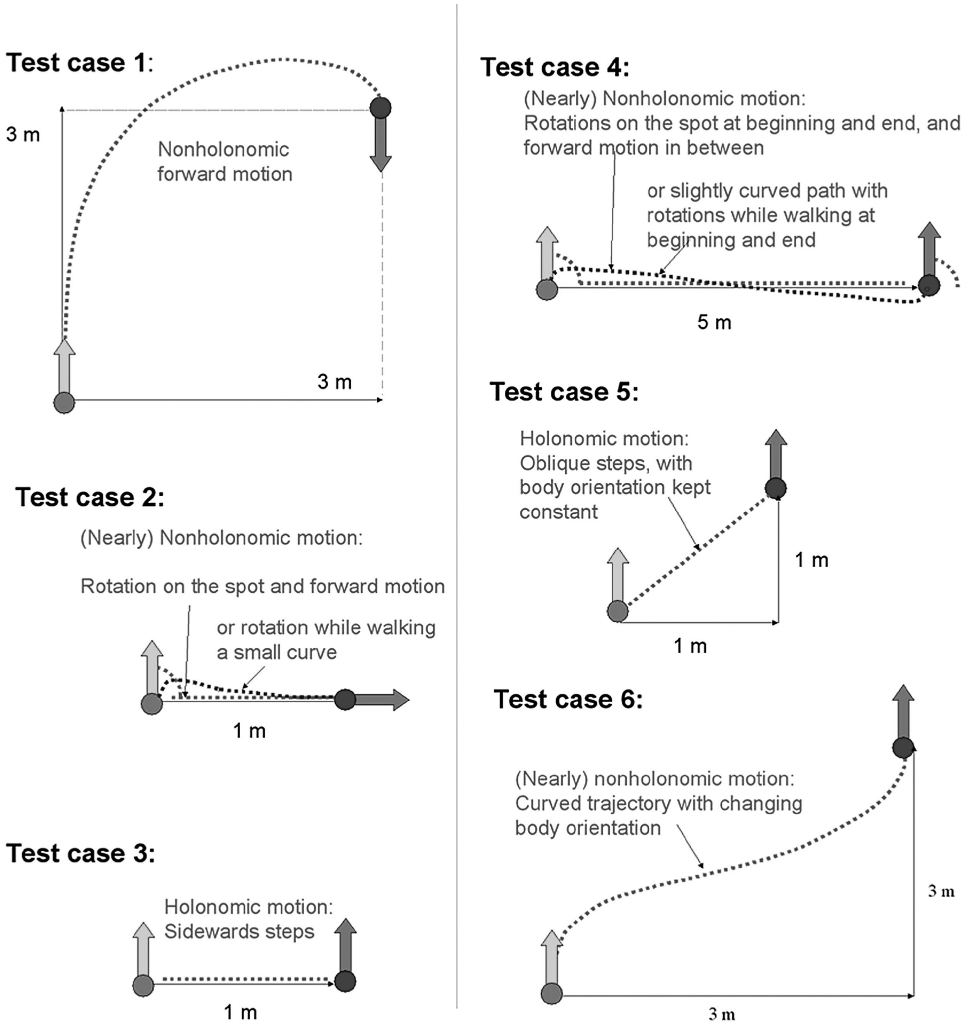

Figure 2. Six test cases for the unified optimal control model of locomotion: light circles and arrows show initial positions and orientations; dark circles and arrows indicate target positions and orientations. Total velocity in both points is zero. The expected 'natural' path is shown as a dotted blue line.

case 4 the goal is at a greater distance, at 5 m to the right, which should naturally lead to most of the motion being performed in forward (non-holonomic) mode, and more or less non-holonomic rotations or small curved paths at the beginning and end. For test cases 5 and 6, the goals are in diagonal positions, and again, the orientation is the same for start and end points. However, while the goal is close in test case 5 (1 m to the right and 1 m to the front) and we expect a couple of oblique steps with body orientation kept more or less constant, the goal is further away in test case 6 (3 m in both directions) and the motion is expected to be a smoothly shaped, (close to) non-holonomic forward motion.

4.2. Influence of the Objective Function Parameters on the Path

Before coming to the actual results we would briefly like to demonstrate, for the example of test cases 1 and 3, how much a change in the objective function parameters modifies the shape of the locomotor path. For each of the two cases, eight paths resulting from the optimization of different objective functions (produced by different sets of parameters) are compared (see Fig. 3). Solid lines denote single base function criteria (where only one of the α_i is non-zero). As Fig. 3 shows, different parameters lead to very different behaviors. We would like to stress the fact that it is important to look not only at the shape of the path, but also at its evolution in time. Some combinations of objective function parameters (notably those with $\alpha_0 = 0$) tend to give unacceptably slow motions, but the shape of the path may look good at first sight.

As this variation of parameters and the resulting very different behaviors show, it was not *a priori* clear if it might be possible to find a unique combination of parameters performing well in all cases. However, in the next section we will show that we were able to identify such a unique parameter set.

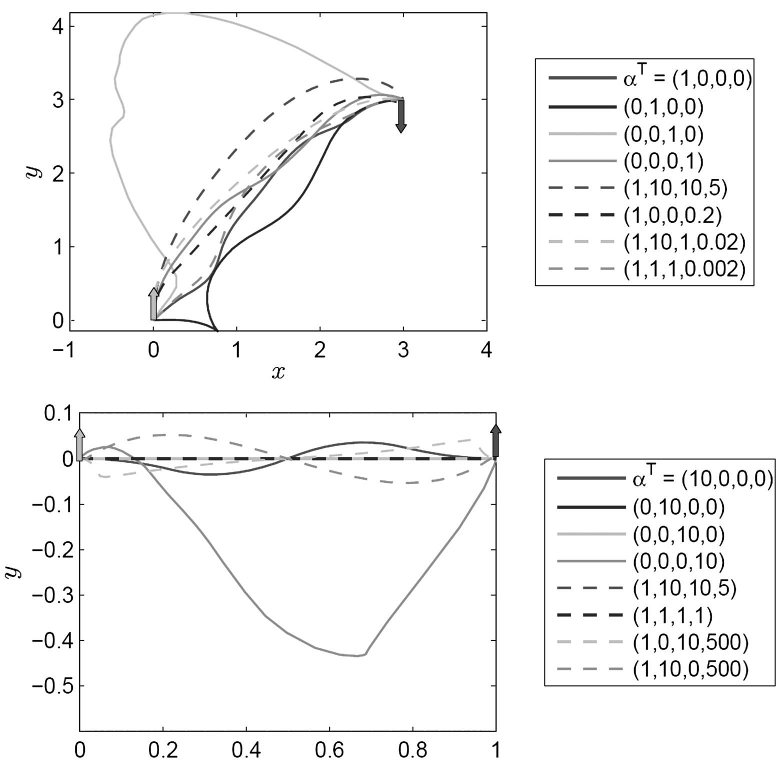

Figure 3. Influence of changes of the objective function parameters $\alpha^{\mathrm{T}} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ for test cases 1 (top) and 3 (bottom). In both cases, solid lines show single base function criteria (only one $\alpha_i \neq 0$) and dashed lines show different combinations of base functions.

4.3. Optimization Results Producing Natural Locomotion

We now present the optimization results for all six test cases. In all six cases, the choice of parameters $\alpha^T = (1, 10, 10, 5)$ of the objective function (7)/(8) lead to a qualitatively good behavior, i.e., they are well capable of reproducing the expected natural motions described above and shown in Fig. 2 (we use the expression 'qualitatively good behavior' in order to distinguish this observation from quantitative measures of natural motions, e.g., the distance of a computed and a measured trajectory in space or time which have not been applied here). In Fig. 4, we present, for every test case, the resulting optimal path in the x, y-plane and the trajectories of all state variables in time, i.e., positions $x(t)$ and $y(t)$, angle $\phi(t)$, and velocities v_f, v_{orth} and $\omega(t)$.

In the context of this paper, we intentionally do not explicitely classify the resulting paths into strictly holonomic and strictly non-holonomic pieces, since it is not clear how to best determine the distinction criterion in the context of natural motion. Is a natural motion only non-holonomic if v_{orth} is exactly zero, or should it be smaller than 10^{-2} or 10^{-1}? Or should a criterion based on the angle between path and velocity be applied, and if yes, which angle corresponds the natural bound? Instead of strictly classifying, we, therefore, refer to the respective velocity plots in forward and orthogonal directions, and by a relative comparison we can check if a path qualifies as clearly non-holonomic, nearly non-holonomic, clearly holonomic, etc.

- *Test Case 1:* $(0, 0, \pi/2) \to (3, 3, -\pi/2)$. In this case, (8) with $d = 3\sqrt{2}$ and $\Delta\phi = \pi$ leads to $\tilde{\alpha}_3 = 703 \cdot \alpha_3$. The first row of Fig. 4 shows the optimal motion for test case 1 corresponding to the determined set of objective function parameters α. It produces the expected natural behavior of a smooth non-holonomic path with all turning being preformed during the forward motion. The orthogonal velocity component is always zero.

- *Test Case 2:* $(0, 0, \pi/2) \to (1, 0, 0)$. Here, we obtain $\tilde{\alpha}_3 = 30 \cdot \alpha_3$. The optimized motion in Fig. 4 corresponds to the desired behavior, performing at first only a rotation without hardly any translational motion and then moving forward to the end point. The motion is not strictly non-holonomic, but as the velocity plot shows, the orthogonal velocity component is very small.

- *Test Case 3:* $(0, 0, \pi/2) \to (1, 0, \pi/2)$. For the factor of the holonomic part, we obtain here a small number $\tilde{\alpha}_3 = 3 \cdot \alpha_3$ from (8). As expected, we find in this case a pure sideward, i.e., orthogonal, motion with forward as well as rotational velocity equal to zero. The motion is clearly holonomic.

- *Test Case 4:* $(0, 0, \pi/2) \to (5, 0, \pi/2)$. For these end values, the factor of the holonomic part is $\tilde{\alpha}_3 = 51 \cdot \alpha_3$, i.e., the weight of the holonomic part in the objective function is increased. As we expected, we find for this stretched version of test case 3 a different behavior: the motion is (nearly) non-holonomic, with

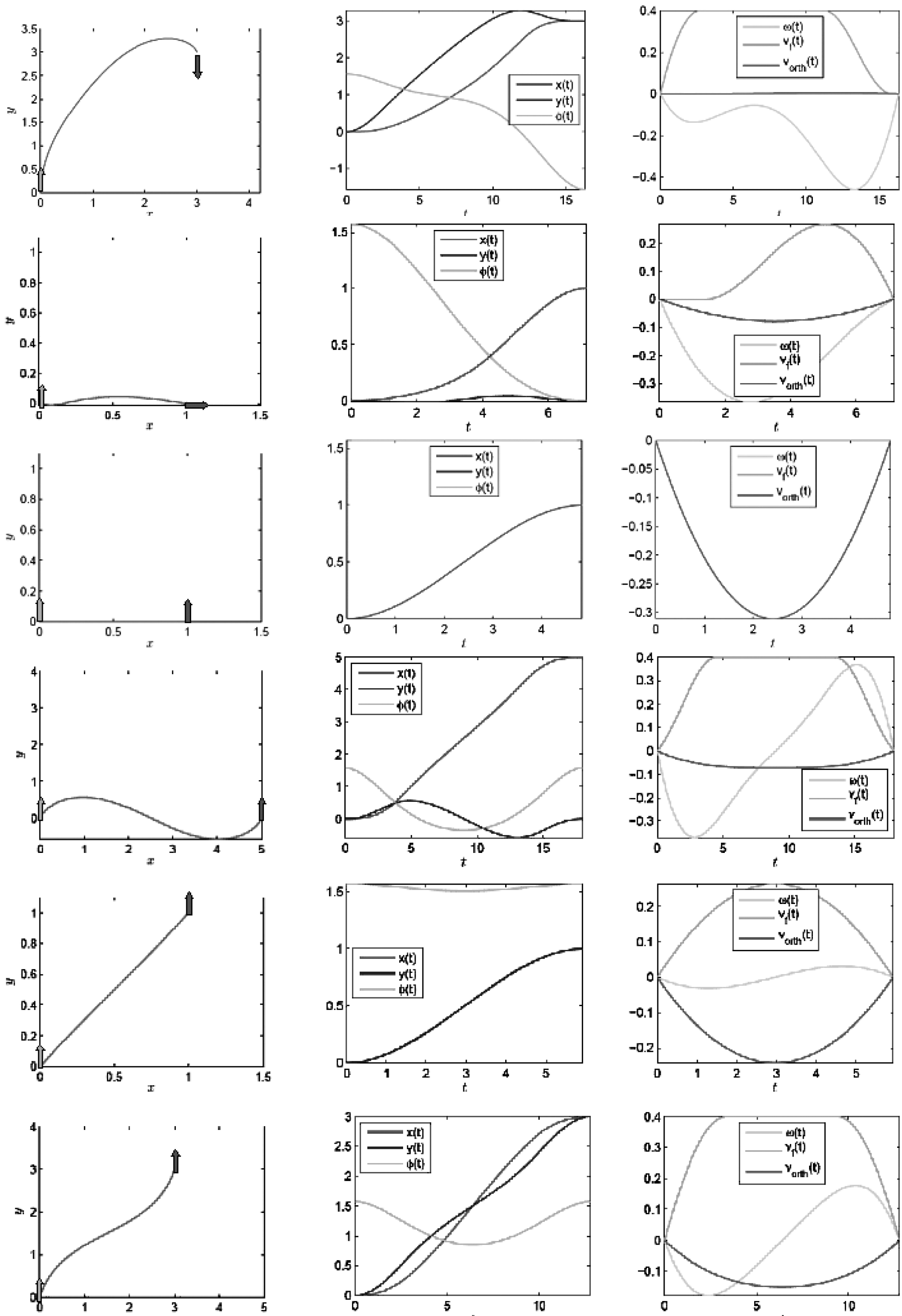

Figure 4. Optimization results in natural behavior for all six test cases, using identical objective function parameters $\alpha^{\mathrm{T}} = (1, 10, 10, 5)$: plots show path in the x, y-plane (left column), and corresponding time histories of positions (middle) and velocities (right).

curved parts of the path at the beginning and end to preform rotations, and a forward motion in between to gain distance.

- *Test Case 5:* $(0, 0, \pi/2) \rightarrow (1, 1, \pi/2)$. Here we have a factor of $\tilde{\alpha}_3 = 5 \cdot \alpha_3$ of the holonomic part. As Fig. 4 shows, the close goal in the diagonal position results in oblique steps with nearly identical forward and orthogonal velocity component and a nearly constant body orientation. The motion is, thus, clearly holonomic with an angle of approximately $\pi/4$ between the tangent and body orientation.
- *Test Case 6:* $(0, 0, \pi/2) \rightarrow (3, 3, \pi/2)$. For the diagonal goal located further away, the factor of the orthogonal acceleration gets larger $\tilde{\alpha}_3 = 37 \cdot \alpha_3$, resulting in a stronger punishment of holonomic motion components. The orthogonal velocity is considerably reduced compared to case 5, and the motion includes mainly forward motions and turns. It is nearly non-holonomic.

We can summarize that in all six test cases, the expected natural behavior could be produced by optimization using the optimal control formulation of Section 2 with identical objective function parameters.

The computation time for these optimization runs was typically far below 1 s (see above, without any code optimization). These same optimization runs could be executed on the robot HRP-2 in order to allow it to autonomously generate, for any given target, a natural path before starting to move. For this, the optimization procedures would have to be implemented on the robot, which is easily feasible, since the systems are compatible and computation times are low enough.

5. Implementation of Optimal Paths on the Humanoid Robot HRP-2

Being able to generate natural paths is an important issue for humanoid robots. The purpose of this section is 2-fold. First, and foremost, we want to 'visualize' the six trajectories computed in this paper by implementing them on the humanoid robot HRP-2 [32] in simulation and experiments. We feel that even though we cannot present movies but only sequences of snapshots of this implementation here, this is will give a more intuitive impression of the resulting trajectories than any of the 2-D plots shown so far.

The second purpose is to briefly outline a straightforward way to transform computed overall locomotion trajectories into motions of HRP-2 based on the humanoid simulator and control software OpenHRP [33]. Figure 5 shows the software architecture required to generate and implement optimal paths on HRP-2.

We use a walking pattern generator based on preview control of the zero moment point (ZMP) and an inverted pendulum proposed by Kajita *et al.* [40]. The computed path is given to the pattern generator as the linear and angular velocity of the pelvis. From those velocities, each next foot step is first determined by taking into account such parameters as step length and time ratio of the double/single support phase that accounts for the capacity of the robot. Then the center of mass

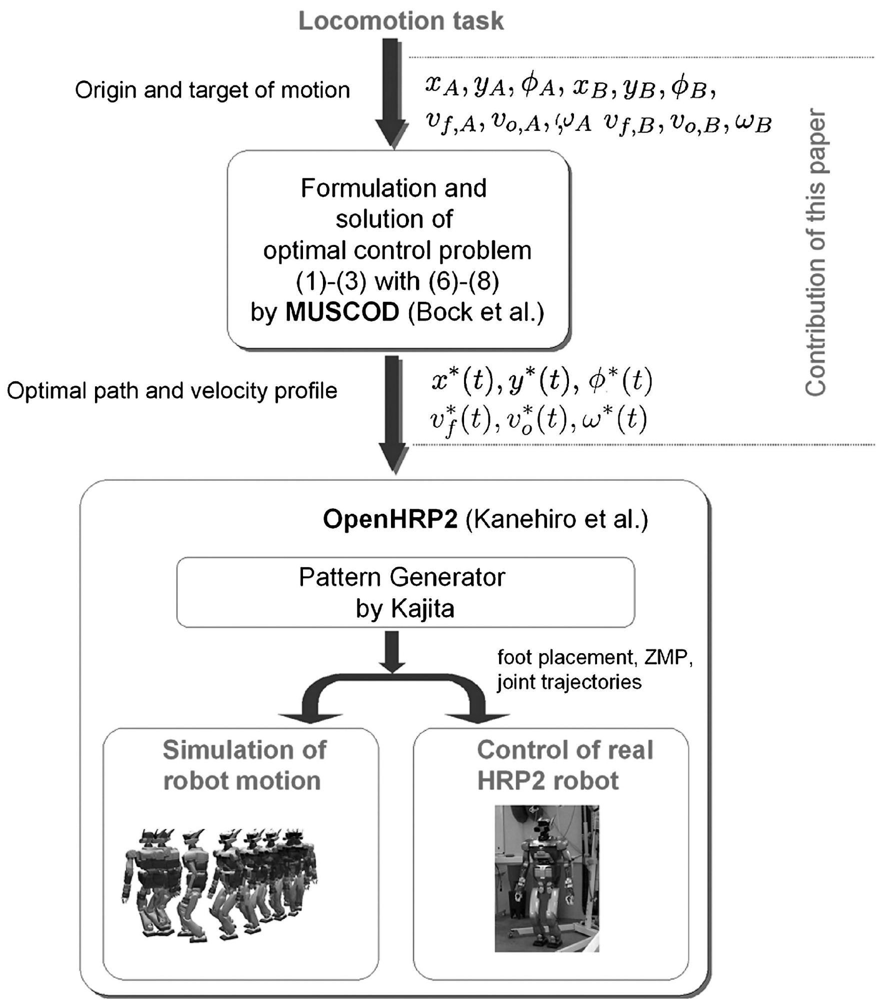

Figure 5. Software architecture for the generation and implementation of optimal paths on the humanoid robot HRP-2.

(CoM) trajectory is derived by applying the preview control scheme to an inverted pendulum model in such a way that the desired discrete footprints are tracked as a continuous ZMP trajectory. Finally, through inverse kinematics, the leg joint angles are computed from the CoM trajectory and the footprints. The resulting biped walking motion is dynamically stable in the sense that the ZMP is always inside the support polygon, whereas the CoM can go outside of it.

We are aware that there is room for improvement in this pattern generation process with respect to producing natural movements. While we have focused on natural locomotion paths in this paper, it will be equally interesting in a next step to generate natural footholds as well as natural whole-body motions on the joint level for a humanoid robot taking into account stability constraints; however, this issue will not be treated in this paper. We assume that the problems of (i) path generation and (ii) whole-body motion generation along this path are hierarchical, and can, therefore, be treated sequentially.

Figures 6 and 7 show animation snapshots of the resulting robot behavior in all six test cases.

OpenHRP also allows us to easily implement the computed paths on the real robot, using the same input data and pattern generation approach as described above. Figure 8 shows HRP-2 performing test cases 2 and 6.

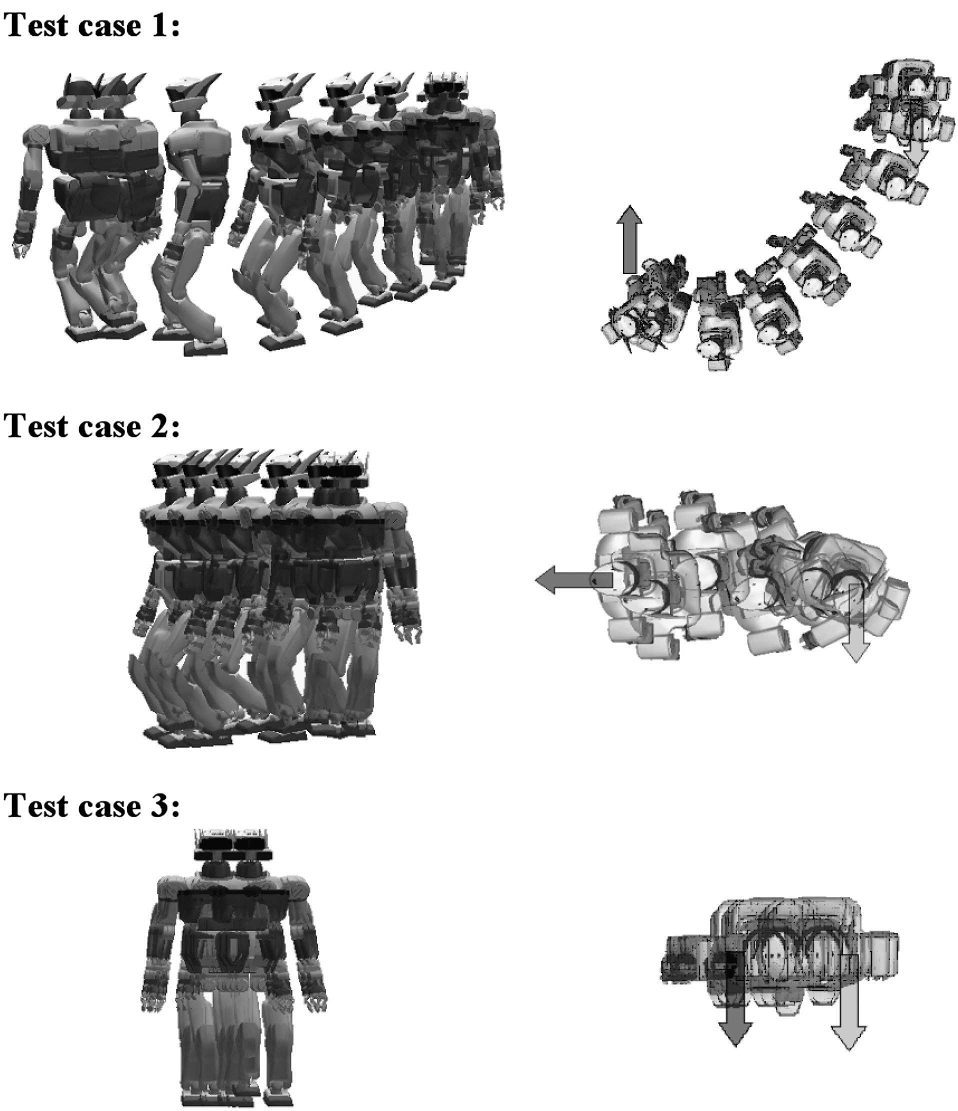

Figure 6. Animations of HRP-2 executing optimized paths (test cases 1–3).

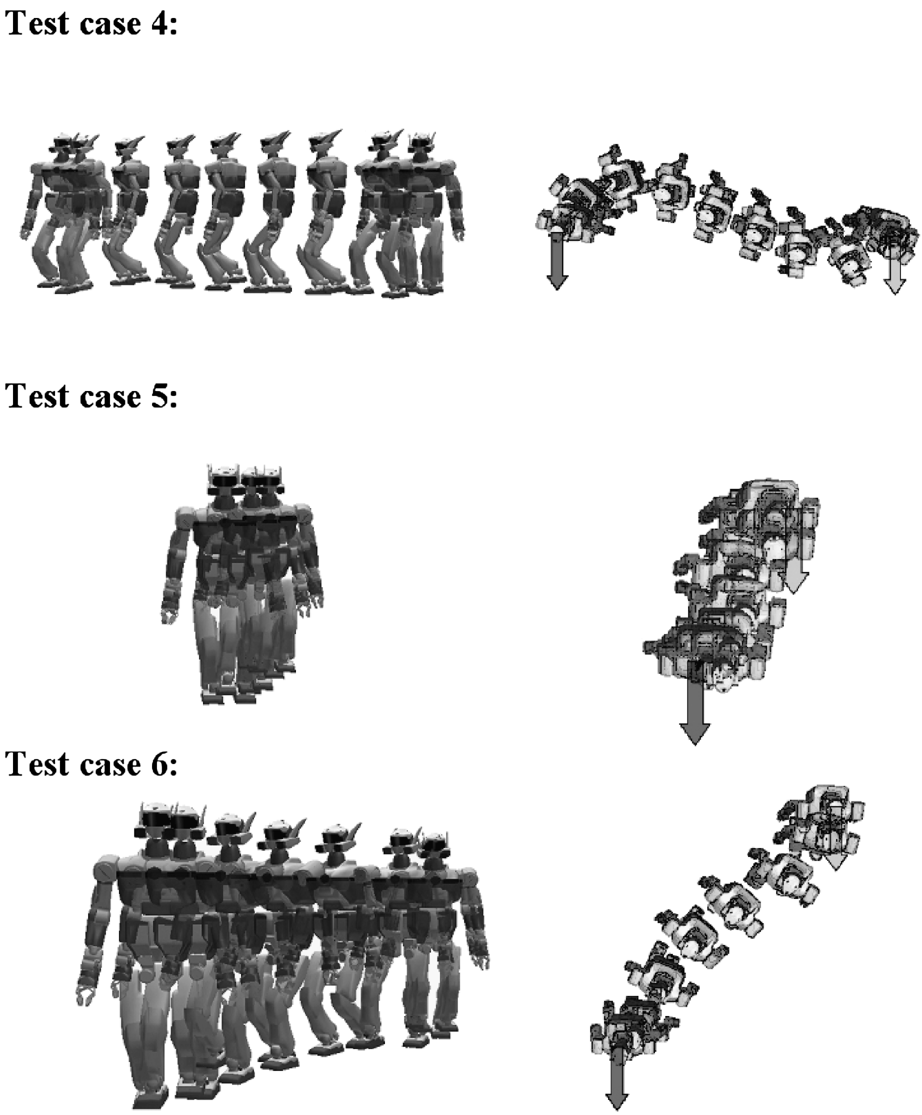

Figure 7. Animations of HRP-2 executing optimized paths (test cases 4–6).

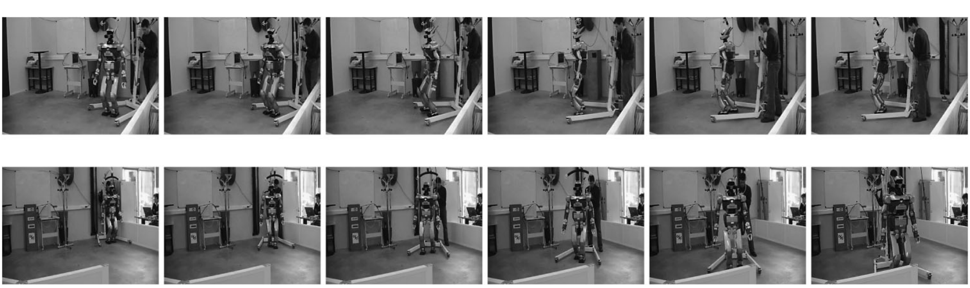

Figure 8. Experimental validation of optimal paths on the HRP-2 robot (test cases 2 and 6).

6. Conclusions

In this paper we have presented a computational model of natural human locomotion as a way to simulate and plan human-like actions for humanoid robots. The key properties of the presented approach are:

- Unification of holonomic and non-holonomic parts of the motion in a single dynamic model with a smooth transfer between the two modes, which has proven to be an adequate description for many locomotion scenarios.
- Formulation of the path planning problem as an optimal control problem with a well-designed objective function that at the same time produces an optimal path and decides about holonomy or non-holonomy of the model.

This research is extended in a couple of directions:

- The objective function proposed in this paper was the result of qualitative reasoning and not of a mathematical identification of human objectives. This is, however, the goal of our current research, where we have applied techniques of inverse optimization to determine objective functions of human path planning from experiments [41].
- In this context it is also interesting to develop a more detailed objective function model, i.e., take more base functions into account. The aim of the optimal control model formulation proposed in this paper was to be as simple as possible and it showed a good qualitative behavior, but for some questions, more detail might be desired. Based on measurements we are also evaluating other base functions related to jerk, velocity or goal orientation in the objective function.
- In a next step, obstacles would have to be taken into account in the problem formulation, since our investigations so far have assumed that the space was empty. One might expect that a combination of the presented objective function with a penalty term to stay (safely) away from constraints might lead to satisfying solutions.
- A logical extension of the *a priori* planning of a full path performed here is real-time path planning in the presence of moving obstacles according to biological criteria. This is a completely new problem setting, and it remains to be investigated according to which criteria the path is reshaped if obstacles move or new obstacles appear and for which horizon re-planning should be performed.

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